Packet #3: Radicals

Name: ____________________________
Teacher: __________________________
Pd: ________
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Day 1: Simplifying Radicals

Warm-Up:

Simplify the following:

<p>| | | | | |</p>
<table>
<thead>
<tr>
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</thead>
<tbody>
<tr>
<td>a) $2^2$</td>
<td>b) $5^2$</td>
<td>c) $8^2$</td>
<td>d) $(-10)^2$</td>
<td>e) $-8^2$</td>
</tr>
<tr>
<td>f) $2^3$</td>
<td>g) $5^3$</td>
<td>h) $(-10)^3$</td>
<td>i) $-10^3$</td>
<td>j) $2^4$</td>
</tr>
</tbody>
</table>

An expression that contains a radical sign $\sqrt{}$ is a **radical expression**. There are many different types of radical expressions, but in this course, you will only study radical expressions that contain square roots.

Examples of radical expressions:

$\sqrt{14}$  $\sqrt{\ell^2 + \omega^2}$  $\sqrt{2gd}$  $\frac{\sqrt{d}}{4}$  $5\sqrt{2}$  $\sqrt{18}$

The expression under a radical sign is the **radicand**. A radicand may contain numbers, variables, or both. It may contain one term or more than one term.
**Example 1:** **A** number is a perfect square when it has two like factors.

Simplify each expression.

A. \( \sqrt{169} = \) ____  

B. \( 7\sqrt{121} = \) ____  

C. \( -\frac{3}{4}\sqrt{64} = \) ____

---

**The Basics:**

Perfect squares (arithmetic): 1, 4, 9, 16, 25, 36, 49, 64… \( (1^2, 2^2, 3^2, 4^2…) \)

Perfect squares (algebraic): \( x^2, x^4, x^6, x^8 \ldots x^{\text{even}}. \)

a) To determine whether a variable is a perfect square: \( \) the exponent by ____.

---

**Example 2:** Find the square root of each.

D. \( \sqrt{x^{24}} = \) ____  

E. \( \sqrt{225n^{38}} = \) ____  

F. \( \pm\sqrt{49b^{580}} = \) ____

---

**Example 3:** Simplifying square roots: Simplify \( \sqrt{75x^4y^3}. \)

Definition: A square root is said to be simplified if there are no perfect square factors of the radicand.

**Method 1:**

Separate the radicand into largest perfect square factors and whatever is left over.

Use \( \sqrt{a \cdot b} = \sqrt{a} \cdot \sqrt{b} \) separate the radical of products into products of radicals (I usually and you can omit this step, but this is really what you are doing…)

\[ \sqrt{-\cdot 3 \cdot -\cdot -\cdot \cdot \cdot y} \]

Each square root of a perfect square is RATIONAL. Re-write as a rational number, remembering that \( \sqrt{x^2} = |x| \)

**Answer:**

**Method 2:**

Separate the radicand into prime factors. You can use a prime number tree for the coefficient (not shown here)

\[ \sqrt{5 \cdot 5 \cdot 3 \cdot x \cdot x \cdot x \cdot y \cdot y \cdot y} \]

Since, for example, \( \sqrt{5 \cdot 5} = 5, \) the rule is, if there are a pair of factors in the radicand, they can “escape” and be written as one factor outside the radicand. Circle the pairs, and “free” them as one outside the radical.

\[ \sqrt{5 \cdot 5 \cdot 3 \cdot x \cdot x \cdot x \cdot y \cdot y \cdot y} \]

Simplify the expression, understanding that \( |a| \cdot |b| = |ab|. \)

**Answer:**
**Practice:** Simplify each.

Perfect Squares: 1, 4, 9, 16, 25, 36, 49, 64, 81, 100, 121, 144, 169, 196, 225, 256, 289, 324, 361, 400

1) \(\sqrt{200}\)  
2) \(\sqrt{12}\)  

3) \(3\sqrt{50}\)  
4) \(\sqrt{32(x + 2)^2}\)  

5) \(\sqrt{x^7}\)  
6) \(\sqrt{18x^9}\)

**** A number is a perfect cube when it has three like factors.

Ex: \((2)(2)(2) = 8\) therefore 8 is a perfect cube  
\((-2)(-2)(-2) = -8\) therefore -8 is a perfect cube

**The Basics:**

Perfect cubes (arithmetic): 1, 8, 27, 64, 125... \((1^3, 2^3, 3^3, 4^3...\))

Perfect cubes (algebraic): \(x^3, x^6, x^9, x^{12}... x^{\text{multiple of 3}}\).

b) To determine whether a variable is a perfect cube: _____________ the exponent by ____.

Perfect 4\(^{th}\) powers (arithmetic): 1, 16, 81, 256... \((1^4, 2^4, 3^4, 4^4...\))

Perfect 4\(^{th}\) powers (algebraic): \(x^4, x^8, x^{12}, x^{16}... x^{\text{multiple of 4}}\).

c) To determine whether a variable is a root of 4: _____________ the exponent by ____.

*Ad infinitum*....

**Example 4: Simplifying Cube Roots**  
Simplify \(\sqrt[3]{32x^4y^6}\)

Definition: A cube root is said to be simplified if there are no perfect cube factors of the radicand.
<table>
<thead>
<tr>
<th>Method 1:</th>
<th>Method 2:</th>
</tr>
</thead>
<tbody>
<tr>
<td>Separate the radicand into largest perfect cube factors and whatever is left over.</td>
<td>Separate the radicand into prime factors. You can use a prime number tree for the coefficient (not shown here)</td>
</tr>
<tr>
<td>Use ( \sqrt[3]{a \cdot b} = \sqrt[3]{a} \cdot \sqrt[3]{b} ) to separate the radical of products into products of radicals (I usually and you can omit this step, but this is really what you are doing…)</td>
<td>Since, for example, ( \sqrt[3]{2 \cdot 2 \cdot 2} = 2 ), the rule is, if there are a TRIPLE of factors in the radicand, they can “escape” and be written as one factor outside the radicand. Circle the triples, and “free” them as one outside the radical.*</td>
</tr>
<tr>
<td>( \sqrt[3]{\phantom{a \cdot b}} \cdot \sqrt[3]{\phantom{a \cdot b}} \cdot \sqrt[3]{\phantom{a \cdot b}} \cdot \sqrt[3]{\phantom{a \cdot b}} )</td>
<td>( \sqrt[3]{2 \cdot 2 \cdot 2 \cdot 4 \cdot x \cdot x \cdot x \cdot y \cdot y \cdot y \cdot y \cdot y} )</td>
</tr>
<tr>
<td>Each square root of a perfect square is RATIONAL. Re-write as a rational number*</td>
<td>Answer:</td>
</tr>
</tbody>
</table>

*Please note that for cube roots (or any odd root), it is possible to take an odd root of a negative number.

These methods can be extended for any \( n^{th} \) root.

**Practice: Simplify each.**

Perfect cubes: 1, 8, 27, 64, 125, 216, 343, 512, 729, 1000…
Perfect 4\(^{th}\) powers 1, 16, 81, 256, 625, 1296, 2401, 4096, 6561, 10,000…

7) \( \sqrt[3]{-64} \)
8) \( \sqrt[3]{\frac{54a^3}{8b^6}} \)

9) \( \sqrt[3]{24x^{11}} \)
10) \( \sqrt[5]{x^{20}y^{10}} \)

11) \( -\sqrt[3]{320(y + 2)^3} \)
12) \( \sqrt[4]{256x^4y^{16}} \)
Challenge: Solve for x.

\[ 4x - \sqrt{8} = \sqrt{72} \]

Summary:

Simplifying square roots. Example: Simplify \( \sqrt{75x^4y^3} \).

Method 1:
Separate the radicand into largest perfect square factors and whatever is left over.

\[ \sqrt{25 \cdot 3 \cdot x^4 \cdot y^2 \cdot y} \]

Use \( \sqrt{a \cdot b} = \sqrt{a} \cdot \sqrt{b} \) separate the radical of products into products of radicals (I usually and you can omit this step, but this is really what you are doing...)

Each square root of a perfect square is RATIONAL. Re-write as a rational number, remembering that \( \sqrt{x^2} = |x| \)

\[ 5 \cdot |x^2| \cdot |y| \cdot \sqrt{3 \cdot y} \]

Use \( \sqrt{a \cdot b} = \sqrt{a} \cdot \sqrt{b} \) to re-write any remaining radicals as one radical

\[ 5|x^2| |y| \sqrt{3y} \]

Because \( x^2 \) is always always positive, we don’t actually need the absolute value symbol around it at all.

\[ 5x^2 |y| \sqrt{3y} \]

Simplifying Cube Roots. Example: Simplify \( \sqrt[3]{32x^4y^6} \).

Separate the radicand into largest perfect cube factors and whatever is left over.

\[ \sqrt[3]{8 \cdot 4 \cdot x^3 \cdot x \cdot y^6} \]

Use \( \sqrt[3]{a \cdot b} = \sqrt[3]{a} \cdot \sqrt[3]{b} \) to separate the radical of products into products of radicals (I usually and you can omit this step, but this is really what you are doing...)

Each square root of a perfect square is RATIONAL. Re-write as a rational number

\[ \sqrt[3]{8} \cdot \sqrt[3]{4} \cdot \sqrt[3]{x^3} \cdot \sqrt[3]{x} \cdot \sqrt[3]{y^6} \]

Use \( \sqrt[3]{a \cdot b} = \sqrt[3]{a} \cdot \sqrt[3]{b} \) to re-write any remaining radicals as one radical

\[ 2xy^2 \sqrt[3]{4x} \]

Exit Ticket:

Simplify: \( \sqrt[3]{-32x^4y^6} \)

A) \( 3xy \sqrt[3]{5xy^2} \)
B) \( -5 \sqrt[3]{4x^2y} \)
C) \( 4x^2y^2 \sqrt[3]{6x^2y^2} \)
D) \( -2xy^2 \sqrt[3]{4x} \)
Day 2: Operations with Radicals

Warm-Up

Simplify the radical.

$$\sqrt[3]{56mn^4n^7}$$

A) \(2mn^2\sqrt[3]{7mn}\)
B) \(6m^2n^2\sqrt[3]{m^2n^2}\)
C) \(3m^2n^2\sqrt[3]{3n}\)
D) \(-3\sqrt[3]{7mn}\)

Adding and Subtracting Radicals

You can only add “like” radicals. “Like” radicals have the same radicand and index. To add or subtract “like radicals, KEEP the like radical, and add (or subtract, if it is a subtraction problems) the coefficients.

Example: \(2\sqrt{5} + 7\sqrt{5} = (\_ + \_ )\sqrt{5}\)

If the radicals are NOT “like radicals,” you must simplify each radical first to see if you can then add/subtract them.

Example: \(\sqrt{25} + \sqrt{16} \neq \sqrt{25+16}\)
\[5 + 4 \neq \sqrt{41}\]
\[9 \neq \sqrt{41}\]

Example: \(2\sqrt{48} + 4\sqrt{27}\)
\[= 2\sqrt{16\sqrt{3}} + 4\sqrt{9\sqrt{3}}\]
\[= 2 \cdot 4\sqrt{3} + 4 \cdot 3\sqrt{3}\]
\[= 8\sqrt{3} + 12\sqrt{3}\]
\[= 20\sqrt{3}\]
Perfect Squares: 1, 4, 9, 16, 25, 36, 49, 64, 81, 100, 121, 144, 169, 196, 225, 256, 289, 324, 361, 400
Perfect cubes: 1, 8, 27, 64, 125, 216, 343, 512, 729, 1000...

Practice: Add and Subtract. Make sure your radical is in simplest form.

1) $3\sqrt{3} + 2\sqrt{3} - 3\sqrt{3} = \underline{\_\_\_\_\_\_\_}$

2) $-2\sqrt{6} - 2\sqrt{6} - \sqrt{4} = \underline{\_\_\_\_\_\_\_}$

3) $\sqrt{45x^2} - \sqrt{20x^2}$

4) $\sqrt{128x^9} + \sqrt{50x^9}$

5) $\sqrt[3]{24} - \sqrt[3]{81}$

6) $2\sqrt[3]{128x^6} + 7\sqrt[3]{1024x^6}$

*7) $26x - \sqrt{18} = \sqrt{200}$

Multiplying Radicals: $\sqrt[n]{a} \cdot \sqrt[n]{b} = \sqrt[n]{ab}$

If the indices (plural of index) of two radicals are the same, you can multiply the radicands.

Example: $\sqrt[3]{4x^3y} \cdot \sqrt[3]{16xy^6} = \sqrt[3]{64x^4y^7}$. Then simplify using the methods for simplification.

\[
\begin{align*}
\sqrt[3]{64x^4y^7} &= \sqrt[3]{64} \cdot \sqrt[3]{x^3} \cdot \sqrt[3]{x} \cdot \sqrt[3]{y^6} \cdot \sqrt[3]{y} \\
&= 4xy^2\sqrt[3]{xy}
\end{align*}
\]

Dividing Radicals: $\frac{\sqrt[n]{a}}{\sqrt[n]{b}} = \sqrt[n]{\frac{a}{b}}$

If the indices (plural of index) of two radicals are the same, you can divide the radicands. Or, you can separate the radicand of a fraction into a fraction of radicands. Apply whichever is easiest to use at the time.

Example: $\frac{\sqrt[3]{16x^3y^5}}{\sqrt[3]{2xy^4}} = \sqrt[3]{8x^2y}$. Then simplify using the methods for simplification.

\[
\begin{align*}
\sqrt[3]{8x^2y} &= \sqrt[3]{8} \cdot \sqrt[3]{x^2} \cdot \sqrt[3]{y} \\
&= 2\sqrt[3]{x^2y}
\end{align*}
\]
Perfect Squares: 1, 4, 9, 16, 25, 36, 49, 64, 81, 100, 121, 144, 169, 196, 225, 256, 289, 324, 361, 400
Perfect cubes: 1, 8, 27, 64, 125, 216, 343, 512, 729, 1000…

Practice: Multiply and divide. Express your answer in simplest radical form.

8) \(5\sqrt{2x^3} \cdot 5\sqrt{6x}\)  
9) \(-\sqrt[3]{81} \cdot \frac{2\sqrt[3]{x^7}}{3}\)

10) \((\sqrt{5})^2\)  
11) \((2\sqrt{7})^2\)

12) \(\sqrt{3}(2 + \sqrt{3})\)  
13) \((4 + \sqrt{5})(-2 + \sqrt{5})\)

14) \(\sqrt{250x^8} \div \sqrt{5x^2}\)  
15) \(\frac{\sqrt{15}}{\sqrt{45}}\)

16) \(\frac{\sqrt{90x^5}}{\sqrt{2x^4}}\)  
17) \(\frac{12\sqrt{10a} + 8\sqrt{15b}}{4\sqrt{5ab}}\)
Challenge:

If \( \frac{(x - 3)^2}{\sqrt{x^2 - 6x + 9}} = 2 \) and \( x > 0 \), then \( x = \)

Summary

Adding and Subtracting Radicals

You can only add "like" radicals. "Like" radicals have the same radicand and index. To add or subtract "like radicals, KEEP the like radical, and add (or subtract, if it is a subtraction problems) the coefficients.

Example: \( 2\sqrt{5} + 7\sqrt{5} = 9\sqrt{5} \)

If the radicals are NOT "like radicals," you must simplify each radical first to see if you can then add/subtract them.

Example: \( 2\sqrt{48} + 4\sqrt{27} = 2\sqrt{16\sqrt{3}} + 4\sqrt{9\sqrt{3}} = 2 \cdot 4\sqrt{3} + 4 \cdot 3\sqrt{3} = 8\sqrt{3} + 12\sqrt{3} = 20\sqrt{3} \)

Multiplying Radicals: \( \sqrt{a} \cdot \sqrt{b} = \sqrt{ab} \)

If the indices (plural of index) of two radicals are the same, you can multiply the radicands.

Example: \( \sqrt[3]{x^3} \cdot \sqrt[6]{16xy^6} = \sqrt[3]{64x^4y^7} \). Then simplify using the methods for simplification.

\[ = \frac{1}{64} \cdot \sqrt[3]{x^3} \cdot \sqrt[6]{x} \cdot \sqrt[6]{y^6} \cdot \sqrt[6]{y} \]

\[ = 4xy^2\sqrt[6]{xy} \]

Dividing Radicals: \( \frac{\sqrt{a}}{\sqrt{b}} = \sqrt{\frac{a}{b}} \)

If the indices (plural of index) of two radicals are the same, you can divide the radicands. Or, you can separate the radicand of a fraction into a fraction of radicands. Apply whichever is easiest to use at the time.

Example: \( \frac{\sqrt[3]{8x^4y^3}}{\sqrt[3]{2xy^2}} = \sqrt[3]{\frac{8x^4y^3}{2xy^2}} \). Then simplify using the methods for simplification.

\[ = \frac{3}{8} \cdot \sqrt[3]{x^2} \cdot \sqrt[3]{y} \]

\[ = 2\sqrt[3]{x^2y} \]

Exit Ticket

Multiply and Simplify.

\[ \frac{3\sqrt{45}}{3\sqrt{12}} \]

A) \( \frac{3\sqrt{30}}{3\sqrt{20}} \)  B) \( 3\sqrt{20} \)  C) \( \frac{3\sqrt{57}}{3\sqrt{7}} \)  D) 540
Yesterday, we discussed not being allowed to leave a radical in the denominator of a fraction.

**Rationalizing Monomial Denominators**
If the denominator of the fraction is a monomial, multiply the fraction by a Fancy Form of 1 of JUST the radical that is in the bottom of the fraction. Simplify as normally required.

Example: $\frac{12}{3\sqrt{2}} = \frac{12}{3\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{12\sqrt{2}}{6} = \frac{6\sqrt{2}}{3} = \frac{2\sqrt{2}}{1}$

**Rationalizing Binomial Denominators**
If the denominator of the fraction is a binomial, multiply the fraction by a Fancy Form of 1 of the CONJUGATE of the radical that is in the bottom of the fraction. Simplify as normally required.

The conjugate of a binomial is the result of reversing the sign between the two terms. Example: $(\quad)$ and $(\quad)$ are conjugates.

Example: $\frac{12}{3-\sqrt{2}}$

$$= \frac{12}{3-\sqrt{2}} \cdot \frac{(3+\sqrt{2})}{(3+\sqrt{2})} = \frac{12(3+\sqrt{2})}{(3-\sqrt{2})(3+\sqrt{2})}$$
Examples: Rationalize each.

<table>
<thead>
<tr>
<th>a) $\sqrt{\frac{5}{2}}$</th>
<th>b) $\frac{7}{2\sqrt{7}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>c) $\frac{3}{2+\sqrt{6}}$</td>
<td>d) $\frac{3+\sqrt{2}}{3-\sqrt{2}}$</td>
</tr>
<tr>
<td>e) $\frac{\sqrt{2}}{3-\sqrt{6}} - \frac{6}{\sqrt{8}}$</td>
<td>f) $x\sqrt{2} = 3 - x$</td>
</tr>
</tbody>
</table>
Challenge

If \( \left( \frac{x}{100} - 1 \right) \left( \frac{x}{100} + 1 \right) = kx^2 - 1 \), then
the value of \( k \) is

Summary:

**Rationalizing Monomial Denominators**

If the denominator of the fraction is a monomial, multiply the fraction by a Fancy Form of 1 of JUST the radical that is in the bottom of the fraction. Simplify as normally required.

Example: \( \frac{12}{3\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{12\sqrt{2}}{3 \cdot 2} = \frac{12\sqrt{2}}{6} = 2\sqrt{2} \)

**Rationalizing Binomial Denominators**

If the denominator of the fraction is a binomial, multiply the fraction by a Fancy Form of 1 of the CONJUGATE of the radical that is in the bottom of the fraction. Simplify as normally required.

The conjugate of a binomial is the result of reversing the sign between the two terms.

Example: \( (a - b) \) and \( (a + b) \) are conjugates.

Example: \( \frac{12}{3-\sqrt{2}} \cdot \frac{3+\sqrt{2}}{3+\sqrt{2}} = \frac{12(3+\sqrt{2})}{(3-\sqrt{2})(3+\sqrt{2})} = \frac{36+12\sqrt{2}}{9+3\sqrt{2}-3\sqrt{2}-4} = \frac{36+12\sqrt{2}}{5} \)

Exit Ticket:

The expression \( \frac{2}{\sqrt{3} + 1} \) is equivalent to

1) \( \frac{\sqrt{3}}{2} \)

2) \( \frac{2\sqrt{3} + 2}{4} \)

3) \( \sqrt{3} - 1 \)

4) \( 1 - \sqrt{3} \)
Day 4: SWBAT: Solve Radical Equations

Warm - Up:

1) Rationalize: \( \frac{6}{\sqrt{6}} \)  
2) Rationalize: \( \frac{5}{4 + \sqrt{3}} \)

Solving Radical Equations

- Isolate the radical so that it is the only term on one side of the equation.
- If the radical is a square root, square both sides of the equation. Use PARENTHESES!
- Solve the derived equation. Perhaps the result will be linear, or it might be quadratic. Solve as necessary
- CHECK your answers

Example

\[ 10 + 2\sqrt{x - 2} = 20 \]
\[ -10 = -10 \]
\[ 2\sqrt{x - 2} = 10 \]
\[ \sqrt{x - 2} = 5 \]
\[ (\sqrt{x - 2})^2 = (5)^2 \]
\[ x - 2 = 25 \]
\[ x = 27 \]

Example 1: \( \sqrt{2x - 3} = 5 \)

Example 2: \( \sqrt{x - 2} - 7 = 2 \)
<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>a) $\sqrt{6} - x = \sqrt{2} - 2x$</td>
<td>b) $\sqrt{20} - 2x = \sqrt{5x} - 8$</td>
</tr>
<tr>
<td>c) $\frac{3}{4} - 2\bar{a} = -2$</td>
<td>d) $2 + \frac{3}{\sqrt{3x}} - 2 = 6$</td>
</tr>
<tr>
<td>e) $x = 1 + \sqrt{15} - 7x$</td>
<td>f) $x = -3 + \sqrt{3x + 7}$</td>
</tr>
</tbody>
</table>
Challenge:
If \( x + \frac{1}{x} = 30 \), then \( x^2 + \frac{1}{x^2} \) is equal to what number?

Summary:
**Solving Radical Equations**
- Isolate the radical so that it is the only term on one side of the equation.
- If the radical is a square root, square both sides of the equation. Use PARENTHESES! 
- Solve the derived equation. Perhaps the result will be linear, or it might be quadratic. Solve as necessary.
- CHECK your answers

<table>
<thead>
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<tbody>
<tr>
<td>( 10 + 2\sqrt{x-2} = 20 )</td>
</tr>
<tr>
<td>(-10)</td>
</tr>
<tr>
<td>( 2\sqrt{x-2} = 10 )</td>
</tr>
<tr>
<td>( \sqrt{x-2} = 5 )</td>
</tr>
<tr>
<td>( (\sqrt{x-2})^2 = (5)^2 )</td>
</tr>
<tr>
<td>( x - 2 = 25 )</td>
</tr>
<tr>
<td>( x = 27 )</td>
</tr>
</tbody>
</table>

Exit Ticket:
Solve the equation for \( x \).
\[
\sqrt{22 - 3x - 2} = x
\]

A) \( \{2\} \)  
B) \( \{-2, -9\} \)  
C) \( \{-2, 9\} \)  
D) \( \{-9\} \)
HOMEWORK
ANSWERS
Answer Key

3-3 Simplifying Radicals (pages 93–94)

Writing About Mathematics
1. $-\sqrt{36}$ is the negative of the square root of 36, which is a real number, simplifying to $-6$. $\sqrt{-3i}$ is the square root of a negative number and is not real.
2. Negative. If $a$ is negative, $-\sqrt{a^2}$ will be positive and its cube root will be also positive. The negative sign in front makes the whole expression negative.

Developing Skills

<table>
<thead>
<tr>
<th>Expression</th>
<th>Answer</th>
</tr>
</thead>
<tbody>
<tr>
<td>$2\sqrt{3}$</td>
<td>4. $5\sqrt{2}$</td>
</tr>
<tr>
<td>$4\sqrt{2}$</td>
<td>6. $2\sqrt{2b}$</td>
</tr>
<tr>
<td>$7c^2\sqrt{2}$</td>
<td>8. $6y^2\sqrt{5y}$</td>
</tr>
<tr>
<td>$50y\sqrt{2x}$</td>
<td>10. $44x^3y^3\sqrt{3xy}$</td>
</tr>
<tr>
<td>$3b^2\sqrt{2ab}$</td>
<td>12. $2\sqrt{2}$</td>
</tr>
<tr>
<td>$2\sqrt[3]{3}$</td>
<td>14. $2a\sqrt[3]{5a}$</td>
</tr>
<tr>
<td>$5x^2\sqrt[3]{3x^5}$</td>
<td>16. $2a^2\sqrt[3]{ab^3}$</td>
</tr>
<tr>
<td>$2\sqrt[3]{x}$</td>
<td>18. $\frac{b}{5}\sqrt[3]{5}$</td>
</tr>
<tr>
<td>$\sqrt[3]{y}$</td>
<td>20. $\frac{c}{5}\sqrt[3]{a}$</td>
</tr>
<tr>
<td>$2.5\sqrt[3]{5ab}$</td>
<td>22. $\sqrt{\frac{5x^3}{3x}}$</td>
</tr>
<tr>
<td>$\sqrt[3]{3x^2y^2}$</td>
<td>24. $\sqrt[6]{\frac{4b}{5}}$</td>
</tr>
<tr>
<td>$\sqrt[3]{10}$</td>
<td>26. $\sqrt[3]{\frac{3}{2}}$</td>
</tr>
<tr>
<td>$\frac{3}{2}\sqrt[3]{5ab}$</td>
<td>28. $\frac{\sqrt[3]{100xy}}{2y}$</td>
</tr>
<tr>
<td>$\sqrt[3]{10x^2}$</td>
<td>30. $\sqrt[3]{5}$</td>
</tr>
<tr>
<td>$\frac{3}{2}\sqrt[3]{10x}$</td>
<td>32. $\frac{11b\sqrt[3]{5ab}}{10}$</td>
</tr>
<tr>
<td>$10\sqrt[3]{5}$</td>
<td>34. $8x\sqrt[3]{2}$</td>
</tr>
<tr>
<td>$\frac{8\sqrt[3]{2}}{bc}$</td>
<td>36. $\frac{3\sqrt[3]{2}}{10}$</td>
</tr>
<tr>
<td>$2xy\sqrt[3]{2x}$</td>
<td>38. $2xy\sqrt[3]{2x}$</td>
</tr>
</tbody>
</table>

Applying Skills

<table>
<thead>
<tr>
<th>Expression</th>
<th>Answer</th>
</tr>
</thead>
<tbody>
<tr>
<td>$4\sqrt{13}$ cm</td>
<td>40. $6\sqrt{5}$ in.</td>
</tr>
<tr>
<td>$12\sqrt{3}$ m</td>
<td>42. $5\sqrt{13}$ ft</td>
</tr>
<tr>
<td>$5\sqrt{6}$ ft</td>
<td>44. $xy^2\sqrt{5}$ m</td>
</tr>
<tr>
<td>$6b^2$ units</td>
<td>46. The longest diagonal of the trunk is $\sqrt{1604}$ or approximately 40.05 inches. Thus, everything but the walking stick will fit.</td>
</tr>
</tbody>
</table>

Page 97

3-4 Adding and Subtracting Radicals (pages 97–98)

Writing about Mathematics
1. Yes, for $x > 0$, $(3x)^2 = 9x^2$, and $\sqrt{9x^2} = 3x$. Her substitution is correct.
2. No. We do not add radicands. $\sqrt{16} + \sqrt{48} = 4 + 4\sqrt{3}$, which is not equal to $\sqrt{64} = 8$.

Developing Skills

<table>
<thead>
<tr>
<th>Expression</th>
<th>Answer</th>
</tr>
</thead>
<tbody>
<tr>
<td>$6\sqrt{2}$</td>
<td>4. $2\sqrt{5}$</td>
</tr>
<tr>
<td>$9\sqrt{3}$</td>
<td>6. $4\sqrt{7}$</td>
</tr>
<tr>
<td>$6\sqrt{2}$</td>
<td>8. $\sqrt{5y}$</td>
</tr>
<tr>
<td>$6a\sqrt{10}$</td>
<td>10. $5b\sqrt{11}$</td>
</tr>
<tr>
<td>$5y\sqrt{6x}$</td>
<td>12. $5a^2\sqrt{2a}$</td>
</tr>
<tr>
<td>$4x^2\sqrt{2y}$</td>
<td>14. $12x\sqrt{2x}$</td>
</tr>
<tr>
<td>$11b\sqrt{5b}$</td>
<td>16. $22x^3\sqrt{5}$</td>
</tr>
<tr>
<td>$6\sqrt{5}$</td>
<td>18. $3\sqrt{6}$</td>
</tr>
<tr>
<td>$4\sqrt{7}$</td>
<td>20. $\sqrt{6x}$</td>
</tr>
<tr>
<td>$5a\sqrt{5} - 5\sqrt{2a}$</td>
<td>22. $22x\sqrt{6}$</td>
</tr>
<tr>
<td>$6\sqrt{3y} + y^2$</td>
<td>24. $6b\sqrt{2b} + 2$</td>
</tr>
<tr>
<td>$9\sqrt{3} - \sqrt{6}$</td>
<td>26. $3\sqrt{3} - \frac{\sqrt{10}}{10}$</td>
</tr>
<tr>
<td>$\frac{1}{2}\sqrt{6}$</td>
<td>28. $3\sqrt{2}$</td>
</tr>
<tr>
<td>$7\sqrt{2}$</td>
<td>30. $\sqrt{3}$</td>
</tr>
<tr>
<td>$8\sqrt{x}$</td>
<td>32. $5\sqrt{y}$</td>
</tr>
<tr>
<td>$\sqrt{2a}$</td>
<td>34. $2ab\sqrt{2}$</td>
</tr>
<tr>
<td>$3a\sqrt{3} - 3a\sqrt{3}$ or $3a(\sqrt{3} - \sqrt{3})$</td>
<td></td>
</tr>
<tr>
<td>$b\sqrt{a}$</td>
<td>36. $15x\sqrt{2x}$</td>
</tr>
<tr>
<td>$3x\sqrt{x}$</td>
<td>38. $\frac{b}{2}$</td>
</tr>
<tr>
<td>$\sqrt{3}$</td>
<td>40. $\sqrt{3}$</td>
</tr>
<tr>
<td>$\frac{2}{3}$</td>
<td>41. $\sqrt{b}$</td>
</tr>
</tbody>
</table>

Applying Skills

<table>
<thead>
<tr>
<th>Expression</th>
<th>Answer</th>
</tr>
</thead>
<tbody>
<tr>
<td>$14\sqrt{3}$ in.</td>
<td></td>
</tr>
</tbody>
</table>

17
3-5 Multiplying Radicals (pages 100–101)

Writing About Mathematics

1. Yes. $\sqrt{2}$ is a positive real number.
2. Yes. We can simplify by dividing the exponent of the radicand by the index.

Developing Skills

3. $4$  
4. $15$
5. $9$  
6. $4\sqrt{6}$
7. $-6\sqrt{5}$  
8. $6\sqrt{5}$
9. $2\sqrt{2}$  
10. $2\sqrt{7}$
11. $4\sqrt{10}$  
12. $12$
13. $27$  
14. $20$
15. $2x^2$  
16. $4ab\sqrt{5}$
17. $2y^2\sqrt{5}$  
18. $x^2y^2\sqrt{3}$
19. $\sqrt[3]{a}$  
20. $\frac{1}{3}\sqrt[3]{x}$
21. $\frac{2}{3}\sqrt{5a}$  
22. $2$
23. $3a^2\sqrt{5}$  
24. $3$
25. $2\sqrt{2} + 2$  
26. $\sqrt{5} - 5\sqrt{2}$
27. $12\sqrt{2} + 4$  
28. $5a - 3\sqrt{5a}$
29. $6xy^2 + 6y\sqrt{3xy}$  
30. $-2 + 2\sqrt{5}$
31. $9 + 10\sqrt{2b} + 2b$  
32. $21 - 4\sqrt{5}y - 5y$
33. $49 - 5b$  
34. $2x^3 - 3x\sqrt{3}y + \sqrt{3}y$
35. $-36 - \sqrt{6}$  
36. $6 - 36c^2$
37. $a^2 - b$  
38. $4 - 2\sqrt{3}$
39. $9 + 6b\sqrt{5ab} + 5ab^3$  
40. $-6 - 6\sqrt{7}$
41. $-2 - \sqrt{5}$

Applying Skills

42. $4,608\text{ m}^2$  
43. $120\text{ ft}^2$
44. $\sqrt{3}\text{ in.}^3$
45. $a. 2\sqrt{6}\text{ ft} \quad b. 6 + 2\sqrt{6}\text{ ft} \quad c. 3\text{ ft}^2$
46. $\pi(4 + xy^2 + 4y^2\sqrt{xy})\text{ m}^2$
3.7 Rationalizing a Denominator

**Writing About Mathematics**

1. a. If Juan writes \( \sqrt{49} \) as \( \frac{\sqrt{49}}{2} \), the fraction becomes
   \( \frac{\sqrt{49}}{2} \), which simplifies to \( \frac{7}{2} \).
   b. No, Juan’s procedure cannot be applied to \( \frac{2}{2\sqrt{3}} \)
      because 5 is not a factor of 49.
2. Brittany took the fraction at face value and
   multiplied by the conjugate of the denominator.
   Justin saw that the denominator factored to
   \( 2(1 + \sqrt{3}) \) is a factor of the numerator, so the
   fraction is equivalent to \( \frac{2}{1 + \sqrt{3}} \).

**Developing Skills**

3. \( \frac{\sqrt{5}}{3} \)
4. \( \frac{2}{2\sqrt{2}} \)
5. \( \sqrt{5} \)
6. \( \frac{\sqrt{2}}{2} \)
7. \( \sqrt{3} \)
8. \( \frac{2}{3} \)
9. \( \frac{\sqrt{2}}{3} \)
10. \( \sqrt{3} \)
11. \( \frac{\sqrt{2}}{6} \)
12. \( \frac{\sqrt{6}}{6} \)
13. \( \frac{\sqrt{10}}{5} \)
14. \( \frac{1}{2} \)
15. \( \frac{3 - \sqrt{2}}{4} \)
16. \( \frac{5 + \sqrt{2}}{23} \)
17. \( \frac{\sqrt{2} - 2}{2} \)
18. \( \frac{5 + 3\sqrt{2}}{4} \)
19. \( \sqrt{5} - 3 \)
20. \( \frac{16 - 4\sqrt{3}}{3} \)
21. \( 3\sqrt{5} + 6 \)
22. \( 3\sqrt{5} - 6 \)
23. \( 2\sqrt{5} + 1 \)
24. \( 3 - \sqrt{5} \)
25. \( \frac{2a + 2\sqrt{2}}{3x - 4} \)
26. \( \frac{10\sqrt{3} - 2\sqrt{2}}{5x^2 - 1} \)
27. \( \frac{5 + 3\sqrt{2}}{2\sqrt{7}} \)
28. \( \frac{11 - 2\sqrt{2}}{9} \)
29. \( \frac{27 + 4\sqrt{3}}{22} \)
30. \( \frac{2a + 2\sqrt{2}}{a - 4} \)
31. \( \frac{5a - 2\sqrt{2}}{a - b} \)
32. \( \frac{2\sqrt{2} - 20}{\sqrt{2} - 4} \)
33. \( \frac{4\sqrt{3} - 22}{\sqrt{2} - 4} \)
34. \( \frac{2\sqrt{2} + 12}{\sqrt{2} - 4} \)
35. \( \frac{2\sqrt{3} + 2\sqrt{5}}{\sqrt{2}} \)
36. \( \frac{3\sqrt{2} + (3a - 6)x\sqrt{3} - 18}{3x - 10} \)
37. \( \frac{2a + 2b + 2\sqrt{2}}{a + b} \)
38. \( \frac{2a^2 - 4x\sqrt{2} + (3a^2 - 10)\sqrt{2}}{a + 2\sqrt{3}} \)
39. \( \frac{\sqrt{2}}{a - 4} \)
40. \( \frac{\sqrt{6}}{a - 4} \)
41. \( \frac{\sqrt{2}}{a - 4} \)
42. \( \frac{\sqrt{3}}{a - 4} \)
43. \( \sqrt{2} \)
44. \( \sqrt{3} \)
45. \( \sqrt{3} - 1 \)
46. \( \sqrt{3} - \sqrt{3} \)

3.8 Solving Radical Equations

**Writing About Mathematics**

1. There is no real number for which the square root
   is negative. If \( x = -3 \) the radicand will not
   be negative, so there will be a solution in the set
   of real numbers.
2. No. Once we square the equation it has two real
   roots. One is the root of the given equation and
   the other is the root of \( \sqrt{15 - 7x} = 1 - x \).

**Developing Skills**

3. 25
4. 49
5. 9
6. 36
7. 16
8. 32
9. 4
10. 8
11. 4
12. 44
13. 4
14. 2
15. 2
16. 25
17. 4
18. 4
19. 5
20. 2
21. -1
22. 5
23. 5
24. \{ \} (no solution)
25. 5
26. 3
27. 1
28. 3
29. \{ 1, 2 \}
30. 8
31. -10
32. 22
33. -34
34. 18
35. 4
36. \{ 0, 8 \}
37. 5
38. 5

**Applying Skills**

39. 8 units each
40. Width = 2, length = 1
41. a. \( AB = BC = 2\sqrt{10}, AC = \sqrt{10} \)
b. \( 5\sqrt{10} \)
Review Exercises (pages 114–116)

In 1–8, answers will be graphs of number lines.

1. \(-\frac{1}{2} < x < \frac{3}{4}\) 
2. \(-6.5 \leq x \leq 2.5\)
3. \(-10.5 \leq x \leq 11.5\) 
4. \(x < -2 \text{ or } x > 4.5\)
5. \(x \leq \frac{-3}{5} \text{ or } x \geq 2\frac{1}{2}\) 
6. \(x > 1 \text{ or } x < -13\)
7. \(-1 \leq x \leq 2\frac{1}{2}\) 
8. \(0 < x < 2\)
9. \(8\sqrt{2}\) 
10. \(\sqrt{10}\)
11. \(15\sqrt{3}\) 
12. \(\sqrt{3}\)
13. \(21\sqrt{3}\) 
14. \(4\sqrt{2}\)
15. \(5\sqrt{6} + 2\sqrt{3}\) 
16. \(12\)
17. \(7\sqrt{5}\) 
18. \(4 + 3\sqrt{2}\)

19. \(-20 - 20\sqrt{2}\) 
20. \(6\sqrt{2} + 60\)
21. \(-1\) 
22. \(22\)
23. \(5 + 3\sqrt{3}\) 
24. \(\sqrt{2}\)
25. \(2\sqrt{5} + 1\) 
26. \(4\sqrt{2} - 2\)
27. \(\frac{4\sqrt{7}}{13} + \frac{1}{3}\) 
28. \(8\sqrt{7}\)
29. \(7b\sqrt{2}b\) 
30. \(x^\sqrt{12x}\)
31. \(\sqrt{ab}\) 
32. \(12x^2\sqrt{x}\)
33. \(4x - x\sqrt{50}\) 
34. \(x^3y^2\sqrt{y}\)
35. \(3a^3\sqrt{2}\) 
36. \(4a^2\)
37. \(b^2\sqrt{5}\) 
38. \(\frac{2 + \sqrt{x}}{y - x}\)
39. \(\frac{16 + a - 8\sqrt{a}}{16 - a}\) 
40. \(a\)
41. \(x^2\) 
42. \(13\)
43. \(5\) 
44. \(18\)
45. \(14\) 
46. \([3, 4]\)
47. \(5\) 
48. \(-5\)
49. \([-2, 3]\) 
50. \(12 + 7\sqrt{3} \text{ ft}\)

51. a. \(4 \text{ m}\) 
   b. \(4 + 4\sqrt{2} \text{ m}\)

52. Show that these values satisfy the Pythagorean Theorem:

\[
x^2 + (\sqrt{2x - 1})^2 = (x + 1)^2
\]

\[
x^2 + 2x + 1 = x^2 + 2x + 1
\]

True for all values of \(x > 0\) because of the radical.

53. a. \(24 - 12\sqrt{2} \text{ ft}\) 
   b. \(52 - 22\sqrt{3} \text{ ft}\)