CHAPTER 6

Polygons, Quadrilaterals, and Special Parallelograms

Name:__________________________________________

Teacher:________________________________________

Pd: _______
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Chapter 6 (Section 1) – Day 1

Angles in polygons

A polygon is a closed plane figure formed by three or more segments that intersect only at their endpoints. Each segment that forms a polygon is a side of the polygon. The common endpoint of two sides is a vertex of the polygon. A segment that connects any two nonconsecutive vertices is a diagonal.

![Diagram of a polygon with labeled sides, vertices, and diagonal]

You can name a polygon by the number of its sides. The table shows the names of some common polygons.

<table>
<thead>
<tr>
<th>Number of Sides</th>
<th>Name of Polygon</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>Triangle</td>
</tr>
<tr>
<td>4</td>
<td>Quadrilateral</td>
</tr>
<tr>
<td>5</td>
<td>Pentagon</td>
</tr>
<tr>
<td>6</td>
<td>Hexagon</td>
</tr>
<tr>
<td>7</td>
<td>Heptagon</td>
</tr>
<tr>
<td>8</td>
<td>Octagon</td>
</tr>
<tr>
<td>9</td>
<td>Nonagon</td>
</tr>
<tr>
<td>10</td>
<td>Decagon</td>
</tr>
<tr>
<td>12</td>
<td>Dodecagon</td>
</tr>
<tr>
<td>n</td>
<td>n-gon</td>
</tr>
</tbody>
</table>

All the sides are congruent in an equilateral polygon. All the angles are congruent in an equiangular polygon. A regular polygon is one that is both equilateral and equiangular. If a polygon is not regular, it is called irregular.

A polygon is concave if any part of a diagonal contains points in the exterior of the polygon. If no diagonal contains points in the exterior, then the polygon is convex. A regular polygon is always convex.

Warm – Up

Tell whether the following polygons are concave or convex and regular or irregular.

1. :     2. :     

3. :     4. :     

Warm – Up
Sum of Interior Angles in Polygons

To find the sum of the interior angle measures of a convex polygon, draw all possible diagonals from one vertex of the polygon. This creates a set of triangles. The sum of the angle measures of all the triangles equals the sum of the angle measures of the polygon.

<table>
<thead>
<tr>
<th>Polygon</th>
<th>Number of Sides</th>
<th>Number of Triangles</th>
<th>Sum of Interior Angle Measures</th>
</tr>
</thead>
<tbody>
<tr>
<td>Triangle</td>
<td>3</td>
<td>1</td>
<td>(1) $180^\circ = 180^\circ$</td>
</tr>
<tr>
<td>Quadrilateral</td>
<td>4</td>
<td>2</td>
<td>(2) $180^\circ = 360^\circ$</td>
</tr>
<tr>
<td>Pentagon</td>
<td>5</td>
<td>3</td>
<td>(3) $180^\circ = 540^\circ$</td>
</tr>
<tr>
<td>Hexagon</td>
<td>6</td>
<td>4</td>
<td>(4) $180^\circ = 720^\circ$</td>
</tr>
<tr>
<td>(n)-gon</td>
<td>(n)</td>
<td>(n - 2)</td>
<td>((n - 2)180^\circ)</td>
</tr>
</tbody>
</table>

In each convex polygon, the number of triangles formed is two less than the number of sides \(n\). So the sum of the angle measures of all these triangles is \((n - 2)180^\circ\).

**Example 1: Calculating the Sum of Interior Angles**

Find the sum of the interior angles of a decagon.

**You Try It!**
Find the sum of the interior angles of a 14-gon.
Example 2: Calculating the number of sides of a polygon
given the sum of the interior angles

The sum of the interior angles of a convex regular polygon measure $1980^\circ$, how many sides does the polygon have?

You Try It!
The sum of the interior angles of a convex regular polygon measure $3240^\circ$, how many sides does the polygon have?
Example 3: Calculating the measure of each of interior Angle of any regular polygon

What is the measure of each interior angle of a regular octagon?

---

You Try It!
What is the measure of each interior angle of a regular 12-gon?

---

You Try It!
How many sides does a regular polygon have if each interior angle measures 160°?
Exterior Angles

Refer to the two polygons below. What do you notice about the exterior angles of any polygon?

Example 4: Calculating the measure of an exterior angle given the number of sides or Vice Versa

Find the measure of each exterior angle of a polygon with 18 sides.

You Try It!
Find the measure of each exterior angle of a polygon with 36 sides.
You Try It!
The measure of an exterior angle of a convex regular polygon is 45°. Find the number of sides of the polygon.

Example 5: Solving Algebraic Problems
Find the value of x.

Find x:
Challenge

In Exercises 1, find each lettered angle measure.

\[ a = \_, b = \_, c = \_, d = \_, e = \_ \]

Summary

Properties and Attributes of Polygons

**Why?** Understanding properties of polygons and their angle sums is fundamental to successful work with quadrilaterals.

**Theorem**
The sum of the interior angle measures of a convex polygon with \( n \) sides

\[ S_i = (n - 2)180 \]

**Theorem**
The sum of the exterior angle measures, one angle at each vertex, of a convex polygon is 360°.

\[ S_e = 360 \text{ (always – no matter what the polygon) } \]

**Theorem**

\[ e = \frac{360}{n} \]

**Theorem**

\[ i = \frac{180(n - 2)}{n} \]

Exit Ticket

A stop sign in the shape of a regular octagon is resting on a brick wall, as shown in the accompanying diagram.

What is the measure of angle \( x \)?
1) 45°
2) 60°
3) 120°
4) 135°
Day 1: Homework

The number of sides of a convex polygon is given. Find the sum of the measures of the interior angles of each polygon.

1) 8  
2) 12

The sum of the measures of the interior angles of a convex polygon is given. Find the number of sides of each polygon.

6) 7020°  
7) 1980°

The number of sides of a regular polygon is given. Find the measure of each interior angle of each polygon.

11) 7  
12) 9

Find the exact measure of each exterior angle of the regular polygon.

19) 18-gon  
20) 20-gon
27) In quadrilateral ABCD the measures of \( \angle A, \angle B, \angle C, \) and \( \angle D \) are the ratio of 1:2:3:4, respectively. Find the measures of the four angles.

Find the value of \( x \).

28) \[ \begin{array}{ccc}
75^\circ & \text{ } & \\
97^\circ & \text{ } & 105^\circ \\
\end{array} \]

29) \[ \begin{array}{ccc}
139^\circ & \text{ } & 13^\circ \\
5x^\circ & \text{ } & 9x^\circ \\
92^\circ & \text{ } & 71^\circ \\
\end{array} \]

30) \[ \begin{array}{ccc}
138^\circ & \text{ } & 18x^\circ \\
167^\circ & \text{ } & 115^\circ \\
133^\circ & \text{ } & 151^\circ \\
14x^\circ & \text{ } & 120^\circ \\
\end{array} \]
34) **Light Fixture** The side view of a light fixture is shown below. Find the value of $x$. Then determine the measure of each angle.

```
   C
  / |
 /  |
B  (10x - 1)°
 /  |
 (6x + 5)°  (6x + 5)°
   |
   A
```

35) **Tent** The front view of a camping tent is shown below. Find the value of $x$. Then determine the measure of each angle.

```
   J
 /      |
(9x - 14)°      (9x + 1)°
 /      |
H  (9x + 1)°  (9x + 1)°
 /      |
G 10x°
 /      |
F 5x°  10x°
 /      |
   M
```

10
Chapter 6 (Section 2) – Day 2
Parallelograms

Warm-Up

The measures of five of the interior angles of a hexagon are 150°, 100°, 80°, 165°, and 150°. What is the measure of the sixth interior angle?
1) 75° 2) 80° 3) 105° 4) 180°

Any polygon with four sides is called a Quadrilateral. However, some quadrilaterals have special properties. These special quadrilaterals are given their own names.

A quadrilateral with two pairs of parallel sides is a parallelogram. To write the name of a parallelogram, you use the symbol \( \square \).

Given: \( \square ABCD \)
Given: $\square ABCD$

Identify all angles that are congruent.

Properties of Parallelograms
- If a quadrilateral is a parallelogram, then its **opposite angles** are congruent.

- If a quadrilateral is a parallelogram, then its **consecutive angles are supplementary**.

Example 1:
If $m\angle A = 5x$ and $m\angle B = 25x$ solve for $x$ and the following angle measures.

$$x = \text{_____} \quad m\angle A = \text{______} \quad m\angle B = \text{______} \quad m\angle C = \text{______} \quad m\angle D = \text{______}$$
Example 2:
Given: $\square ABCD$, $\angle BAC = 25^\circ$ and $\angle D = 135^\circ$. Find the measure of all the other angles.

$\angle ACD = \underline{\quad}^\circ$
$\angle B = \underline{\quad}^\circ$
$\angle BCA = \underline{\quad}^\circ$
$\angle CAD = \underline{\quad}^\circ$

- If a quadrilateral is a parallelogram, then its opposite sides are congruent.

Example 3: Solve for $x$ and $y$.
$AB = 4(x + 3)$; $BC = 6 - (2 + y)$
$DC = 12(x - 5)$; $AD = 3y$

$x = \underline{\quad}$
$y = \underline{\quad}$
If a quadrilateral is a parallelogram, then its **diagonals bisect each other**.

**Example 4: Solve for** \(x\) **and** \(y\).

\[
\begin{align*}
BE &= 6y - 22, \\
DE &= 4y - 4, \\
AE &= 3x - 2, \\
AC &= 8x - 20
\end{align*}
\]

\[
\begin{align*}
x &= \phantom{0} \\
y &= \phantom{0}
\end{align*}
\]
You Try It!

TX = 4y, \quad YX = 30
XW = y + 3, \quad XZ = 3y + 12

Solve for x and y.

\[ x = \underline{\quad} \]
\[ y = \underline{\quad} \]

\[ \measuredangle TXW = \frac{12y}{4(2y + 7)} \]
\[ \measuredangle XZW = \frac{(2x + 3)^\circ}{(3x - 7)^\circ} \]

\[ \measuredangle NML = \underline{\quad} \]
\[ \measuredangle NKM = \underline{\quad}^\circ \]

\[ \overline{NM} \parallel \overline{KL}, \quad \text{find } x, \quad m \angle N, \quad \text{and } m \angle K. \]
**Challenge**

In the accompanying diagram of parallelogram $ABCD$, side $AD$ is extended through $D$ to $E$ and $DB$ is a diagonal. If $m\angle EDC = 65$ and $m\angle CBD = 85$, find $m\angle CDB$.

![Parallelogram Diagram](image)

**Summary**

**Properties of Parallelograms**

*Why?* The properties of parallelograms make these figures useful in mechanics and construction.

A quadrilateral is a parallelogram → all of these properties are true.

- Opposite sides are parallel.
- Opposite sides are congruent.
- Opposite angles are congruent.
- Consecutive angles are supplementary.
- Diagonals bisect each other.

**Exit Ticket**

Which statement is *not* always true about a parallelogram?

1) The diagonals are congruent.
2) The opposite sides are congruent.
3) The opposite angles are congruent.
4) The opposite sides are parallel.

Find the value of $x$ in parallelograms $ABCD$.

A. 70     B. 75     C. 105     D. 110
Homework

Find the measurement indicated in each parallelogram.

1) \( \angle RST = 135^\circ \)
2) \( \angle DCE = 70^\circ \)
3) \( \angle WXV = 80^\circ \)
4) \( \angle LMN = 98^\circ \)
5) \( \angle RSP = 23^\circ \)
6) \( \text{side } ED = 12.8 \)
7) \( RT = 19.8 \)
   Find \( RP \)
8) \( \angle TXY = 52^\circ \)
9) \( \angle WXY = 83^\circ \)
10) \( \angle DEF = 44^\circ \)
Solve for $x$. Each figure is a parallelogram.

11) $Z \quad 11x - 10$
12) $2x + 15$
13) $9x + 15 \quad 6x + 15$
14) $S \quad 35^\circ \quad 14x + 5$
15) $x + 13$
16) $L \quad 50x \quad 130x$
17) $UH = 19$
18) $KU = 3x + 3$
   $FH = 5x - 7$
   $UM = 4x - 4$

Find the measurement indicated in each parallelogram.

19) Find $RQ$

20) Find $m\angle G$

21) $TE = 4 + 2x$
    $EV = 4x - 4$
    Find $TE$

22) $DB = 5x - 1$
    $BF = 5 + 3x$
    Find $DB$
Chapter 6 (Section 4) – Day 3
Rectangles

Warm - Up

The measures of two consecutive angles of a parallelogram are in the ratio 5:4. What is the measure of an obtuse angle of the parallelogram?
1) 20°
2) 80°
3) 100°
4) 160°

Definition: A rectangle is a parallelogram with one right angle.

Properties of a Rectangle

1. A rectangle has all the properties of a parallelogram.
2. A rectangle contains four right angles and is therefore equiangular.
3. The diagonals of a rectangle are congruent.

<table>
<thead>
<tr>
<th>Properties of Rectangles</th>
</tr>
</thead>
<tbody>
<tr>
<td><img src="Diagram.png" alt="Diagram" /></td>
</tr>
<tr>
<td>GHJK is a parallelogram.</td>
</tr>
<tr>
<td>If a quadrilateral is a rectangle, then it is a parallelogram.</td>
</tr>
<tr>
<td><img src="Diagram.png" alt="Diagram" /></td>
</tr>
<tr>
<td>HJ ≡ HK</td>
</tr>
<tr>
<td>If a parallelogram is a rectangle, then its diagonals are congruent.</td>
</tr>
</tbody>
</table>

Since a rectangle is a parallelogram, a rectangle also has all the properties of parallelograms.
Let's explore the Properties of the rectangle!

- The diagonals of a rectangle are congruent.

<table>
<thead>
<tr>
<th>If WY = 19, then ZX = ?</th>
</tr>
</thead>
<tbody>
<tr>
<td>If WY = 19, then WT = ?</td>
</tr>
<tr>
<td>If TX = 4.5, then WY = ?</td>
</tr>
</tbody>
</table>

Rectangle GALS has diagonals GL and AS. If GL = 3a + 6 and AS = 5a − 18, then a = ?

- The angles of a rectangle are all right angles.

➢ If \( m \angle 1 = 55^\circ \), find all the missing angle measures.

| \( m \angle 2 = \) ____ ° |
| \( m \angle 3 = \) ____ ° |
| \( m \angle 4 = \) ____ ° |

➢ Quadrilateral \( ABCD \) is a rectangle.

If \( m \angle BDC = 7x + 1 \) and \( m \angle ADB = 9x − 7 \), find \( m \angle BDC \).
Practice Problems

a. If $AE = 5$, and $DC = 8$, find $AC$, $BD$, $AD$, and $AB$.

\[ \text{AC} = \underline{\hspace{2cm}} \]
\[ \text{BD} = \underline{\hspace{2cm}} \]
\[ \text{AD} = \underline{\hspace{2cm}} \]
\[ \text{AB} = \underline{\hspace{2cm}} \]

b. If $BD = 3x - 7$ and $CA = x + 5$, find $BD$, $ED$, $CA$, and $AE$.

\[ \text{BD} = \underline{\hspace{2cm}} \]
\[ \text{ED} = \underline{\hspace{2cm}} \]
\[ \text{CA} = \underline{\hspace{2cm}} \]
\[ \text{AE} = \underline{\hspace{2cm}} \]

c. Find the measures of the numbered angles in each rectangle.

\[ \text{m} \angle 1 = \underline{\hspace{2cm}} ^\circ \]
\[ \text{m} \angle 2 = \underline{\hspace{2cm}} ^\circ \]
\[ \text{m} \angle 3 = \underline{\hspace{2cm}} ^\circ \]

\[ \text{m} \angle 1 = \underline{\hspace{2cm}} ^\circ \]
\[ \text{m} \angle 2 = \underline{\hspace{2cm}} ^\circ \]
\[ \text{m} \angle 3 = \underline{\hspace{2cm}} ^\circ \]
\[ \text{m} \angle 4 = \underline{\hspace{2cm}} ^\circ \]

d. If $m \angle DAC = 2x + 4$ and $m \angle BAC = 3x + 1$, find $m \angle BAC$.

\[ \text{m} \angle DAC = 2x + 4 \]
\[ \text{m} \angle BAC = 3x + 1 \]
**CHALLENGE**

In a rectangle, the length is twice the width, and the perimeter is 48. Find the area of the rectangle.

**SUMMARY**

**Properties of Rectangles** A rectangle is a quadrilateral with four right angles. Here are the properties of rectangles.

A rectangle has all the properties of a parallelogram.

- Opposite sides are parallel.
- Opposite angles are congruent.
- Opposite sides are congruent.
- Consecutive angles are supplementary.
- The diagonals bisect each other.

Also:

- All four angles are right angles. \( \angle UTS, \angle TSR, \angle SRU, \) and \( \angle RUT \) are right angles.
- The diagonals are congruent. \( TR \cong US \)

**Example 1** Quadrilateral RUTS above is a rectangle. If \( US = 6x + 3 \) and \( RT = 7x - 2 \), find \( x \).

The diagonals of a rectangle are congruent, so \( US = RT \).

\[
6x + 3 = 7x - 2 \\
3 = x - 2 \\
5 = x
\]

**Example 2** Quadrilateral RUTS above is a rectangle. If \( m \angle STR = 8x + 3 \) and \( m \angle UTR = 16x - 9 \), find \( m \angle STR \).

\( \angle UTS \) is a right angle, so

\[
m \angle STR + m \angle UTR = 90. \\
8x + 3 + 16x - 9 = 90 \\
24x - 6 = 90 \\
24x = 96 \\
x = 4
\]

\( m \angle STR = 8x + 3 = 8(4) + 3 \) or 35

**Exit Ticket**

In rectangle \( ABCD \), \( \overline{AC} \) and \( \overline{BD} \) are diagonals. If \( m \angle 1 = 55 \), find \( m \angle ABD \).

1. 20
2. 35
3. 55
4. 65
Homework

Use rectangle $ABCD$ and the given information to solve each problem.

1. If $AC = 4x - 60$ and $BD = 30 - x$, find $BD$.

2. If $AC = 4x - 60$ and $AE = x + 5$, find $EC$.

3. If $m \angle BAC = 4x + 5$ and $m \angle CAD = 5x - 14$, find $m \angle CAD$.

4. If $AE = 2x + 3$ and $BE = 12 - x$, find $BD$.

5. If $m \angle BAC = 3x + 5$ and $m \angle ACD = 40 - 2x$. Find $m \angle AED$.

6. In the diagram, $ABCD$ is a rectangle, $E$ is a point on $\overline{CD}$, $m \angle DAE = 30$, and $m \angle CBE = 20$. What is $m \angle x$?
   
   1. 25
   2. 50
   3. 60
   4. 70

 Quadrilateral $GHJK$ is a rectangle. Find each measure if $m \angle 1 = 37$.

7. $m \angle 2$  

8. $m \angle 3$

9. $m \angle 4$  

10. $m \angle 5$

11. $m \angle 6$  

12. $m \angle 7$
Chapter 6 (section 4) – Day 4
Rhombi and Squares

Warm - Up

1. Quadrilateral $DEFG$ is a rectangle.
   
   If $FD = 3x - 7$ and $EG = x + 5$, find $EG$.

2. Quadrilateral $ABCD$ is a rectangle. Find each measure if $m \angle 2 = 40$.

   - $m \angle 1 = \_\_\_\_\_\_^\circ$
   - $m \angle 5 = \_\_\_\_\_\_^\circ$
   - $m \angle 2 = 40^\circ$
   - $m \angle 6 = \_\_\_\_\_\_^\circ$
   - $m \angle 3 = \_\_\_\_\_\_^\circ$
   - $m \angle 7 = \_\_\_\_\_\_^\circ$
   - $m \angle 4 = \_\_\_\_\_\_^\circ$
   - $m \angle 8 = \_\_\_\_\_\_^\circ$
**Rhombus**

**Definition:** A quadrilateral with 4 congruent sides.

1) Opposite sides are congruent (they equal each other).
2) Opposite angles are congruent (they equal each other).
3) Consecutive angles are supplementary (they add up to 180).
4) Diagonals bisect each other (the parts are equal).
5) Diagonals perpendicular (the form right angles in the middle).
6) Diagonals bisect angles (the angles are equal to each other).
7) All four sides are congruent.
8) The diagonals are NOT congruent.

**Square**

**Definition:** A square is a rectangle with 2 congruent consecutive sides.

**Squares:**

1) Opposite sides are congruent (they equal each other).
2) Opposite angles are congruent (they equal each other).
3) Consecutive angles are supplementary (they add up to 180).
4) Diagonals bisect each other (the parts are equal).
5) Diagonals are congruent (they equal each other).
6) All four corner angles are 90°.
7) Diagonals perpendicular (the form right angles in the middle).
8) Diagonals bisect angles (the angles equal to each other).
Problems Involving the Squares

- If $AB = 2x + 4$ and $CD = 3x - 5$, find $BC$ and $BD$.

- If $m\angle AEB = (3x)^\circ$, find ‘$x$’.

- If $m\angle BAC = (9x)^\circ$, find ‘$x$’.

- The perimeter of the square is 32 cm. Find the length of diagonal $DB$. 
Problems Involving the Rhombus

➢ If \( DM = 6y + 4 \) and \( ML = 5y + 8 \), find the length of KL.

---

Find each variable in the rhombus.

---

➢ Find the measures of the numbered angles in each rhombus.

\[
\begin{align*}
\text{m} \angle 1 &= \_\_\_\_\_^\circ \\
\text{m} \angle 2 &= \_\_\_\_\_^\circ \\
\text{m} \angle 3 &= \_\_\_\_\_^\circ \\
\text{m} \angle 4 &= \_\_\_\_\_^\circ \\
\end{align*}
\]

➢ The diagonals of a Rhombus are 10, and 24 cm. Find the length of the side of the rhombus.
**Challenge**

SNOW is a rhombus with SN = 2x^2 - 20, NO = x^2 + 9x + 16, and OW = y. Find the value of x and y.

**SUMMARY**

**SQUARES:**

1) Opposite sides are congruent (they equal each other).
2) Opposite angles are congruent (they equal each other).
3) Consecutive angles are supplementary (they add up to 180).
4) Diagonals bisect each other (the parts are equal).
5) Diagonals are congruent (they equal each other).
6) All four corner angles are 90°.
7) Diagonals perpendicular (the form right angles in the middle).
8) Diagonals bisect angles (the angles equal to each other).

**RHOMBI:**

1) Opposite sides are congruent (they equal each other).
2) Opposite angles are congruent (they equal each other).
3) Consecutive angles are supplementary (they add up to 180).
4) Diagonals bisect each other (the parts are equal).
5) Diagonals perpendicular (the form right angles in the middle).
6) Diagonals bisect angles (the angles are equal to each other).
7) All four sides are congruent.
8) The diagonals are NOT congruent.

**Exit Ticket**

<table>
<thead>
<tr>
<th>Property</th>
<th>Rectangle</th>
<th>Rhombus</th>
<th>Square</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. All the properties of a parallelogram?</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2. Equiangular (4 right corner angles?)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3. Equilateral (4 congruent sides?)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>4. Diagonals bisect angles?</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>5. Diagonals congruent?</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>6. Diagonals perpendicular?</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Homework

Find the value of each variable in each rhombus.

1. \[ \text{a} = \_, \text{b} = \_, \text{c} = \_, \text{d} = \_ \]

2. \[ \text{a} = \_, \text{b} = \_, \text{c} = \_, \text{d} = \_ \]

3. \[ \text{a} = \_, \text{b} = \_, \text{c} = \_, \text{d} = \_ \]

4. \[ \text{a} = \_, \text{b} = \_, \text{c} = \_, \text{d} = \_ \]

5. \( KLMN \) is a square
   \[ \overline{NM} = 14 \]
   \[ m \angle LMO = \]
   \[ m \angle KLM = \]
   Perimeter \( KLMN = \)

6. \( EFGH \) is a square. If \( EF = 10 \), find the following:
   a. \( FG = \) 
   b. \( \angle EFG = \) 
   c. \( EG = \) 
   d. \( EI = \) 
   e. \( IF = \) 
   f. \( \angle{EIF} = \) 
   g. \( \angle 1 = \) 
   h. \( \angle 3 = \) 
   i. \( HF = \)
11. The diagonals of a rhombus are 12 and 16 cm. Find the length of a side of the rhombus.

12. The shorter diagonal of a rhombus measures 18 cm. The side of the rhombus measures 41 cm. Find the length of the longer diagonal.
Warm-Up

1. $ABCD$ is a rhombus. If $PB = 12$, $AB = 15$, and $m\angle ABD = 21$, find each measure.

23. $AP$  
24. $CP$  
25. $m\angle BDA$  
26. $m\angle ACB$

2. $WXYZ$ is a square. If $WT = 3$, find each measure.

27. $ZX$  
28. $XY$  
29. $m\angle WTZ$  
30. $m\angle WYX$
Definition: A trapezoid is a quadrilateral with one pair of parallel sides.

---

Definition: An isosceles trapezoid is a trapezoid with non parallel sides congruent.

---

Properties of Isosceles Trapezoids

- If a quadrilateral is an isosceles trapezoid, then each pair of base angles are congruent.

- If a quadrilateral is an isosceles trapezoid, then the diagonals are congruent
Practice Problems

ALGEBRA Find each measure.

1. \( m \angle S \)

2. \( m \angle M \)

3. Trapezoid PORQ. Find the \( m \angle 1 \) and \( m \angle 2 \).

4. Isosceles Trapezoid ABCD.

5. Find the values of the variables.

\[ DF = 4x, \ EG = 2x + 16 \]

6. \[ AC = 7x - 15, \ BD = 4x + 15 \]
The **midsegment of a trapezoid** is the segment that connects the midpoints of the legs of the trapezoid. The theorem below relates the midsegment and the bases of a trapezoid.

\[
\text{Median} = \frac{1}{2} (\text{base}_1 + \text{base}_2) \\
2m = b_1 + b_2
\]

- The midsegment of a trapezoid is parallel to each base. \( AB \parallel MN \) and \( AB \parallel LP \)
- The length of the midsegment is one-half the sum of the length of the bases. 
\[
AB = \frac{1}{2}(MN + LP)
\]

For trapezoid \( QRTU \), \( V \) and \( S \) are midpoints of the legs.

7. If \( QR = 12 \) and \( UT = 22 \), find \( VS \).

8. If \( QR = 4 \) and \( UT = 16 \), find \( VS \).

9. If \( VS = 9 \) and \( UT = 12 \), find \( QR \).

10. If \( RQ = 3x - 3 \), \( UT = 4x + 2 \), and \( VS = 10 \), find \( VS \).
11. In the accompanying figure, isosceles trapezoid $ABCD$ has bases of lengths 9 and 15 and an altitude of length 4. Find $AB$.

![Diagram of an isosceles trapezoid with base lengths 9 and 15 and an altitude of 4 cm.]

12. Find the length of base $AB$ of trapezoid $ABCD$.

![Diagram of a trapezoid with base lengths 12 cm and 10 cm, angles of 45° and 60° at points A and B, and points D and C.]
**CHALLENGE**

Given trapezoid BARK with midsegment $\overline{NO}$.

$BA = c^2$, $NO = 6c$, $KR = c^2 + 18$. Find $c$.

**SUMMARY**

**ISOSCELES TRAPEZIIDS:**

$2m = b_1 + b_2$

1) Lower two base angles are congruent (they equal each other).
2) Upper two base angles are congruent (they equal each other).
3) The diagonals are congruent (they equal each other).
4) Opposite angles are supplementary (they add up to 180).

**Exit Ticket**

Isosceles trapezoid $ABCD$ has diagonals $\overline{AC}$ and $\overline{BD}$. If $AC = 5x + 13$ and $BD = 11x - 5$, what is the value of $x$?

1) 28
2) $10\frac{3}{4}$
3) 3
4) $\frac{1}{2}$
Homework – Trapezoids

Find each measure.

1. \( m \angle T \)

2. \( m \angle Y \)

3. Trapezoid PQRS. Find the \( \angle 1 \) and \( \angle 2 \).

4. ABCD is an isosceles trapezoid. Find the \( \angle 1 \) and \( \angle 2 \).

5. MATH is an isosceles trapezoid with \( \overline{AT} \parallel \overline{MH} \). If \( m \angle M = (3x - 9)^\circ \) and \( m \angle H = (x + 3)^\circ \), find the value of ‘x’.

6. Let AC = 25 and DB = 5x.
7. If \( EH = FG \), and \( m \angle E = 65^\circ \), then \( m \angle G = \) ? and \( m \angle GKJ = \) ?

8. \( KB = 12 \) and \( MF = 30 \). Find \( FB \).

9. \( JN = 10 \), and \( NL = 14 \). Find \( KM \).

10. Find the value of \( x \).
11. The cross section of an attic is in the shape of an isosceles trapezoid, as shown in the accompanying figure. If the height of the attic is 9 feet, $BC = 12$ feet, and $AD = 28$ feet, find the length of $AB$ to the nearest foot.

12. The accompanying diagram shows ramp $RA$ leading to level platform $AM$, forming an angle of $45^\circ$ with level ground. If platform $AM$ measures 2 feet and is 6 feet above the ground. Find RA.

13. If $PQ = 15$, and $SR = 9$, find $ST$ and $PS$. 

---

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Chapter 6 (Review) – Day 6

Warm – Up

In the diagram below of trapezoid $RSUT$, $RS \parallel TU$, $X$ is the midpoint of $RT$, and $V$ is the midpoint of $SU$.

If $RS = 30$ and $XV = 44$, what is the length of $TU$?
1) 37
2) 58
3) 74
4) 118

Write the missing terms in the unlabeled sections.

- Quadrilaterals
  - Rectangles
  - Squares
  - Parallelograms
  - Rhombuses
  - Kites
SUMMARY

PARALLELOGRAMS (rectangles, squares, and rhombi):

1) Opposite sides of a parallelogram are congruent.
2) Opposite angles of a parallelogram are congruent.
3) Consecutive angles in a parallelogram are supplementary.
4) The diagonals of a parallelogram bisect each other.

RECTANGLES:

1) Opposite sides are congruent (they equal each other).
2) Opposite angles are congruent (they equal each other).
3) Consecutive angles are supplementary (they add up to 180).
4) Diagonals bisect each other (the parts are equal).
5) Diagonals are congruent (they equal each other).
6) All four corner angles are 90°.

SQUARES:

1) Opposite sides are congruent (they equal each other).
2) Opposite angles are congruent (they equal each other).
3) Consecutive angles are supplementary (they add up to 180).
4) Diagonals bisect each other (the parts are equal).
5) Diagonals are congruent (they equal each other).
6) All four corner angles are 90°.
7) Diagonals perpendicular (the form right angles in the middle).
8) Diagonals bisect angles (the angles equal to each other).

RHOMBI:

1) Opposite sides are congruent (they equal each other).
2) Opposite angles are congruent (they equal each other).
3) Consecutive angles are supplementary (they add up to 180).
4) Diagonals bisect each other (the parts are equal).
5) Diagonals perpendicular (the form right angles in the middle).
6) Diagonals bisect angles (the angles equal to each other).
7) All four sides are congruent.
8) The diagonals are NOT congruent.

ISOSCELES TRAPEZIODS:

\[ \text{Median} = \frac{1}{2} (\text{base} + \text{base}) \]

\[ 2m = b_1 + b_2 \]

1) Lower two base angles are congruent (they equal each other).
2) Upper two base angles are congruent (they equal each other).
3) The diagonals are congruent (they equal each other).
4) opposite angles are supplementary (they add up to 180).
Chapter 6 – Review

Angles in Polygons

1. Find the sum of the interior angles of a heptagon?

2. Each interior angle of a regular convex polygon measures 150°. How many sides does the polygon have?

3. The sum of the interior angles of a convex regular polygon measures 1620°. How many sides does the polygon have?

4. How many sides does a polygon have if each exterior angle measures 36°?

5. Find the measure of each exterior angle of regular hexagon ABCDEF?

6. Find the value of x.

7. Find the value of x.

Formulas

\[ S_i = (n - 2)180 \]

\[ S_e = 360 \text{ (always – no matter what the polygon)} \]

\[ E = \frac{360}{n} \]

\[ I = \frac{180(n-2)}{n} \]
Properties of Quadrilaterals

List all the properties of a parallelogram below:

1. 
2. 
3. 
4. 
5. 

The rectangle has all the properties of the parallelogram AND

1. 
2. 

The rhombus has all the properties of the parallelogram AND

1. 
2. 
3. 

The square has all the properties of the parallelogram AND

1. 
2. 
3. 
4. 
5. 

Complete the Venn diagram using the letter of the word(s) provided.

A. Trapezoid
B. Parallelogram
C. Rhombus
D. Square
E. Rectangle
F. Quadrilateral
G. Isosceles Trapezoid

1. 
2. 
3. 
4. 
5. 
6. 
7. 
<table>
<thead>
<tr>
<th>Property</th>
<th>Quadrilateral</th>
<th>Parallelogram</th>
<th>Rectangle</th>
<th>Rhombus</th>
<th>Square</th>
<th>Trapezoid</th>
<th>Isosceles Trapezoid</th>
<th>Kite</th>
</tr>
</thead>
<tbody>
<tr>
<td>Both pairs of opposite sides are congruent.</td>
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<tr>
<td>Both pairs of opposite angles are congruent.</td>
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<td>Has 4 sides.</td>
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<td>Has 4 angles.</td>
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<td>Both diagonals are congruent.</td>
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<td>4 congruent sides.</td>
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<td>The diagonals bisect the opposite angles.</td>
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<tr>
<td>All consecutive angles are supplementary.</td>
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<td>Both pairs of opposite sides are parallel.</td>
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<td>Exactly one pair of opposite sides are parallel.</td>
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<tr>
<td>The diagonals are perpendicular.</td>
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<tr>
<td>The diagonals bisect each other.</td>
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<td>Exactly one pair of opposite sides congruent.</td>
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<tr>
<td>4 right angles.</td>
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</tbody>
</table>
Properties of Quadrilaterals – True or False

Mark each as true or false. Remember, that a property is true only if it is true all the time.

1. Parallelogram
   a. both pairs of opposite sides are parallel
   b. both pairs of opposite sides are congruent
   c. both pairs of opposite angles are congruent
   d. all consecutive angles are supplementary
   e. the diagonals are congruent
   f. the diagonals bisect each other

2. Rhombus
   a. all consecutive angles are supplementary
   b. all consecutive sides are congruent
   c. all consecutive sides form right angles
   d. the diagonals are perpendicular
   e. the diagonals bisect the opposite angles
   f. exactly one pair of opposite sides is parallel

3. Rectangle
   a. the diagonals are perpendicular
   b. the diagonals are congruent
   c. the diagonals bisect the opposite angles
   d. the diagonals bisect each other
   e. all consecutive angles are congruent
   f. all consecutive angles are supplementary
5. **Trapezoid**

   ______ a. both pairs of opposite sides are parallel
   ______ b. the diagonals are congruent
   ______ c. the diagonal bisect each other
   ______ d. the diagonal bisect the opposite angles
   ______ e. it can never be drawn as a concave polygon
   ______ f. any pair of upper and lower base angles will be supplementary

6. **Isosceles Trapezoid**

   ______ a. the diagonals are congruent
   ______ b. the diagonals bisect each other
   ______ c. the diagonals bisect the opposite angles
   ______ d. exactly one pair of opposite sides are parallel
   ______ e. exactly one pair of opposite sides is congruent
   ______ f. any pair of upper and lower base angles will be supplementary and the upper and lower base angles will be congruent.

7. **Quadrilateral**

   ______ a. has 3 sides
   ______ b. has 5 angles
   ______ c. can be drawn as either a convex or a concave polygon
   ______ d. has four diagonals
   ______ e. the one shown could be named ABCD

8. **Square**

   ______ a. any two consecutive sides are congruent
   ______ b. any two consecutive angles are supplementary
   ______ c. the slopes of the diagonals are opposite reciprocals
   ______ d. the slopes of any two consecutive sides are opposite reciprocals
   ______ e. the diagonals split opposite angles into two 45° angles.
   ______ f. is both a rectangle and a rhombus at the same time
Parallelograms

1. In \( \square ABCD \), \( \angle A = 68^\circ \). Find the measure of the other angles. Draw a sketch to help you.

   a. \( \angle B = \) _______

   b. \( \angle C = \) _______

   c. \( \angle D = \) _______

2. In \( \square ABCD \), \( \angle A = x \), \( \angle B = 2x + 60 \). \( \angle D = \) _______. Draw a sketch to help you.

3. Use parallelogram CDFB to find each measure.
   
   FD = _______
   BF = _______
   \( m \angle CBF = \) _______
   \( m \angle BCD = \) _______
   GF = _______

4. Given: Parallelogram \( \square ABCD \)
   
   with \( AB = 20 \), \( BD = 18 \), \( AC = 24 \)
   
   Find the perimeter of \( \triangle DEC \). _______
Rectangles

5. In rectangle \(ABCD\), \(CB = 6\), \(AB = 8\). Find the missing lengths:
   a. \(AD\)  
   b. \(CD\)  
   c. \(EC\)  
   d. \(AE\)
   e. \(DE\)  
   f. \(EB\)  
   g. \(DB\)

6. Given: \(JKLM\) is a rectangle. \(JK = 16\), \(KL = 12\), \(m\angle 1 = 53^\circ\)
   a. \(m\angle JKL = \text{_____}\)  
   c. \(m\angle 2 = \text{_____}\)  
   e. \(m\angle JNK = \text{_____}\)
   b. \(JL = \text{_____}\)  
   d. \(m\angle 4 = \text{_____}\)  
   f. \(MN = \text{_____}\)

7. \(ABCD\) is a rectangle. If \(AB = 24\), \(BC = 10\), and \(\angle 1 = 50^\circ\), find the following:
   a. \(CD = \text{_____}\)  
   d. \(BD = \text{_____}\)  
   g. \(\angle DAB = \text{_____}\)
   b. \(AD = \text{_____}\)  
   e. \(AX = \text{_____}\)  
   h. \(\angle 3 = \text{_____}\)
   c. \(AC = \text{_____}\)  
   f. \(BX = \text{_____}\)  
   i. \(\angle AXB = \text{_____}\)

8. Quadrilateral \(ABCD\) is a rectangle. Find each measure if \(m\angle 2 = 70^\circ\).
   \(m\angle 1\)  
   \(m\angle 7\)  
   \(m\angle 3\)
   \(m\angle 5\)  
   \(m\angle 6\)  
   \(m\angle 8\)
Rhombus

9. Use rhombus STPK to find each measure.
   TP = __________  
   PB = __________  
   KT = __________  
   m∠KTP = __________  
   m∠KBP = __________  

10. Given: PQRS is a rhombus. PQ = 4, m∠PQR = 60°
    a. QR = __________  
    b. m∠3 = __________  
    c. m∠2 = __________  
    d. PT = __________  
    e. ST = __________  
    f. m∠SPQ = __________  

11. ABCD is a rhombus. If AB = 6, XC = 3, and ∠DAB = 120°, find the following:
    a. BC = __________  
    b. ∠ADC = __________  
    c. ∠DCB = __________  
    d. ∠AXB = __________  
    e. ∠1 = __________  
    f. ∠2 = __________  
    g. ∠3 = __________  
    h. ∠4 = __________  
    i. AX = __________  

12. The diagonals of a rhombus are 32 cm and 126 cm. Find the perimeter of the rhombus.
13. Use Square SQR to find each measure.
   EQ = 
   SU = 
   m< SQU = 
   m< REU = 
   m< SQE = 

14. KLMN is a square and NO = 6.
    Find:
    m< QKL = 
    m< MOL = 
    Perimeter KLMN = 

15. ABCD is a square. If AB = 16 and AC = 16\sqrt{2}, find the following:
    a. BC = 
    b. BD = 
    c. AD = 
    d. \angle 1 = 
    e. \angle 2 = 
    f. \angle AXB = 
    g. \angle BXC = 
    h. \angle 4 = 

16. The perimeter of a square measures 72cm. Find the length of the diagonal.
17. **WXYZ** is an isosceles trapezoid. What are the values of $x$ and $y$?

18. Find the length of the midsegment.

19. **Given:** An isosceles trapezoid with a perimeter of 120 cm. Find $x$.

20. TRAP is an isosceles trapezoid. $\angle RTP = 70^\circ$, $AS = 4$, and $SP = 7$. Find the following:

   a. $\angle TRA =$  
   b. $\angle RAT =$  
   c. $\angle APT =$  
   d. $\angle TRP =$  
   e. $\angle RAP =$  
   f. $\angle TSP =$  
   g. $RP =$  
   h. $\angle ASP =$  
   i. $AP =$  
   j. $RS =$  

21. Find the length of base $\overline{AB}$ of trapezoid $ABCD$. 

---

**Trapezoids**

---

**WXYZ** is an isosceles trapezoid. What are the values of $x$ and $y$?
### Summary Charts:

<table>
<thead>
<tr>
<th>Special Quadrilateral</th>
<th>Diagonals Congruent</th>
<th>Diagonals Perpendicular</th>
<th>Diagonals Bisect Each Other</th>
<th>Diagonals Bisect Angles</th>
</tr>
</thead>
<tbody>
<tr>
<td>Parallelogram</td>
<td>Sometimes</td>
<td>Sometimes</td>
<td>Always</td>
<td>Sometimes</td>
</tr>
<tr>
<td>Rectangle</td>
<td>Always</td>
<td>Sometimes</td>
<td>Always</td>
<td>Sometimes</td>
</tr>
<tr>
<td>Rhombus</td>
<td>Sometimes</td>
<td>Always</td>
<td>Always</td>
<td>Always</td>
</tr>
<tr>
<td>Square</td>
<td>Always</td>
<td>Always</td>
<td>Always</td>
<td>Always</td>
</tr>
<tr>
<td>Trapezoid</td>
<td>Sometimes</td>
<td>Never</td>
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<tr>
<td>Isosceles Trapezoid</td>
<td>Always</td>
<td>Never</td>
<td>Never</td>
<td>Never</td>
</tr>
<tr>
<td>Kite</td>
<td>Never</td>
<td>Always</td>
<td>Only one diagonal</td>
<td>Only one angle</td>
</tr>
</tbody>
</table>

### Property Chart:

<table>
<thead>
<tr>
<th>Property</th>
<th>Rectangle</th>
<th>Rhombus</th>
<th>Square</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. All the properties of a parallelogram?</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
</tr>
<tr>
<td>2. Equiangular (4 right corner angles?)</td>
<td>Yes</td>
<td>No</td>
<td>Yes</td>
</tr>
<tr>
<td>3. Equilateral (4 congruent sides?)</td>
<td>No</td>
<td>Yes</td>
<td>Yes</td>
</tr>
<tr>
<td>4. Diagonals bisect angles?</td>
<td>No</td>
<td>Yes</td>
<td>Yes</td>
</tr>
<tr>
<td>5. Diagonals congruent?</td>
<td>Yes</td>
<td>No</td>
<td>Yes</td>
</tr>
<tr>
<td>6. Diagonals perpendicular?</td>
<td>No</td>
<td>Yes</td>
<td>Yes</td>
</tr>
</tbody>
</table>

### Diagram:

- Quadrilateral: Sum of angles is 360
  - 2 pairs of parallel sides
  - One pair of parallel sides
- Parallelogram: 4 right angles
- Rectangle: 4 right angles and 4 congruent sides
- Rhombus: 4 congruent sides
- Trapezoid: 2 congruent legs
- Isosceles Trapezoid: No parallel sides, 2 pair consecutive sides congruent
- Square: 4 right angles and 4 congruent sides
- Kite: 4 sides, no parallel sides, 2 pair consecutive sides congruent