GEOMETRY
COORDINATE GEOMETRY
Proofs

'I think you should be more explicit here in step two.'

Name ________________________________

Period ______________________________
Coordinate Geometry Proofs

**Slope:** We use slope to show parallel lines and perpendicular lines.

**Parallel Lines have the same slope**

If \( \overrightarrow{AB} \) is parallel to \( \overrightarrow{CD} \), the slope of \( \overrightarrow{AB} = m_1 \), and the slope of \( \overrightarrow{CD} = m_2 \), then:

\[
m_1 = m_2
\]

**Perpendicular Lines have slopes that are negative reciprocals of each other.**

If \( \overrightarrow{AB} \) is perpendicular to \( \overrightarrow{CD} \), the slope of \( \overrightarrow{AB} = m_1 \), and the slope of \( \overrightarrow{CD} = m_2 \), then:

\[
m_1 = -\frac{1}{m_2} \quad \text{or} \quad m_2 = -\frac{1}{m_1} \quad \text{or} \quad m_1 \cdot m_2 = -1
\]

\[
\text{slope} = m = \frac{\Delta y}{\Delta x}
\]

\[
m = \frac{y_2 - y_1}{x_2 - x_1} \quad \text{or} \quad m = \frac{y_1 - y_2}{x_1 - x_2}
\]

**Midpoint:** We use midpoint to show that lines bisect each other.

**Lines With the same midpoint bisect each other**

**Midpoint Formula:**

\[
\text{mid} = \left( \frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)
\]

**Distance:** We use distance to show line segments are equal.

You can use the Pythagorean Theorem or the formula:

\[
d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}
\]
Proving a triangle is a right triangle

**Method:** Show two sides of the triangle are perpendicular by demonstrating their slopes are opposite reciprocals.

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**Example 1: Using Geometer’s Sketchpad**

*Given:* The triangle with vertices $A(4, -1), B(5, 6),$ and $C(1, 3)$.

*Show:* ΔABC is an isosceles right triangle.

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**Question:**

**Formula:**

**Work:**

**Statement:**
**Practice 1:** The vertices of triangle JEN are J(2,10), E(6,4), and N(12,8). Use coordinate geometry to prove that Jen is an isosceles right triangle.

**Practice 2:**
Prove that the polygon with coordinates A(1, 1), B(4, 5), and C(4, 1) is a right triangle.
Practice 3: “Using Geometer’s Sketchpad”
Show that the triangles with the following vertices are isosceles.
(1, 0), (5, 0), (3, 4)

SUMMARY
Homework

1. Prove the polygon with the given coordinates is a right triangle.
   \((-1, 0), (6, 1), (2, 4)\)

2. Show that the triangles with the following vertices are isosceles.
   \((2, 3), (5, 7), (1, 4)\)
3. Show that the line segments joining the points \((-1, 3), (9, 3),\) and \((4, 8)\) form an isosceles right triangle.
Proving a Quadrilateral is a Parallelogram

**Method 1:** Show that the diagonals bisect each other by showing the midpoints of the diagonals are the same

**Method 2:** Show both pairs of opposite sides are parallel by showing they have equal slopes.

**Method 3:** Show both pairs of opposite sides are equal by using distance.

**Method 4:** Show one pair of sides is both parallel and equal.

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**Example Model Problem**
Prove that the quadrilateral with the coordinates P(0, 2), Q(4, 8), R(7, 6) and S(3, 0) is a parallelogram.

**Question:**

**Formula:**

**Work:**

**Statement:**
Practice

1. Prove that the quadrilateral with the coordinates $L(-2,3)$, $M(4,3)$, $N(2,-2)$ and $O(-4,-2)$ is a parallelogram.

![Coordinate Plane](image1.png)

2. Prove that the quadrilateral with the coordinates $P(1,1)$, $Q(2,4)$, $R(5,6)$ and $S(4,3)$ is a parallelogram.

![Coordinate Plane](image2.png)
3. Prove that the quadrilateral with the coordinates R(3,2), S(6,2), T(0,-2) and U(-3,-2) is a parallelogram.

Summary
Homework

Prove that quadrilateral SAND with the vertices S(-2,-2), A(-1,2), N(5,3) and D(4,-1) is a parallelogram.

Prove that quadrilateral LEAP with the vertices L(-3,1), E(2,6), A(9,5) and P(4,0) is a parallelogram.
Proving a Quadrilateral is a Rectangle

Prove that it is a parallelogram first, then:

**Method:** Show that the diagonals are congruent.

**Example Model Problem**
1. Prove a quadrilateral with vertices G(0,5), H(6,9), I(8,6) and J(2,2) is a rectangle.

**Question:**

**Formula:**

**Work:**

**Statement:**
Practice
1. The vertices of quadrilateral COAT are C(0,0), O(5,0), A(5,2) and T(0,2). Prove that COAT is a rectangle.
Practice

2 Prove that quadrilateral GHIJ with the vertices \(G(1,1), H(5,3), I(4,5),\) and \(J(0,3)\) is a rectangle.
Homework

Prove that quadrilateral ABCD with the vertices A(2,1), B(1,3), C(-5,0), and D(-4,-2) is a rectangle.

Prove that quadrilateral PLUS with the vertices P(2,1), L(6,3), U(5,5), and S(1,3) is a rectangle.
Proving a Quadrilateral is a Rhombus

**Method:** Prove that all four sides are equal.

**Example Model Problem**
Prove that a quadrilateral with the vertices A(-2,3), B(2,6), C(7,6) and D(3,3) is a rhombus.

**Question:**

**Formula:**

**Work:**

![Graph](image-url)
1. Prove that the quadrilateral with the vertices D(2,1), A(6,-2), V(10,1) and E(6,4) is a rhombus.

2. Prove that quadrilateral ABCD with the vertices A(8,0), B(0,6), C(-8,0), and D(0,-6) is a rhombus.
Homework

1. Prove that quadrilateral GHIJ with the vertices G(-2,2), H(3,4), I(8,2), and J(3,0) is a rhombus.
Prove that quadrilateral TIME with the vertices T(1,1), I(5,3), M(7,7), and E(3,5) is a rhombus.
Proving that a Quadrilateral is a Square

There are many ways to do this. Prove that the quadrilateral is a rectangle and a rhombus.

Example Model Problem
1. Prove that the quadrilateral with vertices A(0,0), B(4,3), C(7,-1) and D(3,-4) is a square.

Statement:
Practice

1. Prove that the quadrilateral with vertices A(2,2), B(5,-2), C(9,1) and D(6,5) is a square.
2. Prove that quadrilateral PQRS with the vertices P(0,0), Q(4,3), R(7,-1), and S(3,-4) is a square.
Homework
Prove that quadrilateral JKLM with the vertices J(2,-1), K(-1,-4), L(-4,-1), and M(-1,2) is a square.

Prove that quadrilateral ABCD with the vertices A(1,3), B(2,0), C(5,1), and D(4,4) is a square.
Proving a Quadrilateral is a Trapezoid

Show one pair of sides are parallel (same slope) and one pair of sides are not parallel (different slopes).

Example Model Problem
Prove that DEFG a trapezoid with coordinates D(-4,0), E(0,1), F(4,-1) and G(-4,-3).

Question:

Formula:

Work:

Statement:
Proving a Quadrilateral is an Isosceles Trapezoid

Prove that it is a trapezoid first, then:

Method 1: Prove the diagonals are congruent using distance.

Method 2: Prove that the pair of non parallel sides are equal.

Example Model Problem
1. Prove that quadrilateral BCDE with the vertices B(0,4), C(3,1), D(3, -5), and E(0,-8) is an isosceles trapezoid.

Question:

Formula:

Work:

Statement:
Prove that quadrilateral ABCD with the vertices A(1,6), B(7,9), C(13,6), and D(3,1) is a trapezoid.

Quadrilateral ABCD has vertices A(-8,2), B(0,6), C(8,0), and D(-8,-8). Prove that quadrilateral ABCD is an isosceles trapezoid.
The vertices of a trapezoid ABCD are A(2, -4), B(11, -4), C(10, 2), and D(3, 2).
Prove that ABCD is an isosceles trapezoid.
Homework

Quadrilateral *KATE* has vertices *K*(1,5), *A*(4,7), *T*(7,3), and *E*(1,–1). Prove that *KATE* is a trapezoid.

Quadrilateral *PQRS* has vertices *P*(–3,–4), *Q*(9,5), *R*(–1,10), and *S*(–5,7). Prove that quadrilateral *PQRS* is an isosceles trapezoid.
Practice with Coordinate Proofs

1. The vertices of \( \triangle ABC \) are \( A(3,-3), B(5,3) \) and \( C(1,1) \). Prove by coordinate geometry that \( \triangle ABC \) is an isosceles right triangle.

2. Prove that quadrilateral PLUS with the vertices P(2,1), L(6,3), U(5,5), and S(1,3) is a parallelogram.
3. *Given:* The quadrilateral with vertices $A(2, 2)$, $B(5, -2)$, $C(9, 1)$, and $D(6, 5)$.

*Show:* $ABCD$ is a rhombus.

4. The vertices of quadrilateral $ABCD$ are $A(-3, -1)$, $B(6, 2)$, $C(5, 5)$ and $D(-4, 2)$. Prove that quadrilateral $ABCD$ is a rectangle.
5. The vertices of quadrilateral ABCD are A(-3,1), B(1,4), C(4,0) and D(0,-3). Prove that quadrilateral ABCD is a square.

6. Quadrilateral METS has vertices M(-5, -2), E(-5,3), T(4,6) and S(7,2). Prove by coordinate geometry that quadrilateral METS is an isosceles trapezoid.