Dilations and Symmetry - Day #4

Warm Up

What are the coordinates of the image of \((2, -5)\) after a counterclockwise rotation of 90º about the origin?

1) \((-2, 5)\)
2) \((2, 5)\)
3) \((-5, -2)\)
4) \((5, 2)\)

Dilations

- A _______ is used to create an image that is larger or smaller than the original.
- If the new image is larger, it is called an ________.
- If the new image is smaller than the original, it is a ________.
- A dilation always creates ________.
- The ________ of a dilation determines its size.
- If the scale factor is ___ one, the dilation is an enlargement.
- If the scale factor is ___ one, the dilation will be a reduction.
- During dilations, there are some properties that remain the same throughout.
  1.
  2.
  3.
  4.
  5.
- ______ ___ is never kept in dilations unless the scale factor is one
Practice Problems for Dilations

1. What are the coordinates of the point (1,5) after being dilated by \( D_{-2} \)

2. Find the dilation image of the point (-8,12) after being dilated by \( D_{\frac{1}{2}} \)

3. In which quadrant would the image of point \((5,-3)\) fall after a dilation using a factor of \(-3\)?
   A) I  
   B) II  
   C) III  
   D) IV

4. When a dilation is performed on a hexagon, which property of the hexagon will not be preserved in its image?
   (1) parallelism  
   (2) orientation  
   (3) length of sides  
   (4) measure of angles

5. Using a drawing program, a computer graphics designer constructs a circle on a coordinate plane on her computer screen. She determines that the equation of the circle’s graph is \((x - 3)^2 + (y + 2)^2 = 36\). She then dilates the circle with the transformation \( D_{3} \). After this transformation, what is the center of the new circle?
   1) \((6,-5)\)
   2) \((-6,5)\)
   3) \((9,-6)\)
   4) \((-9,6)\)
Symmetry

- ______ symmetry, or ________ symmetry, is if the figure can be reflected onto itself by a line.
- ______ symmetry, or ________ symmetry is if the figure can be mapped onto itself by a rotation that is greater than _____° and less than _____°.
- The number of times a figure can map onto itself is the ______ of symmetry.
- The _____________ of symmetry is the smallest angle that a figure can rotate to map onto itself.
- To calculate the magnitude of symmetry: __________ (n is number of sides)

Practice Problems for Symmetry

1. State whether the following figure has reflectional symmetry, rotational symmetry, both kinds of symmetry, or neither kind of symmetry.

2. A line of symmetry for the figure is shown. Find the coordinates of point A.

3. Regular hexagon HEXAGO is divided into six congruent triangles.
   A Name the image of $E$ under a $60^\circ$ rotation about $N$.
   B Name the image of $X$ under a $180^\circ$ rotation about $N$.
   C Name the image of $O$ under a $120^\circ$ rotation about $N$.
   D Name the image of $A$ under a $240^\circ$ rotation about $N$.
   E Name the image of $H$ under a $300^\circ$ rotation about $N$.
   F Name the image of $G$ under a $360^\circ$ rotation about $N$. 
Summary

**Line Symmetry (Reflection Symmetry)** - divides the figure into two congruent halves.

**Angle of Rotational Symmetry** - The smallest angle through which the figure is rotated to coincide with itself.

**Order of Rotational Symmetry** - the number of times that you can get an identical figure when repeating the degree of rotation.

**Order of Symmetry** - number of times a figure maps onto itself as it rotates from 0° to 360°.

**Properties preserved under dilation:**
1. Angle measures *(remain the same)*
2. Colinearity *(points stay on the same lines)*
3. Midpoint *(midpoints remain the same in each)*
4. Parallelism *(parallel lines remain parallel)*
5. Orientation *(lettering order remains the same)*
6. Isometry is **NOT** preserved *(lengths of segments are **NOT** the same in all cases except scale factor or 1.)*

**Dilation** is used to create an image that is larger or smaller than the original.

- **Smaller image** - reduction has occurred with a dilation with 0 < k < 1
- **Larger image** - Enlargement is a dilation with k > 1
- **Scale Factor** - determine its size K represents the scale factor D_k (x,y) = (kx,ky)
Challenge

The rectangular prism shown is enlarged by dilation with scale factor 4. Find the surface area and volume of the image.

Exit Ticket

1) The image of (-2,6) after a dilation with respect to the origin is (-10,30). What is the constant of the dilation?

2) Which special quadrilaterals have both rotational and line symmetry?

3) The image of point A after a dilation of 3 is (6, 15). What was the original location of point A?
Part I- Foundations of Dilation

Fill in the blanks:

Enlargement- __________________________________________________________
Reduction- __________________________________________________________

Written Equation (Write it): _____________________________________________

What are the five properties that are preserved with under a dilation?

1. _____________________________________________________________________
2. _____________________________________________________________________
3. _____________________________________________________________________
4. _____________________________________________________________________
5. _____________________________________________________________________

What is never preserved under a dilation?

6. _____________________________________________________________________

Part II- Practice Problems

In 7–10, find the coordinates of the image of each given point under $D_5$.

7. (2, 2)  8. (1, 10)
9. (−3, 5)  10. (0, 4)
15 On the accompanying set of axes, graph $\triangle ABC$ with coordinates $A(-1, 2)$, $B(0, 6)$, and $C(5, 4)$. Then graph $\triangle A'B'C'$, the image of $\triangle ABC$ after a dilation of 2.

16 On the accompanying grid, graph and label quadrilateral $ABCD$, whose coordinates are $A(-1, 3)$, $B(2, 0)$, $C(2, -1)$, and $D(-3, -1)$. Graph, label, and state the coordinates of $A'B'C'D'$, the image of $ABCD$ under a dilation of where the center of dilation is the origin.
Part I - Foundations of Symmetry

Fill in the blanks:

Line Symmetry:

______________________________________________________________________________

______________________________________________________________________________

Rotation Symmetry:

______________________________________________________________________________

______________________________________________________________________________

Part II - Practice Problems

Draw lines of symmetry for each figure:

1. 

2. 

3. 

4. 
5. For each figure, state if it has rotational symmetry and estimate the angle of rotation.

6. Regular octagon \( EIGHTSUP \) is divided into eight congruent triangles. Find the image of each point or segment.

A. 45° rotation of \( G \) about \( Z \)
B. 225° rotation of \( U \) about \( Z \)
C. 315° rotation of \( E \) about \( Z \)
D. 270° rotation of \( EI \) about \( Z \)
E. 135° rotation of \( S \) about \( Z \)
F. 360° rotation of \( ST \) about \( Z \)
Part I - Foundations of Dilation

Fill in the blanks:

Enlargement - A dilation with \( k > 1 \), larger than original

Reduction - A dilation with \( k < 1 \), smaller than original

Written Equation (Write it):

\[
D_k(x, y) = (kx, ky)
\]

What are the five properties that are preserved with under a dilation?

1. **Measures**
2. **Parallel lines**
3. **Collinearity**
4. **Midpoints**
5. **Congruence**

What is **never** preserved under a dilation?

6. **Distance**

Part II - Practice Problems

In 7–10, find the coordinates of the image of each given point under \( D_3 \):

7. \((2, 2)\) \(\rightarrow (0, 10)\)
8. \((1, 10)\) \(\rightarrow (5, 50)\)
9. \((-3, 5)\) \(\rightarrow (-15, 7.5)\)
10. \((0, 4)\) \(\rightarrow (0, 3.5)\)
15 On the accompanying set of axes, graph \( \triangle ABC \) with coordinates \( A(-1.2), B(0,6), \) and \( C(5,4). \) Then graph \( \triangle A'B'C' \), the image of \( \triangle ABC \) after a dilation of 2.

\[
\begin{align*}
A \ (-1,2) & \rightarrow ( -2, 4) \\
B \ (0,6) & \rightarrow (0,12) \\
C \ (5,4) & \rightarrow (10,8)
\end{align*}
\]

16 On the accompanying grid, graph and label quadrilateral \( ABCD \) whose coordinates are \( A(-1.3), B(2.0), C(2,-1), \) and \( D(-3,-1). \) Graph, label, and state the coordinates of \( A'B'C'D' \), the image of \( ABCD \) under a dilation of \( \sqrt{2} \) where the center of dilation is the origin.

\[
\begin{align*}
A \ (-1.3) & \rightarrow (-0.5, 4.5) \\
B \ (2,0) & \rightarrow (1, 0) \\
C \ (2,-1) & \rightarrow (1, -0.5) \\
D \ (-3,-1) & \rightarrow (-1.5, -0.5)
\end{align*}
\]
Part I- Foundations of Symmetry

Fill in the blanks:

Line Symmetry - The figure can be mapped onto itself by a reflection in the line of symmetry.

Rotation Symmetry - The figure can be mapped onto itself by a rotation between $0^\circ$ and $360^\circ$ about the center of symmetry.

Part II- Practice Problems

Draw lines of symmetry for each figure:

1. 

2. 

3. 

4.
5. For each figure, state if it has rotational symmetry and estimate the angle of rotation.

Yes, 120°  Yes 180°  Yes, 120°  Yes, 90°

6. Regular octagon EIGHTSUP is divided into eight congruent triangles. Find the image of each point or segment.

A. 45° rotation of G about Z
B. 225° rotation of U about Z
C. 315° rotation of E about Z
D. 270° rotation of EH about Z
E. 135° rotation of S about Z
F. 360° rotation of ST about Z