A **transformation** is a function that moves or changes a figure in some way to produce a new figure call an **image**. Another name for the original figure is the **pre-image**.

We will study the following transformations:

<table>
<thead>
<tr>
<th>Reflections</th>
<th><img src="image1.png" alt="Image" /></th>
</tr>
</thead>
<tbody>
<tr>
<td>Translations</td>
<td><img src="image2.png" alt="Image" /></td>
</tr>
<tr>
<td>Rotations</td>
<td><img src="image3.png" alt="Image" /></td>
</tr>
<tr>
<td>Dilations</td>
<td><img src="image4.png" alt="Image" /></td>
</tr>
</tbody>
</table>
Vocabulary to know:

A **Rigid Motion** is a transformation that preserves length (distance preserving) and angle measure (angle preserving).

Another name for a rigid motion is an **isometry**.

**Orientation** refers to the arrangement of points relevant to one another, after a transformation has occurred.

A **direct isometry** preserves distance and orientation. Translations and Rotations are direct isometries.

An **opposite isometry** preserves distance but does not preserve orientation. Line reflections are opposite isometries.
Example 1: Is the transformation an isometry? Do the figures have the same or opposite orientation?

Example 2: Is the transformation an isometry? Do the figures have the same or opposite orientation?

Example 3: Match each of the following

<table>
<thead>
<tr>
<th>1) Rotation</th>
<th>a)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2) Translation</td>
<td>b)</td>
</tr>
<tr>
<td>3) Line Reflection</td>
<td>c)</td>
</tr>
<tr>
<td>4) Dilation</td>
<td>d)</td>
</tr>
</tbody>
</table>
Line Reflections

Line reflections are isometries that do not preserve orientation.

Therefore line reflections are opposite isometries.

Thus, a reflected image in a mirror appears "backwards."

- Every point is the same distance from the central line!
- The reflection has the same size as the original image.

A line reflection is a rigid motion.

Example:

Graph ΔABC with vertices A(1, 3), B(5, 2), and C(2, 1) and its image after the reflection described.

a. In the line m: x = 3

b. In the line m: y = 1
**Reflections in the Coordinate Plane**

1. Triangle: A(2, 2), B(3, -5), C(-1, 0)  
   Reflection: reflection across x-axis  
   \[ A'(\_\_, \_\_) \]  
   \[ B'(\_\_, \_\_) \]  
   \[ C'(\_\_, \_\_) \]  
   Rule: \( r_{\text{x-axis}}(x, y) = (\_\_, \_\_) \)

2. Triangle: A(1, 3), B(-2, 6), C(0, 0)  
   Reflection: reflection over y-axis  
   \[ A'(\_\_, \_\_) \]  
   \[ B'(\_\_, \_\_) \]  
   \[ C'(\_\_, \_\_) \]  
   Rule: \( r_{\text{y-axis}}(x, y) \rightarrow (\_\_, \_\_) \)

3. Triangle: A(0, 0), B(5, 2), C(4, 4)  
   Reflection: reflection over line \( y = x \)  
   \[ A'(\_\_, \_\_) \]  
   \[ B'(\_\_, \_\_) \]  
   \[ C'(\_\_, \_\_) \]  
   Rule: \( r_{\text{y=x}}(x, y) = (\_\_, \_\_) \)

4. Triangle: A(2, 2), B(6, 2), and C(4, 7)  
   Reflection: reflection over the line \( y = -x \)  
   \[ A'(\_\_, \_\_) \]  
   \[ B'(\_\_, \_\_) \]  
   \[ C'(\_\_, \_\_) \]  
   Rule: \( r_{\text{y=-x}}(x, y) = (\_\_, \_\_) \)
5. Triangle: A(1,2), B(1,4) and C(3,3).

Reflection: Origin

\[ A'(\quad) \]
\[ B'(\quad) \]
\[ C'(\quad) \]

Rule: \( r_{\text{origin}} (x, y) = (\quad, \quad) \)

Examples:

1. What is the image of point A(4,-7) after a reflection in the x-axis?

2. What is the image of point B(2,-3) after a reflection in the y-axis?

3. Find the coordinates of C', the image of C(5,-2), after a reflection in \( r_{yx} \).

4. Find the coordinates of the image of point D(-3,7) after a reflection in the line \( y = -x \).

**Constructions and Line Reflections.**

**Example 1:**

\( \triangle ABC \) is reflected across \( DE \) and maps onto \( \triangle A'B'C' \).

Use your compass and straightedge to construct the perpendicular bisector of each of the segments connecting \( A \) to \( A' \), \( B \) to \( B' \), and \( C \) to \( C' \).

What do you notice about these perpendicular bisectors?

Label the point at which \( AA' \) intersects \( DE \) as point \( O \). What is true about \( AO \) and \( A'O \)? How do you know this is true?
**Example 2:**

Construct the segment that represents the line of reflection for quadrilateral $ABCD$ and its image $A'B'C'D'$. What is true about each point on $ABCD$ and its corresponding point on $A'B'C'D'$?

**Examples 3–4**

Construct the line of reflection across which each image below was reflected.

3. 

4. 

You have shown that a line of reflection is the perpendicular bisector of segments connecting corresponding points on a figure and its reflected image. You have also constructed a line of reflection between a figure and its reflected image. Now we need to explore methods for constructing the reflected image itself. The first few steps are provided for you in this next stage.
Example 5

The task at hand is to construct the reflection of \( \Delta ABC \) over line \( DE \). Follow the steps below to get started, then complete the construction on your own.

1. Construct circle \( A \): center \( A \), with radius such that the circle crosses \( DE \) at two points (labeled \( F \) and \( G \)).
2. Construct circle \( F \): center \( F \), radius \( FA \) and circle \( G \): center \( G \), radius \( GA \). Label the [unlabeled] point of intersection between circles \( F \) and \( G \) as point \( A' \). This is the reflection of vertex \( A \) across \( DE \).
3. Repeat steps 1 and 2 for vertices \( B \) and \( C \) to locate \( B' \) and \( C' \).
4. Connect \( A', B', \) and \( C' \) to construct the reflected triangle.

Things to consider:

When you found the line of reflection earlier, you did this by constructing perpendicular bisectors of segments joining two corresponding vertices. How does the reflection you constructed above relate to your earlier efforts at finding the line of reflection itself? Why did the construction above work?

Example 6

Now try a slightly more complex figure. Reflect \( ABCD \) across line \( EF \).
Reflecting Over Vertical and Horizontal Lines

1. Triangle: A(-4, 1), B(-1, 1), and C(-3, 4)

Reflection: reflection over the line \( y = -3 \)

- \( A' \) ( , )
- \( B' \) ( , )
- \( C' \) ( , )

Rule: Think \textbf{Midpoint}

\( A(-4, 1), \ B(-1, 1), \) and \( C(-3, 4) \)

\( ( , ), \ ( , ) \ ( , ) \)

\( A'(\ , \), \ B'( \ , \ ) \ C'( \ , \ ) \)

2. Triangle: A(2, 3), B(5, 6), and C(8, 4)

Reflection: reflection over the line \( x = -1 \)

- \( A' \) ( , , )
- \( B' \) ( , , )
- \( C' \) ( , , )

Rule: Think \textbf{Midpoint}

\( A(2, 3), \ B(5, 6), \) and \( C(8, 4) \)

\( ( , , ), \ ( , , ) \ ( , , ) \)

\( A'(\ , , ), \ B'( \ , , ) \ C'( \ , , ) \)
Identifying a Reflection

Write a rule to describe each transformation.

1. 

2. 

Challenge

Algebra The line \( y = 0.5x - 4 \) is reflected in the line \( y = -2 \). What is the equation of the image?
**Summary**

**Reflecting over the x-axis:**
(the x-axis as the line of reflection)

The reflection of the point \((x, y)\) across the x-axis is the point \((x, -y)\).

\[ P(x, y) \rightarrow P(x, -y) \quad \text{or} \quad r_{x\text{-axis}}(x, y) = (x, -y) \]

**Reflecting over the y-axis:**
(the y-axis as the line of reflection)

The reflection of the point \((x, y)\) across the y-axis is the point \((-x, y)\).

\[ P(x, y) \rightarrow P(-x, y) \quad \text{or} \quad r_{y\text{-axis}}(x, y) = (-x, y) \]

**Reflecting over the line \(y = x\) or \(y = -x\):**
(the lines \(y = x\) or \(y = -x\) as the lines of reflection)

The reflection of the point \((x, y)\) across the line \(y = x\) is the point \((y, x)\).

\[ P(x, y) \rightarrow P'(y, x) \quad \text{or} \quad r_{y=x}(x, y) = (y, x) \]

The reflection of the point \((x, y)\) across the line \(y = -x\) is the point \((-y, -x)\).

\[ P(x, y) \rightarrow P'(-y, -x) \quad \text{or} \quad r_{y=-x}(x, y) = (-y, -x) \]

**Exit Ticket**

What is the reflection image of \((5, -3)\) in the y-axis?
A. \((5, 3)\)  B. \((-5, 3)\)  C. \((-5, -3)\)  D. \((-3, 5)\)

What is the reflection image of \((5, -3)\) in the line \(y = -x\)?
F. \((-3, 5)\)  G. \((-3, -5)\)  H. \((3, -5)\)  I. \((3, 5)\)
Homework

1. Find the reflection of the point (5,7) under a reflection in the x-axis.

2. Find the reflection of the point (6,-2) under a reflection in the y-axis.

3. Find the reflection of the point (-1,-6) under a reflection in the line y =x.

4. Is the transformation an isometry? Do the figures have the same or opposite orientation?

5. Is the transformation an isometry? Do the figures have the same or opposite orientation?
Graphing Reflections

Graph the reflection of the polygon in the given line.

1. $x$-axis
2. $y$-axis
3. $x = -1$
4. $y = 1$
5. $y = -x$
6. $y = x$
9. $x$-axis
10. $y$-axis
11. $x$-axis
Identifying Reflections
Write a rule to describe each transformation.

7) [Diagram of a polygon with labeled vertices before and after a reflection across a line.]

8) [Diagram of a polygon with labeled vertices before and after a reflection across a line.]

9) [Diagram of a polygon with labeled vertices before and after a reflection across a line.]

10) [Diagram of a polygon with labeled vertices before and after a reflection across a line.]

11) [Diagram of a polygon with labeled vertices before and after a reflection across a line.]

12) [Diagram of a polygon with labeled vertices before and after a reflection across a line.]
Construct the line of reflection for each pair of figures below.

1. 

2. 

3. 

4. Reflect the given image across the line of reflection provided.
5. Draw a triangle $ABC$. Draw a line $l$ through vertex $C$ so that it intersects the triangle at more than just the vertex. Construct the reflection across $l$. 
Translations – Day 2

Do Now:

1. What is the image of \((1,-3)\) when reflected in the \(y\)-axis?
   
   \[
   \begin{array}{ll}
   (1) \ (-1,3) & (3) \ (-3,-1) \\
   (2) \ (3,-1) & (4) \ (-1,-3)
   \end{array}
   \]

2. What are the coordinates of the image of point \(A(3,-1)\) after a reflection in the line \(x = 2\)?
Performing Translations

A vector is a quantity that has both direction and magnitude, or size, and is represented in the coordinate plane by an arrow drawn from one point to another.

Vectors

The diagram shows a vector. The initial point, or starting point, of the vector is P, and the terminal point, or ending point, is Q. The vector is named \( \overrightarrow{PQ} \), which is read as “vector \( PQ \).” The horizontal component of \( \overrightarrow{PQ} \) is 5, and the vertical component is 3. The component form of a vector combines the horizontal and vertical components. So, the component form of \( \overrightarrow{PQ} \) is \((5, 3)\).

**Example 1: Identifying Vector Components**

In the diagram, name the vector and write its component form.

Translations

A translation moves every point of a figure the same distance in the same direction. More specifically, a translation maps, or moves, the points \( P \) and \( Q \) of a plane figure along a vector \((a, b)\) to the points \( P' \) and \( Q' \), so that one of the following statements is true.

- \( PP' = QQ' \) and \( PP' \parallel QQ' \), or
- \( PP' = QQ' \) and \( PP' \) and \( QQ' \) are collinear.

Translations map lines to parallel lines and segments to parallel segments. For instance, in the figure above, \( PO \parallel PO' \).

**Example 2: Translating a Figure Using a Vector**

a. The vertices of \( \triangle ABC \) are \( A(0, 3) \), \( B(2, 4) \), and \( C(1, 0) \). Translate \( \triangle ABC \) using the vector \((5, -1)\).

b. What is the magnitude of the vector?
Example 4: Translating a Figure in the Coordinate Plane

Graph quadrilateral $ABCD$ with vertices $A(-1, 2)$, $B(-1, 5)$, $C(4, 6)$, and $D(4, 2)$ and its image after the translation $(x, y) \rightarrow (x + 3, y - 1)$. 

Example 5:

a) Name the vector and write its component form.

b) What is the magnitude of the vector?
1. The vertices of \( \triangle LMN \) are \( L(2, 2), M(5, 3) \) and \( N(9, 1) \). Translate \( \triangle LMN \) using the vector \( < -2, 6 > \).

2. Graph \( \triangle RST \) with vertices \( R(2, 2), S(5, 2) \) and \( T(3, 5) \) and its image under the translation \( (x, y) \rightarrow (x + 1, y + 2) \).

3. \( \triangle DEF \) is a translation of \( \triangle ABC \). Use the diagram to write a rule for the translation of \( \triangle ABC \) to \( \triangle DEF \).
4. Pentagon ABCDE is drawn on the grid below.

On the grid, draw a translation of pentagon ABCDE using the rule \( T_{3,5} \).

5.  
   a) On graph paper, draw the graph of circle \( O \), which is represented by the equation 
   \[(x-1)^2 + (y+3)^2 = 16.\]
   b) On the same set of axes, draw the image of circle \( O \) after the translation
   \[(x,y) \rightarrow (x-2, y+4)\] and label it \( O' \).
   c) Write an equation of circle \( O' \).
   d) Write an equation of the line that passes through the centers of circle \( O \) and circle \( O' \).

6.  
   a) Write a motion rule for the translation.
   b) What is the magnitude of the vector?
7. Describe the composition of transformations shown in the diagram.

8. Name the vector and write its component form.
Homework

1. **Use the translation** \((x, y) \rightarrow (x - 5, y + 8)\).
   
   **A)** What is the image of \(B(4, 2)\)?
   
   **B)** What is the preimage of \(F'(-3, -4)\)?

2. **The vertices of** \(\triangle ABC\) are \(A(-1, 1), B(4, -1),\) and \(C(2, 4)\). Graph the image of the triangle using prime notation. \((x, y) \rightarrow (x - 3, y + 5)\)

3. **a)** Write a motion rule for the translation. **b)** What is the magnitude of the vector?
4. Write a rule for the translation.

5. Find the component form of the vector that translates $P(-3,6)$ to $P'(0,1)$.

6. Describe the composition of transformations shown in the diagram.

7. Name the vector and write its component form.
Rotations – Day 3

Warm-Up

1. A translation moves $A(-1,3)$ to $A'(-3,7)$. What are the coordinates of $B'$, the image of $B(5,-3)$ under the same translation?

2. Find the coordinates of $P'$, the image of $P(-3,4)$ under the translation $T_{4,1}$.

A rotation is a rigid motion.

Because a rotation is a rigid motion, and a rigid motion preserves length and angle measure, the following statements are true for the rotation shown.

- $DE = D'E'$, $EF = E'F'$, $FD = F'D'$
- $m\angle D = m\angle D'$, $m\angle E = m\angle E'$, $m\angle F = m\angle F'$

**Key Concept: Rotation**

A rotation about a fixed point, called the center of rotation, through an angle of $x^\circ$ maps a point to its image such that

- if the point is the center of rotation, then the image and preimage are the same point, or
- if the point is not the center of rotation, then the image and preimage are the same distance from the center of rotation and the measure of the angle of rotation formed by the preimage, center of rotation, and image points is $x$.

The direction of a rotation can be either clockwise or counterclockwise. From this point forward, all rotations will be counterclockwise, unless stated otherwise.
Rules

\((x, y)\)

\(R_{90} = (\text{___}, \text{___})\)

\(R_{180} = (\text{___}, \text{___})\)

\(R_{270} = (\text{___}, \text{___})\)

Practice

3. \((2, 3)\)

\(R_{90} = (\text{___}, \text{___})\)

\(R_{180} = (\text{___}, \text{___})\)

\(R_{270} = (\text{___}, \text{___})\)

4. \((-1, 6)\)

\(R_{90} = (\text{___}, \text{___})\)

\(R_{180} = (\text{___}, \text{___})\)

\(R_{270} = (\text{___}, \text{___})\)

5. \((-7, -4)\)

\(R_{90} = (\text{___}, \text{___})\)

\(R_{180} = (\text{___}, \text{___})\)

\(R_{270} = (\text{___}, \text{___})\)
6. Given triangle X(2,2), Y(4,3) and Z(3,0).
Write the coordinates of its image after a rotation 90°.
Rule: (x,y) \rightarrow (     )
X'(     )
Y'(     )
Z'(     )

7. Given triangle A(1,3), B(4,4) and C(4,0).
Write the coordinates of its image after a rotation 180°.
Rule: (x,y) \rightarrow (     )
A'(     )
B'(     )
C'(     )

8. Given triangle A(-2,-5), B(-1,1) and C(1,-2).
Write the coordinates of its image after a rotation 270°.
Rule: (x,y) \rightarrow (     )
A'(     )
B'(     )
C'(     )
Finding the center of rotation using constructions

1. Follow the directions below to locate the center of rotation taking the figure at the top right to its image at the bottom left.
   
   a. Draw a segment connecting points $A$ and $A'$.
   
   b. Using a compass and straightedge, find the perpendicular bisector of this segment.
   
   c. Draw a segment connecting points $B$ and $B'$.
   
   d. Find the perpendicular bisector of this segment.
   
   e. The point of intersection of the two perpendicular bisectors is the center of rotation. Label this point $P$.

2. Find the center of rotation:
Identifying a Rotation

Write a rule to describe each transformation.

a. 

b. 

Challenge:

What is the value of \( x \)?
1. Graph the image of the figure under the transformation given and write the coordinates of the image.
   a) rotation 180° about the origin

   ![Diagram of a rotated image](image1.png)

   b) rotation 90° counterclockwise about the origin

   ![Diagram of a rotated image](image2.png)

   c) rotation 90° clockwise about the origin

   ![Diagram of a rotated image](image3.png)
2. Write the rule for the transformation.

3. Write the rule for the transformation.

4. Graph the image of triangle DEF after a rotation of 180°.
5. Use a compass and a straightedge to find the center of rotation.

6. On your paper, construct an equilateral triangle. Locate the midpoint of one side using your compass. Rotate the triangle 180° around this midpoint. What figure have you formed?

7. Write the rule for the transformation.
Dilations/Symmetry – Day 4

Warm – Up

1. If the letter $\mathbf{P}$ is rotated 180 degrees, which is the resulting figure?
   A) \( \mathbf{d} \)  
   B) \( \mathbf{b} \)  
   C) \( \mathbf{a} \)  
   D) \( \mathbf{d} \)

2. If the triangle is rotated 90 counterclockwise about the origin, what will the image be?
   A) \( \mathbf{a} \)  
   B) \( \mathbf{b} \)  
   C) \( \mathbf{c} \)  
   D) \( \mathbf{d} \)

---

A dilation used to create an image larger than the original is called an **enlargement**. A dilation used to create an image smaller than the original is called a **reduction**. A figure and its dilation are similar figures.

**Reduction** – a dilation with $0 < k < 1$.

**Enlargement** – a dilation with $k > 1$.

A dilation of scalar factor $k$ whose center of dilation is the origin may be written:

$$D_k(x, y) = (kx, ky)$$
Dilations

A dilation is a transformation in which a figure is enlarged or reduced with respect to a fixed point $C$ called the center of dilation and a scale factor $k$, which is the ratio of the lengths of the corresponding sides of the image and the preimage.

A dilation with center of dilation $C$ and scale factor $k$ maps every point $P$ in a figure to a point $P'$ so that the following are true:

- If $P$ is the center point $C$, then $P = P'$.
- If $P$ is not the center point $C$, then the image point $P'$ lies on $CP$. The scale factor $k$ is a positive number such that $k = \frac{CP'}{CP}$.
- Angle measures are preserved.

A dilation does not change any line that passes through the center of dilation. A dilation maps a line that does not pass through the center of dilation to a parallel line. In the figure above, $PR \parallel P'R'$, $PQ \parallel P'Q'$, and $QR \parallel Q'R'$.

When the scale factor $k > 1$, a dilation is an enlargement. When $0 < k < 1$, a dilation is a reduction.

******CHOOSE AN EXPLANATION

A dilation is NOT a rigid motion.

*It does not preserve distance and is not an isometry.

A dilation preserves:

1) Angle measure
2) Parallelism
3) Colinearity
4) Midpoint
5) Orientation
**EXAMPLE 1** Identifying Dilations

Find the scale factor of the dilation. Then tell whether the dilation is a reduction or an enlargement.

a. 

![Image of triangle with labeled sides: CP = 12, CP' = 8, P'P = 8, PP' = 12]

**SOLUTION**

a. Because $\frac{CP'}{CP} = \frac{12}{8}$, the scale factor is $k = \frac{3}{2}$. So, the dilation is an enlargement.

b. Because $\frac{CP'}{CP} = \frac{18}{30}$, the scale factor is $k = \frac{3}{5}$. So, the dilation is a reduction.

---

**Coordinate Rule for Dilations**

If $(x, y)$ is the preimage of a point, then its image after a dilation centered at the origin $(0, 0)$ with scale factor $k$ is the point $(kx, ky)$.

---

**EXAMPLE 2** Dilating a Figure in the Coordinate Plane

Graph $\triangle ABC$ with vertices $A(2, 1)$, $B(4, 1)$, and $C(4, -1)$ and its image after a dilation with a scale factor of 2.

**SOLUTION**

Use the coordinate rule for a dilation with $k = 2$ to find the coordinates of the vertices of the image. Then graph $\triangle ABC$ and its image.

- $(x, y) \rightarrow (2x, 2y)$
- $A(2, 1) \rightarrow A'(4, 2)$
- $B(4, 1) \rightarrow B'(8, 2)$
- $C(4, -1) \rightarrow C'(8, -2)$

Notice the relationships between the lengths and slopes of the sides of the triangles in Example 2. Each side length of $\triangle A'B'C'$ is longer than its corresponding side by the scale factor. The corresponding sides are parallel because their slopes are the same.
**EXAMPLE 3** Dilating a Figure in the Coordinate Plane

Graph quadrilateral $KLMN$ with vertices $K(-3, 6), L(0, 6), M(3, 3),$ and $N(-3, -3)$ and its image after a dilation with a scale factor of $\frac{1}{2}$. 

**EXAMPLE 4** Using a Negative Scale Factor

Graph $\triangle FGH$ with vertices $F(-4, -2), G(-2, 4),$ and $H(-2, -2)$ and its image after a dilation with a scale factor of $-\frac{1}{2}$.

In the coordinate plane, you can have scale factors that are negative numbers. When this occurs, the figure rotates $180^\circ$. So, when $k > 0$, a dilation with a scale factor of $-k$ is the same as the composition of a dilation with a scale factor of $k$ followed by a rotation of $180^\circ$ about the center of dilation. Using the coordinate rules for a dilation and a rotation of $180^\circ$, you can think of the notation as

$$(x, y) \rightarrow (kx, ky) \rightarrow (-kx, -ky),$$

where $k > 0$.
Example: Use a compass and straightedge to construct the dilation of \( \triangle LMN \) with the given center and scale factor 3.

\[ \bullet_C \]

---

Example: Use a compass and straightedge to construct the dilation of \( \triangle PQR \) with a scale factor of 2. Use a point \( C \) outside the triangle as the center of dilation.

**CONSTRUCTION**

**Constructing a Dilation**

Use a compass and straightedge to construct a dilation of \( \triangle PQR \) with a scale factor of 2. Use a point \( C \) outside the triangle as the center of dilation.

**SOLUTION**

**Step 1**

**Draw a triangle** Draw \( \triangle PQR \) and choose the center of the dilation \( C \) outside the triangle. Draw rays from \( C \) through the vertices of the triangle.

**Step 2**

**Use a compass** Use a compass to locate \( P' \) on \( CP \) so that \( CP' = 2(CP) \). Locate \( Q' \) and \( R' \) using the same method.

**Step 3**

**Connect points** Connect points \( P', Q', \) and \( R' \) to form \( \triangle P'Q'R' \).
Homework:

1. Dilate the following figure by a factor of 3

2. Under a dilation where the center of dilation is the origin, the image of $A(-2,-3)$ is $A'(-6,-9)$. What are the coordinates of $B'$, the image of $B(4,0)$ under the same dilation?
   (1) (-12,0)   (3) (-4,0)   (2) (12,0)   (4) (4,0)

3. In which quadrant would the image of point $(5,-3)$ lie after a dilation using a factor of -3?
   (1) I   (3) III   (2) II   (4) IV

4. Write a single rule for a dilation by which $A'$ is the image of $A$.
   \[ A(5,2) \rightarrow A'(10,4) \]
5. Graph B(-5,-10), C(-5,0) and D(0,5) and its image after a dilation of -1/5.

6. Dilate the following figure by a factor of \( \frac{1}{2} \).
7. Find the scale factor of the dilation for each of the following:

a) 

b) 

c) 

8. Use a compass and straightedge to construct the dilation of \( \triangle LMN \) with the given center and scale factor \( \frac{1}{2} \).
9. Use a compass and straightedge to construct the dilation of \( \triangle LMN \) with the given center and scale factor 2.
Symmetry

**Key Concept**

**Line Symmetry**

A figure in the plane has **line symmetry** (or **reflection symmetry**) if the figure can be mapped onto itself by a reflection in a line, called a **line of symmetry** (or **axis of symmetry**).

**Identify Line Symmetry**

State whether the figure has line symmetry. Write **yes** or **no**. If so, copy the figure, draw all lines of symmetry, and state their number.

1. 
2. 
3. 

Another type of symmetry is rotational symmetry.

**Key Concept**

**Rotational Symmetry**

A figure in the plane has **rotational symmetry** (or **radial symmetry**) if the figure can be mapped onto itself by a rotation between 0° and 360° about the center of the figure, called the **center of symmetry** (or **point of symmetry**).

**Examples**

The figure below has rotational symmetry because a rotation of 90°, 180°, or 270° maps the figure onto itself.

The number of times a figure maps onto itself as it rotates from 0° to 360° is called the **order of symmetry**. The **magnitude of symmetry** (or angle of rotation) is the smallest angle through which a figure can be rotated so that it maps onto itself. The order and magnitude of a rotation are related by the following equation.

\[
magnitude = 360° ÷ \text{order}
\]

The figure above has rotational symmetry of order 4 and magnitude 90°.
Identifying Rotational Symmetry

A figure in the plane has **rotational symmetry** when the figure can be mapped onto itself by a rotation of 180° or less about the center of the figure. This point is the **center of symmetry**. Note that the rotation can be either clockwise or counterclockwise.

For example, the figure below has rotational symmetry, because a rotation of either 90° or 180° maps the figure onto itself (although a rotation of 45° does not).

![Rotational Symmetry Example](image)

The figure above also has **point symmetry**, which is 180° rotational symmetry.

**EXAMPLE 4** Identifying Rotational Symmetry

Does the figure have rotational symmetry? If so, describe any rotations that map the figure onto itself.

- a. parallelogram
- b. regular octagon
- c. trapezoid

**SOLUTION**

a. The parallelogram has rotational symmetry. The center is the intersection of the diagonals. A 180° rotation about the center maps the parallelogram onto itself.

b. The regular octagon has rotational symmetry. The center is the intersection of the diagonals. Rotations of 45°, 90°, 135°, or 180° about the center all map the octagon onto itself.

c. The trapezoid does not have rotational symmetry because no rotation of 180° or less maps the trapezoid onto itself.

Tell whether each figure has rotational symmetry. If so, give the angle of rotational symmetry.

- a. 
- b. 
- c. 

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Tell whether each figure has rotational symmetry. If so, give the angle of rotational symmetry.

4.  

5.  

6.  

Regular hexagon HEXAGO is divided into six congruent triangles.

4. Name the image of $E$ under a $60^\circ$ rotation about $N$.
5. Name the image of $X$ under a $180^\circ$ rotation about $N$.
6. Name the image of $O$ under a $120^\circ$ rotation about $N$.
7. Name the image of $A$ under a $240^\circ$ rotation about $N$.
8. Name the image of $H$ under a $300^\circ$ rotation about $N$.
9. Name the image of $G$ under a $360^\circ$ rotation about $N$.
Summary:

A figure has **symmetry** if there is a transformation of the figure such that the image and preimage are identical. There are two kinds of symmetry.

<table>
<thead>
<tr>
<th>Line Symmetry</th>
<th>The figure has a <strong>line of symmetry</strong> that divides the figure into two congruent halves.</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td><img src="image" alt="Diagram showing line of symmetry" /></td>
</tr>
<tr>
<td></td>
<td>one line of symmetry two lines of symmetry no line symmetry</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Rotational Symmetry</th>
<th>When a figure is rotated between $0^\circ$ and $360^\circ$, the resulting figure coincides with the original.</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>• The smallest angle through which the figure is rotated to coincide with itself is called the <strong>angle of rotational symmetry</strong>.</td>
</tr>
<tr>
<td></td>
<td>• The number of times that you can get an identical figure when repeating the degree of rotation is called the <strong>order</strong> of the rotational symmetry.</td>
</tr>
<tr>
<td></td>
<td><img src="image" alt="Diagram showing rotational symmetry" /></td>
</tr>
<tr>
<td>angle:</td>
<td>$180^\circ$</td>
</tr>
<tr>
<td>order:</td>
<td>2</td>
</tr>
<tr>
<td>angle:</td>
<td>$120^\circ$</td>
</tr>
<tr>
<td>order:</td>
<td>3</td>
</tr>
<tr>
<td>angle:</td>
<td>no rotational symmetry</td>
</tr>
</tbody>
</table>
Compositions and Rigid Motions – Day 5

Warm – Up

1. Which of the following capital letters has both rotational and line symmetry?
   F  N  H  W
   G  T  J  X

2. Square ABCD is inscribed in a circle with the center at O.

Which rotation about point O maps B to D? ______

A composition of transformations is when two or more transformations are combined to find a single transformation.

The composition of two or more rigid motions is a rigid motion.
Example 1:

The symbol for a composition of transformations is an open circle.

The notation $r_{x-axis} \circ T_{3,4}$ is read as a reflection in the $x$-axis following a translation of $(x+3, y+4)$. Be careful!!! The process is done in reverse!!

You may see various notations which represent a composition of transformations:

$$r_{x-axis} \circ r_{y-axis}(P) = P'''$$

could also be indicated by $P \rightarrow P' \rightarrow P'''$
Example 2: On the accompanying grid, graph and label $\triangle ABC$ with vertices $A(3, 1), B(0, 4)$, and $C(-5, 3)$. On the same grid, graph and label $\triangle A''B''C''$, the image of $\triangle ABC$ after the transformation $r_{x-axis} \circ r_{y=x}$.

Identifying Congruent Figures

Two geometric figures are congruent figures if and only if there is a rigid motion or a composition of rigid motions that maps one of the figures onto the other. Congruent figures have the same size and shape.

You can identify congruent figures in the coordinate plane by identifying the rigid motion or composition of rigid motions that maps one of the figures onto the other.

EXAMPLE

Identifying Congruent Figures

Identify any congruent figures in the coordinate plane. Explain.

SOLUTION

Square $NPQR$ is a translation of square $ABCD$ 2 units left and 6 units down. So, square $ABCD$ and square $NPQR$ are congruent.

$\triangle KLM$ is a reflection of $\triangle EFG$ in the $x$-axis. So, $\triangle EFG$ and $\triangle KLM$ are congruent.

$\triangle STU$ is a $180^\circ$ rotation of $\triangle HIJ$. So, $\triangle HIJ$ and $\triangle STU$ are congruent.
Example 3: Identify any congruent figures in the coordinate plane. Explain.

Congruence Transformations
Another name for a rigid motion or a combination of rigid motions is a **congruence transformation** because the preimage and image are congruent. The terms “rigid motion” and “congruence transformation” are interchangeable.

Describing a Congruence Transformation
Describe a congruence transformation that maps \( \square ABCD \) to \( \square EFGH \).

**SOLUTION**
The two vertical sides of \( \square ABCD \) rise from left to right, and the two vertical sides of \( \square EFGH \) fall from left to right. If you reflect \( \square ABCD \) in the y-axis, as shown, then the image, \( \square A'B'C'D' \), will have the same orientation as \( \square EFGH \).

Then you can map \( \square A'B'C'D' \) to \( \square EFGH \) using a translation of 4 units down.
Example 4: Describe a congruence transformation that maps $\triangle JKL$ to $\triangle MNP$.

Using Theorems about Congruence Transformations
Compositions of two reflections result in either a translation or a rotation. A composition of two reflections in parallel lines results in a translation, as described in the following theorem.

**Theorem**

**Reflections in Parallel Lines Theorem**

If lines $k$ and $m$ are parallel, then a reflection in line $k$ followed by a reflection in line $m$ is the same as a translation.

If $A''$ is the image of $A$, then

1. $AA''$ is perpendicular to $k$ and $m$, and
2. $AA'' = 2d$, where $d$ is the distance between $k$ and $m$. 
Example 5: Use the figure. The distance between line k and line m is 1.6 centimeters.

a) The preimage is reflected in line k, then in line m. Describe a single transformation that maps the figure on the far left to the figure on the far right.

b) What is the relationship between PP' and line k? Explain.

c) What is the distance between P and P''?
A composition of two reflections in intersecting lines results in a rotation, as described in the following theorem.

**Theorem**

**Theorem 4.3  Reflections in Intersecting Lines Theorem**

If lines \( k \) and \( m \) intersect at point \( P \), then a reflection in line \( k \) followed by a reflection in line \( m \) is the same as a rotation about point \( P \).

The angle of rotation is \( 2x^\circ \), where \( x^\circ \) is the measure of the acute or right angle formed by lines \( k \) and \( m \).

### Example

**Using the Reflections in Intersecting Lines Theorem**

In the diagram, the figure is reflected in line \( k \). The image is then reflected in line \( m \). Describe a single transformation that maps \( F \) to \( F'' \).

### Solution

By the Reflections in Intersecting Lines Theorem, a reflection in line \( k \) followed by a reflection in line \( m \) is the same as a rotation about point \( P \). The measure of the acute angle formed between lines \( k \) and \( m \) is \( 70^\circ \). So, by the Reflections in Intersecting Lines Theorem, the angle of rotation is \( 2(70^\circ) = 140^\circ \). A single transformation that maps \( F \) to \( F'' \) is a \( 140^\circ \) rotation about point \( P \).

You can check that this is correct by tracing lines \( k \) and \( m \) and point \( F \), then rotating the point \( 140^\circ \).

### Example 6

In the diagram, the preimage is reflected in line \( k \), then in line \( m \). Describe a single transformation that maps the blue figure onto the green figure.
Glide reflections – a transformation in which every point \( P \) is mapped onto a point \( P'' \) by the following two steps.

1. A translation maps \( P \) to \( P' \).
2. A reflection in a line \( k \) parallel to the direction of the translation maps \( P' \) to \( P'' \).

When a translation (a slide or glide) and a reflection are performed one after the other, a transformation called a glide reflection is produced. In a glide reflection, the line of reflection is parallel to the direction of the translation. It does not matter whether you glide first and then reflect, or reflect first and then glide. This transformation is commutative.

Since translations and reflections are both isometries, a glide reflection is also an isometry.

**Examples:**

a) Does this paw print illustration depict a glide reflection?

b) Examine the graph below. Is triangle \( A'B''C'' \) a glide reflection of triangle \( ABC \)?
Homework:

1. Identify any congruent figures in the coordinate plane.

2. Describe a congruence transformation that maps Quadrilateral SPQR to ZWXY.

3. Describe a congruence transformation that maps triangle ABC to triangle EFG.

4. a) Evaluate the composition of $r_x$ o $R_{90} (-3,5)$.
   b) What single transformation would be the same as the composition in part a?
5. Examine the graph below. Is triangle $A''B''C''$ a glide reflection of triangle $ABC$?

6. The vertices of $\triangle ABC$ are $A(3, -1)$, $B(7, 1)$, and $C(5, -4)$. Graph the image of $\triangle ABC$ after a composition of the transformations in the order they are listed.

   **Translation:** $(x, y) \rightarrow (x - 4, y + 1)$

   **Reflection:** in the line $x = 1$
7. The graph below shows $\triangle ABC$ becoming $\triangle A'B'C'$ under a glide reflection.

a) Write a composition of transformations representing this glide reflection.

b) Write one transformation, or mapping, that will accomplish this same glide reflection.

8. The coordinates of $\triangle JRB$ are $J(1, -2), R(-3, 6),$ and $B(4, 5)$. What are the coordinates of the vertices of its image after the transformation $T \circ r$?
9. Determine whether the polygons with the given vertices are congruent. Use transformations to explain your reasoning.

a) $Q(2, 4), R(5, 4), S(4, 1)$ and $T(6, 4), U(9, 4), V(8, 1)$

b) $W(-3,1), X(2,1), Y(4,-4), Z(-5,-4)$ and $C(-1,-3), D(-1,2), E(4,4), F(4,-5)$
10. Find the angle of rotation that maps A to A''.

11. Use a compass and straightedge to construct two lines of reflection that produce a composition of reflections resulting in the same image as the given transformation.

Translation: $\triangle ABC \rightarrow \triangle A''B''C''$
Test Prep Questions

1. Which transformation preserves both distance and angle measure?
   a) Translation  b) horizontal stretch  c) vertical stretch  d) dilation

2. Which description is always the composition of two different types of transformations?
   a) Line reflection  b) rotation  c) translation  d) glide reflection

3. Which of the following transformations preserve angle measure but not distance?
   a) Dilation  b) translation  c) vertical stretch  d) rotation

4. AB has a length of $2\sqrt{10}$. What is the length of A'B', the image of AB after a dilation with a scale factor of 3?
   a) $5\sqrt{10}$  b) $2\sqrt{30}$  c) $6\sqrt{10}$  d) $6\sqrt{30}$

5. Which transformation is represented in the table below?
<table>
<thead>
<tr>
<th>Input</th>
<th>Output</th>
</tr>
</thead>
<tbody>
<tr>
<td>(3,5)</td>
<td>(4,2)</td>
</tr>
<tr>
<td>(1,0)</td>
<td>(2,-3)</td>
</tr>
<tr>
<td>(6,3)</td>
<td>(7,0)</td>
</tr>
</tbody>
</table>
   a) $T_{-1,3}$  b) $T_{1,3}$  c) $T_{-3,1}$  d) $T_{1,-3}$

6. Which rigid motion does not preserve orientation?
   a) line reflection  b) rotation  c) translation  d) dilation

7. Which transformation is not considered a rigid motion?
   a) dilation  b) rotation  c) reflection  d) translation

8. Which transformation is equivalent to a reflection over a point?
   a) $T_{(2,5)}$  b) $R_{90^\circ}$  c) $R_{180^\circ}$  d) r-x-axis

9. Which of the following composition of transformations maps $\triangle ABC$ to $\triangle A'B'C'$?
   1) $R_{180^\circ} \circ T_{-2,2}$
   2) $T_{1,4} \circ r_y$-axis
   3) $T_{3,-2} \circ r_x$-axis
   4) $R_{90^\circ} \circ r_{x+y}$
10. A series of three transformations that, when performed on \( \triangle ABC \), finally resulting in \( \triangle A'''B'''C''' \).
   Identify each of the three transformations:
   a) \( \triangle ABC \) to \( \triangle A'B'C' \)
   b) \( \triangle A'B'C' \) to \( \triangle A''B''C'' \)
   c) \( \triangle A''B''C'' \) to \( \triangle A'''B'''C''' \)
   d) Is \( \triangle ABC \cong \triangle A'''B'''C''' \)? Justify your answer.
   e) Identify a single transformation that, when performed on \( \triangle A'''B'''C''' \), would result in a triangle \( \triangle A''''B''''C'''' \) such that \( \triangle ABC \) is not congruent to \( \triangle A''''B''''C'''' \). Justify your answer.

11. Use a sequence of rigid motions to show that \( \triangle 1 \cong \triangle 2 \). Use coordinate notation. Label the vertices in triangle 2 using \( A'B'C' \) to correspond to the appropriate angles in triangle 1.
12. 
a) On the accompanying grid, graph and label ΔXYZ with coordinates X(0,2), Y(5,0) and Z(1,3).

b) On the same grid graph and label ΔX''Y''Z'', the image of ΔXYZ after the composition of transformations R_{180°} o r_{x-axis}.

c) What single transformation maps ΔXYZ to ΔX''Y''Z''?

13. Given circle A with the equation \((x-2)^2 + (y+3)^2 = 4\).

a) Perform the translation of circle A: T(2,-4) and label the translation as circle A’

b) Perform a dilation of circle A’ about its center with a scale factor of 2. Label the image A’’.

c) Write the equation of circle A’ and circle A’’.
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