4-1 Classifying Triangles (pp. 216–221)

**Remember**

1. An **equiangular** triangle has three congruent angles. - ![Equiangular Triangle](image)
2. An **equilateral** triangle has three congruent sides. - ![Equilateral Triangle](image)

An **acute** triangle has three acute angles. - ![Acute Triangle](image)

A **right** triangle has one right angle. - ![Right Triangle](image)

An **obtuse** triangle has one obtuse angle. - ![Obtuse Triangle](image)

An **isosceles** triangle has at least two congruent sides. - ![Isosceles Triangle](image)

A **scalene** triangle has no congruent sides. - ![Scalene Triangle](image)

If an **isosceles** triangle has exactly two congruent sides, those sides are the legs and the third side is the base. The two base angles are congruent.

Identify the following triangles by their sides and their angles.

a. ![Right Triangle](image)

b. ![Equilateral Triangle](image)

c. ![Obtuse Triangle](image)

d. In the triangle shown of \( \triangle ABC \), \( AB = 4x - 3 \), \( BC = 2x + 7 \), and \( AC = 5x - 1 \), and the perimeter of \( \triangle ABC \) is 58.

   \[ 4x - 3 + 2x + 7 + 5x - 1 = 58 \]

   \[ 11x = 55 \]

   \[ x = 5 \]
4-2 Angle Relationships in Triangles (pp. 223–230)

Remember
1. The three interior angles of a triangle add up to 180°. 
   \( (\angle 1 + \angle 2 + \angle 3 = 180°) \)
2. If each side is extended in one direction, 
   exterior angles are formed. \( (\angle 4, \angle 5, \angle 6) \)
3. An interior angle and its adjacent exterior angle 
   are supplements. \( (\angle 1 + \angle 4 = 180°) \)
4. An exterior angle of a triangle is equal to the sum 
   of the two remote interior angles. \( (\angle 4 = \angle 2 + \angle 3) \)
5. The three exterior angles of a triangle add up to 360°. \( (\angle 4 + \angle 5 + \angle 6 = 360°) \)

\[ 2x + 50 = 180 \]
\[ x = 65 \]

In the diagram of \( \triangle ADB \): \( \overline{DCB} \), \( \overline{CD} \equiv \overline{CA} \), and \( \angle ACB = 130° \). What is \( \angle D \)?

f. Find the value of \( x \).

\[ x = 30 \]

g. Find the measures of angle 1, angle 2, angle 3 and angle 4.

\[ m\angle 1 = 55° \]
\[ m\angle 2 = 60° \]
\[ m\angle 3 = 60° \]
\[ m\angle 4 = 65° \]
h. Find the value of $x$.

\[ \text{ext} \angle = r_1 + r_2 \]
\[ 130 = x + 85 \]
\[ x = 45^\circ \]

\[ \text{m} \angle Q = 5(14) + 3 = 73^\circ \]

J. In \( \triangle ABC \), \( \overline{BC} \) is extended to \( D \), \( \angle B = 2y \), \( \angle BCA = 6y \), and \( \angle ACD = 3y \). What is \( \angle A \)?

\[ 6y + 3y = 180 \]
\[ 9y = 180 \]
\[ y = 20 \]
Example If $\triangle XYZ \cong \triangle RST$, name the pairs of congruent angles and congruent sides.

$\angle X \cong \angle R, \angle Y \cong \angle S, \angle Z \cong \angle T$

$XY \cong RS, XZ \cong RT, YZ \cong ST$

K. If $\triangle JGO \cong \triangle RWI$, which angle corresponds to $\angle I$?

Ans: $\boxed{40}$

L. In the diagram, $\triangle EFG \cong \triangle HIJ$. What is the measure of $\angle H$?

\[ \begin{align*}
\angle F & = 35^\circ \\
\angle G & = 35^\circ \\
\angle J & = 80^\circ \\
\angle I & = 65^\circ
\end{align*} \]

M.N. Given $\triangle HJK \cong \triangle TRS$, find the values of $a$ and $b$.

$H = T$

$51 = 6a - 3$

$54 = 6a$

$9 = a$

$K = 5$

$7b - 10 = 4k$

$7b = 56$

$b = 8$
O. Mark the diagram below to show the following given information:

Given: $\overline{AC} \cong \overline{EC}$
\[ \overline{DC} \cong \overline{BC} \]

$\triangle ACB \cong \triangle ECD$ because of $SSS$.

P. In the diagram below, $\triangle ABC \cong \triangle JKH$ because of $SSS$. 

Given: \( \angle J \cong \angle L, \quad \angle JKM \cong \angle LMK \)

\( \triangle JKM \cong \triangle LMK \) because of AAS.

\[
\text{a. Given: } \angle J \cong \angle L, \quad \angle JKM \cong \angle LMK \]

\( \triangle JKM \cong \triangle LMK \) because of AAS.

R. Given: \( WX \perp XZ, \quad YZ \perp ZX, \quad WZ \cong YX \)

\( \triangle WXZ \cong \triangle ZYX \) because of HL.

S. Mark the diagram below to show the following given information:

\[
\text{Given: } AC \text{ bisects } \angle BCD
\]

\( \angle 1 \cong \angle 2 \)

\( \triangle ACB \cong \triangle CAD \) because of ASA.
Determine if you can use SSS, SAS, ASA, AAS, and HL to prove triangles congruent. If not, say no.

Identifying Additional Congruent Parts

What additional information is needed for a SAS congruence correspondence?

A. $\overline{AN} \cong \overline{OM}$
B. $\overline{DA} \cong \overline{MO}$
C. $\overline{ND} \cong \overline{TO}$
D. $\overline{DA} \cong \overline{TO}$

What additional information is needed for an ASA congruence correspondence?

a. $\angle A \cong \angle O$

b. $\angle N \cong \angle M$

c. $\overline{DA} \cong \overline{TO}$

d. $\angle A \cong \angle M$
What additional information is needed for an AAS congruence correspondence?

a. \( \angle O \cong \angle N \)

b. \( \angle A \cong \angle M \)

c. \( \angle A \cong \angle O \)

d. \( \overline{NA} \cong \overline{MO} \)

What additional information is needed for a SSS congruence correspondence?

a. \( \overline{EL} \cong \overline{KA} \)

b. \( \angle C \cong \angle J \)

c. \( \angle E \cong \angle K \)

d. \( \angle L \cong \angle A \)

What additional information is needed for a HL congruence correspondence?

a. \( \overline{AD} \cong \overline{AD} \)

b. \( \angle BAD \cong \angle CAD \)

c. \( \angle B \cong \angle C \)

d. \( \overline{AB} \cong \overline{AC} \)
Section: 4 – 8/Isosceles and Equilateral Triangles

<table>
<thead>
<tr>
<th>Theorem</th>
<th>Examples</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Isosceles Triangle Theorem</strong></td>
<td>![Diagram](36x289 to 577x756)</td>
</tr>
<tr>
<td>If two sides of a triangle are congruent, then the angles opposite the sides are congruent.</td>
<td>If $RT \equiv RS$, then $\angle T \equiv \angle S$.</td>
</tr>
<tr>
<td><strong>Converse of Isosceles Triangle Theorem</strong></td>
<td>![Diagram](36x106 to 510x288)</td>
</tr>
<tr>
<td>If two angles of a triangle are congruent, then the sides opposite those angles are congruent.</td>
<td>If $\angle N \equiv \angle M$, then $\overline{LN} \equiv \overline{LM}$.</td>
</tr>
</tbody>
</table>

Find each angle measure.

1. $\angle C = \boxed{51^\circ}$

   $$2x + 78 = 180$$
   $$x = 51$$

2. $\angle H = \boxed{72^\circ}$

   $$6x + 18 = 8x$$
   $$9 = x$$

3. In the diagram, $\overline{BC} \parallel \overline{CD}$, $\overline{AC} \perp \overline{DE}$, and $\angle A = 50$. Find $\angle E$. $\angle E = 40^\circ$
7. In the accompanying diagram of \( \triangle BCD \), \( \triangle ABC \) is an equilateral triangle and \( AD = AB \). What is the value of \( x \), in degrees?

\[
\angle x + 120^\circ = 180^\circ
\]

\[
x = 30^\circ
\]

---

Equilateral Triangle Corollary
If a triangle is equilateral, then it is equiangular.

\((\text{equilateral } \triangle \rightarrow \text{equiangular } \triangle)\)

Equiangular Triangle Corollary
If a triangle is equiangular, then it is equilateral.

\((\text{equiangular } \triangle \rightarrow \text{equilateral } \triangle)\)
## Proofs

1. \[
\begin{array}{|l|l|}
\hline
\text{Statements} & \text{Reasons} \\
\hline
1. Quadrilateral ABCD, AFEC & 1. Given \\
2. \(AB \cong CD, AD \cong CB\) & 2. Given \\
3. \(DF \perp AC, BE \perp AC\) & 3. Given \\
4. \(\overline{AC} \cong \overline{AC}\) & 4. Reflexive Property \\
5. \(\triangle ADC \cong \triangle CBA\) & 5. SSS (2, 2, 4) \\
6. \(\triangle DAC \cong \triangle BCA\) & 6. CPCTC \\
7. \(\angle DFA \text{ and } \angle BEC \text{ are right angles}\) & 7. Def of perpendicular lines \\
8. \(\angle DFA \cong \angle BEC\) & 8. Right angles are congruent \\
9. \(\triangle AFD \cong \triangle CEB\) & 9. AAS (6, 8, 2) \\
10. \(\overline{DF} \cong \overline{BE}\) & 10. CPCTC \\
\hline
\end{array}
\]

2. \[
\begin{array}{|l|l|}
\hline
\text{Statements} & \text{Reasons} \\
\hline
1. \(\triangle ABC, BD\) is median and altitude to \(\overline{AC}\) & 1. Given \\
2. \(D\) is the midpoint of \(\overline{AC}\) & 2. Def of median \\
3. \(\overline{AD} \cong \overline{CD}\) & 3. Def of midpoint \\
4. \(\overline{BD} \perp \overline{AC}\) & 4. Def of altitude \\
5. \(\angle ADB \text{ and } \angle CDB \text{ are right angles}\) & 5. Def of Perpendicular Lines \\
6. \(\angle ADB \cong \angle CDB\) & 6. Right angles are congruent \\
7. \(\overline{BD} \cong \overline{BD}\) & 7. Reflexive Property \\
8. \(\triangle ADB \cong \triangle CDB\) & 8. SAS (3, 6, 7) \\
9. \(\overline{BA} \cong \overline{BC}\) & 9. CPCTC \\
\hline
\end{array}
\]

3. \[
\begin{array}{|l|l|}
\hline
\text{Statements} & \text{Reasons} \\
\hline
1. \(\angle 1 \cong \angle 5, \angle 2 \cong \angle 6\) & 1. Given \\
2. \(\angle 1 + \angle 2 \cong \angle 5 + \angle 6\) & 2. Addition Postulate (1) \\
3. \(\angle DBA \cong \angle ECA\) & 3. Substitution Property \\
4. \(\angle DBA \supset \angle 3; \angle ECA \supset \angle 4\) & 5. Linear Pair Theorem \\
6. \(\angle 3 \cong \angle 4\) & 6. Congruent Supplements Theorem \\
7. \(\overline{AB} \cong \overline{AC}\) & 7. \text{If } \triangle \text{ then } \triangle \text{ is isosceles.} \\
8. \(\triangle ABC \text{ is isosceles}\) & 8. If the legs of a triangle are congruent, then triangle is isosceles. \\
\hline
\end{array}
\]
4. Given: \( \overline{FJ} \) is the base of an isos \( \triangle \)
   \( FG = JH \)
   \( O \) is mdpt of \( MF \)
   \( K \) is mdpt of \( MJ \)

Concl: \( \overline{OH} = \overline{KG} \)

1. \( \triangle FJM \) is isos
2. \( FG = JH \)
3. \( O \) is mdpt of \( MF \)
4. \( K \) is mdpt of \( MJ \)
5. \( GH = GH \)
6. \( FH = JG \)
7. \( \angle MFJ = \angle MJF \)
8. \( OF = KJ \)
9. \( \triangle OFH = \triangle KJG \)
10. \( \triangle \) CPCTC

5. Given: \( \overline{AC} = \overline{BC} \)
    \( \angle 1 = \angle 3 \)

Prove: \( \triangle DFE \) is isos.

1. \( \overline{AC} = \overline{BC} \)
2. \( \angle 1 = \angle 3 \)
3. \( \overline{CD} = \overline{CE} \)
4. \( \angle C = \angle C \)
5. \( \triangle ACE = \triangle BCD \)
6. \( \angle AEC = \angle BDC \)
7. \( \angle FDE = \angle FED \)
8. \( DF = FE \)
9. \( \triangle DFE \) is isos.

1. Given
2. Given
3. If \( \triangle \) then \( \triangle \)
4. Reflexive prop
5. SAS
6. CPCTC
7. Subtraction prop
8. If \( \triangle \) then \( \triangle \)
9. If at least 2 sides of a \( \triangle \) are \( \equiv \), the \( \triangle \) is isos.
Given: \( \overline{AB} = \overline{AC} \)
\( \overline{BD} \) bis \( \angle ABE \).
\( \overline{CD} \) bis \( \angle ACE \).

Concl: \( \overline{AE} \) bis \( \overline{BC} \).

1. \( \overline{AB} = \overline{AC} \)
2. Given
3. \( \overline{BD} \) bis \( \angle ABE \).
4. Given
5. \( \angle ACD = \angle DCE \)
6. Same as 4
7. \( \angle ABC = \angle ACB \)
8. If \( \triangle \) then \( \triangle \)
9. \( \angle DBE = \angle DCE \)
10. Division prop
11. \( \angle AD = \angle AD \)
12. Reflexive prop
13. \( \triangle ADB = \angle ADC \)
14. SSS
15. \( \triangle ABD = \angle ACD \)
16. Division prop
17. \( \triangle ADB = \angle ADC \)
18. CPCTC
19. \( \triangle ADB \) supp \( \angle BDE \)
20. If 2 \( \angle \)s form a st \( \angle \), the \( \angle \)s are supp.
21. \( \triangle ADC \) supp \( \angle CDE \)
22. Same as 13
23. \( \triangle CDE = \angle BDE \)
24. Supps of = \( \angle \)s are =.
25. \( \triangle DBE = \angle CDE \)
26. ASA
27. \( \overline{BE} = \overline{EC} \)
28. CPCTC
29. \( \overline{AE} \) bis \( \overline{BC} \).
30. If a seg divides a seg into 2 = parts, it bis the seg.
9. Given: △ APB with perpendicular bisector \( PM \)

Prove: \( AR \cong BR \)

<table>
<thead>
<tr>
<th>Statements</th>
<th>Reasons</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. △ APB with perpendicular bisector ( PM )</td>
<td>1. Given</td>
</tr>
<tr>
<td>2. M is the midpoint of ( AB )</td>
<td>2. Def of segment bisector</td>
</tr>
<tr>
<td>3. ( AM \cong BM )</td>
<td>3. Def of midpoint</td>
</tr>
<tr>
<td>4. ( \angle AMP ) and ( \angle BMP ) are right angles</td>
<td>4. Def of perpendicular lines</td>
</tr>
<tr>
<td>5. ( \angle AMP \cong \angle BMP )</td>
<td>5. Right angles are congruent</td>
</tr>
<tr>
<td>6. ( RM \cong RM )</td>
<td>6. Reflexive Property</td>
</tr>
<tr>
<td>7. ( \triangle ARM \cong \triangle BRM )</td>
<td>7. SAS (3, 5, 6)</td>
</tr>
<tr>
<td>8. ( AR \cong RB )</td>
<td>8. CPCTC</td>
</tr>
</tbody>
</table>
10. **Given:**
   - $AF \parallel BC$
   - $BE \parallel AC$
   - $CD \parallel AB$
   - $AF \parallel BC$
   - $BE \parallel AC$
   - $CD \parallel AB$

   **Prove:** $\triangle ABC$ is equilateral.

   1. $AF \parallel BC$  
      - Given
   2. $BE \parallel AC$  
      - Given
   3. $CD \parallel AB$  
      - Given
   4. $AF \parallel BC$  
      - Given
   5. $BE \parallel AC$  
      - Given
   6. $CD \parallel AB$  
      - Given
   7. $AF \perp BC$  
      - Alt. $\perp$ to side
   8. $BE \perp AC$  
      - Same as 7
   9. $CD \perp AB$  
      - Same as 7
   10. $AB = AC$  
       - Pt on $\perp$ bis is $=$ dist from endpts.
   11. $AB = CB$  
       - Same as 10
   12. $CA = CB$  
       - Same as 10
   13. $AB = BC = CA$  
       - Transitive prop
   14. $\triangle ABC$ is equilateral.  
       - An equilateral $\triangle$ has all

11. **Given:** $O, \angle 1 = \angle 2$

   **Concl:** $OY \perp WX$

   1. $O, \angle 1 = \angle 2$  
      - Given
   2. $XY = WY$  
      - If $\triangle$ then $\triangle$
   3. Draw $OX, OW$  
      - Two pts determine a line.
   4. $OX = OW$  
      - Radii of a $\odot$ are $=$
   5. $OY \perp WX$  
      - Two pts $=$ dist from endpts of a seg determine

12. **Given:** $AB = AF$

   **Concl:** $AD \perp CE$

   1. $AB = AF, BD = DF$  
      - Given
   2. $\angle 1 = \angle 2$  
      - Given
   3. $\overline{AD} = \overline{AD}$  
      - Reflexive prop
   4. $\triangle AFD = \triangle ABD$  
      - SSS
   5. $\angle 3 = \angle 4$  
      - CPCTC
   6. $\angle DAE = \angle CDA$  
      - Addition prop
   7. $\overline{AD} \perp \overline{CE}$  
      - $= \text{adj } \angle s \text{ form } \perp \text{ lines.}$
13. Given: \( \overline{BD} \) bis \( \angle ABC \).
Prove: \( \overline{BD} \perp \overline{AC} \).

1. \( \overline{BD} \) bis \( \angle ABC \).
2. Draw \( \overline{OA} \) and \( \overline{OC} \).
3. \( \overline{OA} = \overline{OB} \).
4. \( \angle OAB = \angle OBA \).
5. \( \overline{OC} = \overline{OE} \).
6. \( \angle OCB = \angle OBC \).
7. \( \angle OBC = \angle OBA \).
8. \( \angle OAB = \angle OCB \).
9. \( \overline{OA} = \overline{OC} \).
10. \( \angle OAD = \angle OCD \).
11. \( \angle BAD = \angle BCA \).
12. \( \overline{EA} = \overline{BC} \).
13. \( \overline{BD} \perp \overline{AC} \).

1. Given
2. Two pts determine a line.
3. Radii of a \( \odot \) are \( = \).
4. If \( \triangle \) then \( \triangle \).
5. Same as 3
6. Same as 4
7. Bis divides \( \angle \) into 2 \( = \) \( \angle \)s.
8. Transitive prop
9. Same as 3
10. Same as 4
11. Addition prop
12. If \( \triangle \) then \( \triangle \)
13. Two pts =dist from endpts of a seg determine the \( \perp \) bis.

14. Given: \( \overline{AB} = \overline{AF} \)
\( \overline{BC} = \overline{FE} \)
Concl: \( \overline{CD} = \overline{DE} \)

1. \( \overline{AB} = \overline{AF}, \overline{BC} = \overline{FE} \)
2. \( \overline{AC} = \overline{AE} \)
3. \( \angle A = \angle A \)
4. \( \triangle ACF = \triangle AEB \)
5. \( \angle C = \angle E \)
6. \( \triangle AFC = \triangle ABE \)
7. \( \angle CBD \) supp to \( \angle ABE \)
8. \( \angle EFD \) supp to \( \angle AFC \)
9. \( \angle CBD = \angle EFD \)
10. \( \triangle BCD = \triangle FED \)
11. \( \overline{CD} = \overline{DE} \)

1. Given
2. Addition prop
3. Reflexive prop
4. SAS
5. CPCTC
6. CPCTC
7. Supp \( \angle \)s form \( \perp \) \( \angle \)
8. Same as 7
9. Supps of \( \angle \)s are \( = \)
10. ASA
11. CPCTC
Given: $\triangle ABC$ is isosceles with base $\overline{CB}$
$\overline{CM}$ is a median
$\overline{BD}$ is a median
Prove: $\overline{AR} \perp \overline{CB}$

<table>
<thead>
<tr>
<th>Statements</th>
<th>Reasons</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. $\triangle ABC$ is isosceles with base $\overline{CB}$</td>
<td>1. Given</td>
</tr>
<tr>
<td>2. $\overline{AC} \cong \overline{AB}$</td>
<td>2. Definition of Isosceles Triangles</td>
</tr>
<tr>
<td>3. $\angle ACB \cong \angle ABC$</td>
<td>3. If $\triangle \text{ then } \triangle$</td>
</tr>
<tr>
<td>4. $\overline{CM}$ is a median and $\overline{BD}$ is a median</td>
<td>4. Given</td>
</tr>
<tr>
<td>5. $D$ is the midpoint of $\overline{AC}$ and $M$ is the midpoint of $\overline{AB}$</td>
<td>5. Def of median</td>
</tr>
<tr>
<td>6. $\overline{DC} \cong \frac{1}{2}\overline{AC}$, $\overline{DA} \cong \frac{1}{2}\overline{AC}$; $\overline{MB} \cong \frac{1}{2}\overline{AB}$, $\overline{MA} \cong \frac{1}{2}\overline{AB}$</td>
<td>6. Def of midpoint</td>
</tr>
<tr>
<td>7. $\overline{DC} \cong \overline{MB} \cong \overline{DA} \cong \overline{MA}$</td>
<td>7. Division Postulate</td>
</tr>
<tr>
<td>8. $\overline{CB} \cong \overline{CB}$</td>
<td>8. Reflexive Property</td>
</tr>
<tr>
<td>9. $\triangle DCB \cong \triangle MBC$</td>
<td>9. SAS (7, 3, 8)</td>
</tr>
<tr>
<td>10. $\overline{CDB} \cong \overline{BMC}$</td>
<td>10. CPCTC (9)</td>
</tr>
<tr>
<td>11. $\triangle DEC \cong \triangle MEB$</td>
<td>11. Vertical angles are congruent</td>
</tr>
<tr>
<td>12. $\triangle DCE \cong \triangle MBE$</td>
<td>12. AAS (10, 11, 7)</td>
</tr>
<tr>
<td>13. $\overline{DE} \cong \overline{ME}$</td>
<td>13. CPCTC (12)</td>
</tr>
<tr>
<td>14. $\angle DCB$ supp $\angle ADE$; $\angle BMC$ supp $\angle AME$</td>
<td>14. Linear Pair Theorem</td>
</tr>
<tr>
<td>15. $\angle ADE \cong \angle AME$</td>
<td>15. Congruent Supplements Theorem</td>
</tr>
<tr>
<td>16. $\angle ADE \cong \angle AME$</td>
<td>16. SAS (7, 15, 13)</td>
</tr>
<tr>
<td>17. $\angle DAE \cong \angle MAE$</td>
<td>17. CPCTC (16)</td>
</tr>
<tr>
<td>18. $\overline{AR} \cong \overline{AR}$</td>
<td>18. Reflexive Property</td>
</tr>
<tr>
<td>19. $\angle CAR \cong \angle BAR$</td>
<td>19. SAS (2, 17, 18)</td>
</tr>
<tr>
<td>20. $\angle CRA \cong \angle BRA$</td>
<td>20. CPCTC (19)</td>
</tr>
<tr>
<td>21. $\angle CRA$ supp $\angle BRA$</td>
<td>21. Linear Pair Theorem</td>
</tr>
<tr>
<td>22. $\angle CRA$ and $\angle BRA$ are right angles</td>
<td>22. If 2 angles are congruent and supplementary, then the angles are right.</td>
</tr>
<tr>
<td>23. $\overline{AR} \perp \overline{CB}$</td>
<td>23. If right angles are formed, then the lines that intersect are perpendicular.</td>
</tr>
</tbody>
</table>
16. Given: \( \triangle ABC \) is isosceles with base \( BC \), \( X, Y, M \) are midpoints of \( AB, AC, BC \), respectively.

Prove: \( XM \cong YM \)

<table>
<thead>
<tr>
<th>Statements</th>
<th>Reasons</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. ( \triangle ABC ) is isosceles with base ( BC )</td>
<td>1. Given</td>
</tr>
<tr>
<td>2. ( AB \cong AC )</td>
<td>2. Definition of Isosceles Triangles</td>
</tr>
<tr>
<td>3. ( X, Y, M ) are midpoints of ( AB, AC, BC ), respectively</td>
<td>3. Given</td>
</tr>
<tr>
<td>4. ( BX \cong \frac{1}{2} AB; \ CY \cong \frac{1}{2} AC )</td>
<td>4. Def of midpoint</td>
</tr>
<tr>
<td>5. ( BX \cong CY )</td>
<td>5. Division Postulate</td>
</tr>
<tr>
<td>6. ( 4B \cong 4C )</td>
<td>6.</td>
</tr>
<tr>
<td>7. ( BM \cong CM )</td>
<td>7. Def of midpoint</td>
</tr>
<tr>
<td>8. ( \triangle XBM \cong \triangle YCM )</td>
<td>8. SAS (5, 6, 7)</td>
</tr>
<tr>
<td>9. ( XM \cong YM )</td>
<td>9. CPCTC</td>
</tr>
</tbody>
</table>

17. Given: \( \overline{PT} = \overline{FU} \), \( \overline{PR} = \overline{PS} \)

Prove: \( FQ \) bis \( \angle RPS \)

<table>
<thead>
<tr>
<th>Statements</th>
<th>Reasons</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. ( \overline{PT} = \overline{FU}, \overline{PR} = \overline{PS} )</td>
<td>1. Given</td>
</tr>
<tr>
<td>2. ( \angle TPS = \angle UPR )</td>
<td>2. Reflexive prop</td>
</tr>
<tr>
<td>3. ( \triangle TPS = \triangle UPR )</td>
<td>3. SAS</td>
</tr>
<tr>
<td>4. ( \angle PRU = \angle PST )</td>
<td>4. CPCTC</td>
</tr>
<tr>
<td>5. ( \angle T = \angle U )</td>
<td>5. CPCTC</td>
</tr>
<tr>
<td>6. ( \angle TRQ \supseteq \angle PRU )</td>
<td>6. If 2 ( \angle )s form a st ( \angle ), the ( \angle )s are supp.</td>
</tr>
<tr>
<td>7. ( \angle USQ \supseteq \angle PST )</td>
<td>7. Same as 6</td>
</tr>
<tr>
<td>8. ( \angle TRQ = \angle USQ )</td>
<td>8. Supps of = ( \angle )s are =.</td>
</tr>
<tr>
<td>9. ( \overline{TR} = \overline{US} )</td>
<td>9. Subtraction prop</td>
</tr>
<tr>
<td>10. ( \triangle TRQ = \triangle USQ )</td>
<td>10. ASA</td>
</tr>
<tr>
<td>11. ( RQ = SQ )</td>
<td>11. CPCTC</td>
</tr>
<tr>
<td>12. ( \triangle PRQ = \triangle PSQ )</td>
<td>12. SAS</td>
</tr>
<tr>
<td>13. ( \angle RQP = \angle SPQ )</td>
<td>13. CPCTC</td>
</tr>
<tr>
<td>14. ( FQ ) bis ( \angle RPS ).</td>
<td>14. If a ray divides an ( \angle ) into 2 = ( \angle )s, then it bis the ( \angle ).</td>
</tr>
</tbody>
</table>