Honors Packet on Polygons, Quadrilaterals, and Special Parallelograms

Diagram of quadrilaterals:
- Quadrilateral
  - Kite
  - Trapezoid
    - Isosceles Trapezoid
  - Rhombus
    - Rectangle
      - Square

Name:________________________________________

Teacher:_______________________________________

Pd: ______
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Chapter 6 (Section 1) – Day 1

Angles in polygons

A polygon is a closed plane figure formed by three or more segments that intersect only at their endpoints. Each segment that forms a polygon is a side of the polygon. The common endpoint of two sides is a vertex of the polygon. A segment that connects any two nonconsecutive vertices is a diagonal.

You can name a polygon by the number of its sides. The table shows the names of some common polygons.

<table>
<thead>
<tr>
<th>Number of Sides</th>
<th>Name of Polygon</th>
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<tbody>
<tr>
<td>3</td>
<td>Triangle</td>
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<tr>
<td>4</td>
<td>Quadrilateral</td>
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<td>Pentagon</td>
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<td>6</td>
<td>Hexagon</td>
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<td>Heptagon</td>
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<td>9</td>
<td>Nonagon</td>
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<td>10</td>
<td>Decagon</td>
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<td>12</td>
<td>Dodecagon</td>
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<td>n</td>
<td>n-gon</td>
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All the sides are congruent in an equilateral polygon. All the angles are congruent in an equiangular polygon. A regular polygon is one that is both equilateral and equiangular. If a polygon is not regular, it is called irregular.

A polygon is concave if any part of a diagonal contains points in the exterior of the polygon. If no diagonal contains points in the exterior, then the polygon is convex. A regular polygon is always convex.

Warm – Up

Tell whether the following polygons are concave or convex and regular or irregular.

1. : __________
   : __________

2. : __________
   : __________

3. : __________
   : __________

4. : __________
   : __________
Sum of Interior Angles in Polygons

To find the sum of the interior angle measures of a convex polygon, draw all possible diagonals from one vertex of the polygon. This creates a set of triangles. The sum of the angle measures of all the triangles equals the sum of the angle measures of the polygon.

<table>
<thead>
<tr>
<th>Polygon</th>
<th>Number of Sides</th>
<th>Number of Triangles</th>
<th>Sum of Interior Angle Measures</th>
</tr>
</thead>
<tbody>
<tr>
<td>Triangle</td>
<td>3</td>
<td>1</td>
<td>$180^\circ = 180^\circ$</td>
</tr>
<tr>
<td>Quadrilateral</td>
<td>4</td>
<td>2</td>
<td>$180^\circ = 360^\circ$</td>
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<tr>
<td>Pentagon</td>
<td>5</td>
<td>3</td>
<td>$180^\circ = 540^\circ$</td>
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<tr>
<td>Hexagon</td>
<td>6</td>
<td>4</td>
<td>$180^\circ = 720^\circ$</td>
</tr>
<tr>
<td>$n$-gon</td>
<td>$n$</td>
<td>$n - 2$</td>
<td>$(n - 2)180^\circ$</td>
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</table>

In each convex polygon, the number of triangles formed is two less than the number of sides $n$. So the sum of the angle measures of all these triangles is $(n - 2)180^\circ$.

Theorem 6-1-1  Polygon Angle Sum Theorem

Example 1: Calculating the Sum of Interior Angles

Find the sum of the interior angles of a decagon.
Example 2: Calculating the measure of each of interior Angle of any regular polygon

What is the measure of each angle in a regular octagon?

Example 3: Calculating the number of sides of a polygon given the sum of the interior angles

The sum of the interior angles of a convex regular polygon measure 1980°, how many sides does the polygon have?
Exterior Angles

Refer to the two polygons below. What do you notice about the exterior angles of any polygon?

\[ 147° + 81° + 132° = 360° \]

\[ 43° + 111° + 41° + 55° + 110° = 360° \]

**Theorem 6.2**

**Polygon Exterior Angles Sum**

The sum of the exterior angle measures of a convex polygon, one angle at each vertex, is 360.

**Example**

\[ m\angle 1 + m\angle 2 + m\angle 3 + m\angle 4 + m\angle 5 + m\angle 6 = 360 \]

**Example 4:** Calculating the measure of an exterior angle given the number of sides or Vice Versa

Find the measure of each exterior angle of a polygon with 18 sides.
You Try It!
The measure of an exterior angle of a convex regular polygon is $45^\circ$. Find the number of sides of the polygon.

You Try It!
How many sides does a regular polygon have if each interior angle measures $160^\circ$?

Example 5: Solving Algebraic Problems
Find the value of $x$.

Find $x$:
Number of Diagonals in a Polygon

\[ d = \frac{n(n-3)}{2} \]

1. Find the number of diagonals in a pentadecagon.

2. What is the name of the polygon with 14 diagonals?

**Challenge**

In Exercises 1, find each lettered angle measure.

\[ a = \_\_\_, \quad b = \_\_\_, \quad c = \_\_\_, \quad d = \_\_\_, \quad e = \_\_\_ \]

**Summary**

**Properties and Attributes of Polygons**

Why? Understanding properties of polygons and their angle sums is fundamental to successful work with quadrilaterals.

**Theorem**

The sum of the interior angle measures of a convex polygon with \( n \) sides

\[ S_i = (n-2)180 \]

**Theorem**

The sum of the exterior angle measures, one angle at each vertex, of a convex polygon is 360°.

\[ S_e = 360 \] (always – no matter what the polygon)

**Theorem**

\[ e = \frac{360}{n} \]
Interior and Exterior Angles of Polygons
Day 2 - Practice

Warm - Up

One piece of the birdhouse that Natalie is building is shaped like a regular pentagon, as shown in the accompanying diagram.

If side $AE$ is extended to point $F$, what is the measure of exterior angle $DEF$?
1) 36°
2) 72°
3) 108°
4) 144°

The number of sides of a convex polygon is given. Find the sum of the measures of the interior angles of each polygon.

1) 8
2) 12

The sum of the measures of the interior angles of a convex polygon is given. Find the number of sides of each polygon.

6) 7020°
7) 1980°
The number of sides of a regular polygon is given. Find the measure of each interior angle of each polygon.

11) 7  

12) 9

Find the exact measure of each exterior angle of the regular polygon.

19) 18-gon  

20) 20-gon

23) How many sides does a regular polygon have if each exterior angle has a measure of 15°?

24) How many sides does a regular polygon have if each interior angle has a measure of 108°?
27) In quadrilateral ABCD the measures of $\angle A$, $\angle B$, $\angle C$, and $\angle D$ are the ratio of 1:2:3:4, respectively. Find the measures of the four angles.

Find the value of $x$.

28)  
\[
\begin{array}{c}
75^\circ \\
97^\circ \\
105^\circ \\
\end{array}
\]

29)  
\[
\begin{array}{c}
139^\circ \\
9^\circ \\
71^\circ \\
5^\circ \\
92^\circ \\
\end{array}
\]

30)  
\[
\begin{array}{c}
138^\circ \\
133^\circ \\
14^\circ \\
167^\circ \\
120^\circ \\
\end{array}
\]
34) **Light Fixture** The side view of a light fixture is shown below. Find the value of $x$. Then determine the measure of each angle.

35) **Tent** The front view of a camping tent is shown below. Find the value of $x$. Then determine the measure of each angle.
2. How many diagonals can be drawn in each figure below?
   
   a) Octagon  
   b) Decagon  
   c) Pentagon  
   D) 33-gon

3. In what polygon is the sum of the measures of the angles of the polygon equal to twice the sum of the measures of the exterior angles, one per vertex?

4. Given: \( \angle 1 = 70^\circ \)
   \( \angle 7 = 130^\circ \)

   Find measures of \( \angle 2, \angle 3, \angle 4, \angle 5, \angle 6 \)

5. What are the names of the polygons that contain the following numbers of diagonals?
   
   a) 20  
   b) 77  
   c) 170
Warm-Up

The measures of five of the interior angles of a hexagon are 150°, 100°, 80°, 165°, and 150°. What is the measure of the sixth interior angle?
1) 75° 2) 80° 3) 105° 4) 180°

Any polygon with four sides is called a Quadrilateral. However, some quadrilaterals have special properties. These special quadrilaterals are given their own names.

A quadrilateral with two pairs of parallel sides is called a parallelogram. To write the name of a parallelogram, you use the symbol \( \square \).

Parallelogram \( ABCD \)

\( \square ABCD \)

\( AB \parallel CD, BC \parallel DA \)
Properties of Parallelograms

• If a quadrilateral is a parallelogram, then its **opposite sides** are congruent.

```
Q  R
P  S
```

• If a quadrilateral is a parallelogram, then its **opposite angles** are congruent.

```
Q  R
P  S
```

• If a quadrilateral is a parallelogram, then its **consecutive angles are supplementary**.

```
Q  R
P  S
```

• If a quadrilateral is a parallelogram, then its **diagonals bisect each other**.

```
Q  R
P  S
```
In exercises 14 – 16, each quadrilateral is a parallelogram. Find the indicated values.

14. \( a = \) \_
    \( b = \) \_
    \( x = \) \_
    \( y = \) \_

15. \( a = \) \_
    \( b = \) \_
    \( x = \) \_
    \( y = \) \_

16. \( a = \) \_
    \( b = \) \_
    \( x = \) \_
    \( y = \) \_

In exercises 17 – 19, what values must ‘x’ and ‘y’ have to make each quadrilateral a parallelogram?

17. \( x = \) \_
    \( y = \) \_

18. \( x = \) \_
    \( y = \) \_

19. \( x = \) \_
    \( y = \) \_
Level B:
In the accompanying diagram of parallelogram \(ABCD\), side \(AD\) is extended through \(D\) to \(E\) and \(DB\) is a diagonal. If \(m\angle EDC = 65\) and \(m\angle CBD = 85\), find \(m\angle CDB\).

---

In parallelogram \(LMNO\), an exterior angle at vertex \(O\) measures \(72^\circ\). Find the measure, in degrees, of \(\angle L\).

---

Proofs

Given: \(ABCD\) is a \(\square\) (parallelogram).
\(\angle GHA \cong \angle FEC,\)
\(\overline{HB} \cong \overline{DE}\)

Conclusion: \(GH \cong EF\)

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</table>
Given: PQRS is a parallelogram.
\( YQ \) bisects \( \angle PQR \).
\( XS \) bisects \( \angle PSR \).

Prove: \( SY \equiv XQ \).

**Summary**

**Properties of Parallelograms**

Why? The properties of parallelograms make these figures useful in mechanics and construction.

A quadrilateral is a parallelogram \( \rightarrow \) all of these properties are true.

- Opposite sides are parallel.
- Opposite sides are congruent.
- Opposite angles are congruent.
- Consecutive angles are supplementary.
- Diagonals bisect each other.

**Exit Ticket**

Which statement is *not* always true about a parallelogram?
1) The diagonals are congruent.
2) The opposite sides are congruent.
3) The opposite angles are congruent.
4) The opposite sides are parallel.
Practice with Parallelograms – Day 4

Warm – Up

1. In the accompanying diagram of parallelogram $ABCD$, $\overline{DE}$ bisects $\angle ADC$ and $m\angle A = 44$. Find $m\angle CDE$.

2. In the accompanying diagram, $ABCD$ is a parallelogram, $\overline{DA} \parallel \overline{DE}$, and $m\angle B = 70$. Find $m\angle E$.

3. The measures of angles $A$ and $B$ of parallelogram $ABCD$ are in the ratio $2:7$. Find the degree measure of angle $A$.

4. If the given quadrilateral is a parallelogram, find the value of $x$ and $y$.

$$2x + 4y$$

$$21$$

$$3x + 3y$$

$$6y + \frac{1}{2}x$$

$x = \underline{\hspace{2cm}}$

$y = \underline{\hspace{2cm}}$
5.  ABCD is a parallelogram.  \( AB = 2x \), \( AD = 4y - 6 \), \( BC = 3x \).  If the perimeter of \( \square ABCD \) is 180, find the values of \( x \) and \( y \).

\[ x = \underline{\phantom{00}} \]

\[ y = \underline{\phantom{00}} \]

6.  Let \( E \) be intersection of \( \overline{AC} \) and \( \overline{DB} \)

\( AE = 2x \);  \( EC = y + 7 \)

\( DE = x \);  \( DB = 31 - 7y \)

\[ x = \underline{\phantom{00}} \]

\[ y = \underline{\phantom{00}} \]

7.  MERY is a parallelogram.  Find the value of \( x \) and \( y \).

\[ x = \underline{\phantom{00}} \]

\[ y = \underline{\phantom{00}} \]

8.  Determine the values of \( x \) and \( y \) for which \( ABCD \) is a parallelogram.

\[ x = \underline{\phantom{00}} \]

\[ y = \underline{\phantom{00}} \]
9. Complete the statement for parallelogram $BCDE$. Then state a definition or theorem as the reason.

$BC \parallel \underline{\text{_____}}$

10. Complete the statement for parallelogram $FGHI$. Then state a definition or theorem as the reason.

$FO \cong \underline{\text{_____}}$

11. Complete the statement for parallelogram $BCDE$. Then state a definition or theorem as the reason.

$CD \cong \underline{\text{_____}}$

12. Complete the statement for parallelogram $HIJK$. Then state a definition or theorem as the reason.

$\angle K \cong \underline{\text{_____}}$
13. Given: $\square ABCD$, $BF \perp AE$, $CE \perp AB$  
Prove: $AF \cong DE$

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14. Prove that the diagonals of a parallelogram bisect each other.

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15.

Given:  \( ABCD \) is a parallelogram.
\[ BP \cong DQ \]

Prove: \( X \) is the midpoint of \( AC \).

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<th>Statements</th>
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Day 5 – Proving Quadrilaterals are Parallelograms

Warm – Up

Given: \( \text{ABCD is a parallelogram} \)
\( \text{FG bisects DB} \)
Prove: \( \overline{FE} \cong \overline{EG} \)

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Parallelogram*

- Sides:
  - 2 sets of opposite parallel sides
  - 2 sets of opposite congruent sides
- Angles:
  - 2 sets of opposite angles congruent
  - Consecutive angles supplementary
- Diagonals:
  - Diagonals bisect each other
  - Diagonals form 2 congruent triangles

Ways to prove a quadrilateral is a parallelogram
1.
2.
3.
4.
5.
Determining if a quadrilateral is a parallelogram

Analyzing a Diagram

1. In (a) to (e), the given is marked on the figure. Tell why each quadrilateral ABCD is a parallelogram.

a. [Diagram of quadrilateral ABCD]

b. [Diagram of quadrilateral ABCD]

c. [Diagram of quadrilateral ABCD]

d. [Diagram of quadrilateral ABCD]

e. [Diagram of quadrilateral ABCD]

Based on the markings, decide if each figure is a parallelogram. Justify your answer.

2. [Diagram]

3. [Diagram]

4. [Diagram]

5. [Diagram]

6. [Diagram]

7. [Diagram]

8. [Diagram]

9. [Diagram]

State whether the information given about quadrilateral RAND is sufficient to determine that it is a parallelogram.

10. $\angle RDC \cong \angle NAC$, $\angle ARC \cong \angle DNC$

11. $\overline{RD} \cong \overline{AN}$, $\overline{RN} \cong \overline{RA}$

12. $\angle ACN \cong \angle RCD$, $\angle RCA \cong \angle DCN$

13. $\overline{RD} \cong \overline{AN}$, $\overline{RA} \cong \overline{DN}$
Proofs

   \[ DE \perp AC; \quad BF \perp AC. \]
   \[ AE \cong CF; \quad DE \cong BF. \]
   Prove: $ABCD$ is a parallelogram.

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15. Given: Quadrilateral $ABCD$ with diagonals $\overline{AC}$ and $\overline{BD}$. 
   \[ \overline{BD} \text{ bisects } \overline{AC} \text{ at } E; \quad \angle CAD \cong \angle BCA. \]
   Prove: $ABCD$ is a parallelogram.

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Given: ACDF is a □.
∠AFB ≅ ∠ECD
Prove: FBCE is a □.

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Proving a Quadrilateral Is a Parallelogram - Day 6

Ways to prove a quadrilateral is a parallelogram

1. If quad has opp. sides $\parallel \rightarrow$ (by definition)
2. If quad has opp. sides $\cong \rightarrow$
3. If quad has opp. angles $\cong \rightarrow$
4. If quad has diags. bisecting each other $\rightarrow$
5. If quad has an $\angle$ suppl. to cons. $\angle$s $\rightarrow$
6. If quad has a pair of opposite sides $\cong$ and $\parallel \rightarrow$

1. Given: $JKLM$ is a parallelogram;
   \[ PX \cong QX \]
Prove: $JPLQ$ is a parallelogram

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2. Given: $\square ABCD$, $\overline{ED} \equiv \overline{FB}$
Prove: $\square AECF$ is a $\square$.

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3. Given: $\square ABCD$, $E$ and $F$ are midpoints of $\overline{AD}$ and $\overline{BC}$.
Prove: $\square EFDE$ is a $\square$.

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4. Given: \( \square ABCD; \overline{AP} \text{ bisects } \angle CAB; \overline{CQ} \text{ bisects } \angle ACD. \)
Prove: \( \square CQAP \) is a \( \square \).

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5. Given: \( \square ABDE, \angle 1 \equiv \angle 2 \)
Prove: \( \square ACDF \) is a \( \square \).

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Chapter 5 (section 5-7) – Day 7
Rectangles

Warm-Up

The measures of two consecutive angles of a parallelogram are in the ratio 5:4. What is the measure of an obtuse angle of the parallelogram?

1) 20°
2) 80°
3) 100°
4) 160°

Definition: A rectangle is a parallelogram with one right angle.

Properties of a Rectangle

1. A rectangle has all the properties of a parallelogram.
2. A rectangle contains four right angles and is therefore equiangular.
3. The diagonals of a rectangle are congruent.

<table>
<thead>
<tr>
<th>Properties of Rectangles</th>
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<tbody>
<tr>
<td>If a quadrilateral is a rectangle, then it is a parallelogram.</td>
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<tr>
<td>If a parallelogram is a rectangle, then its diagonals are congruent.</td>
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Since a rectangle is a parallelogram, a rectangle also has all the properties of parallelograms.
Let’s explore the Properties of the rectangle!

- The diagonals of a rectangle are congruent.

<table>
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<th>If WY = 19, then ZX = ?</th>
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<tr>
<td>If WY = 19, then WT = ?</td>
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<td>If TX = 4.5, then WY = ?</td>
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</table>

Rectangle GALS has diagonals GL and AS. If GL = 3a + 6 and AS = 5a − 18, then a = ?

- The angles of a rectangle are all right angles.

➤ If m∠1 = 55°, find all the missing angle measures.

m 4 | 2 = ____°
m 4 | 3 = ____°
m 4 | 4 = ____°

➤ Quadrilateral ABCD is a rectangle.

If m∠BDC = 7x + 1 and m∠ADB = 9x − 7, find m∠BDC.
Practice Problems

a. If $AE = 5$, $BC = 6$, and $DC = 8$, find $AC$, $BD$, $AD$, and $AB$.

\[ AC = \boxed{\phantom{0000}} \]
\[ BD = \boxed{\phantom{0000}} \]
\[ AD = \boxed{\phantom{0000}} \]
\[ AB = \boxed{\phantom{0000}} \]

b. If $BD = 3x - 7$ and $CA = x + 5$, find $BD$, $ED$, $CA$, and $AE$.

\[ BD = \boxed{\phantom{0000}} \]
\[ ED = \boxed{\phantom{0000}} \]
\[ CA = \boxed{\phantom{0000}} \]
\[ AE = \boxed{\phantom{0000}} \]

c. Find the measures of the numbered angles in each rectangle.

\[ m\angle 1 = \boxed{\phantom{0000}^\circ} \]
\[ m\angle 2 = \boxed{\phantom{0000}^\circ} \]
\[ m\angle 3 = \boxed{\phantom{0000}^\circ} \]

\[ m\angle 1 = \boxed{\phantom{0000}^\circ} \]
\[ m\angle 2 = \boxed{\phantom{0000}^\circ} \]
\[ m\angle 3 = \boxed{\phantom{0000}^\circ} \]
\[ m\angle 4 = \boxed{\phantom{0000}^\circ} \]

d. If $m\angle DAC = 2x + 4$ and $m\angle BAC = 3x + 1$, find $m\angle BAC$.

\[ m\angle DAC = \boxed{\phantom{0000}^\circ} \]
\[ m\angle BAC = \boxed{\phantom{0000}^\circ} \]
Rectangle Proofs

Given: Rectangle WXYZ, M is the midpoint of WX.
Prove: \( \triangle ZMY \) is isosceles.

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Given: Parallelogram ABCD with AB extended to P so that \( \overline{CP} \perp \overline{AP} \) and \( \overline{DQ} \perp \overline{AP} \).
Prove: QPCD is a rectangle.

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**CHALLENGE**
In a rectangle, the length is twice the width, and the perimeter is 48. Find the area of the rectangle.

**SUMMARY**

Properties of Rectangles A rectangle is a quadrilateral with four right angles. Here are the properties of rectangles.
A rectangle has all the properties of a parallelogram.
- Opposite sides are parallel.
- Opposite angles are congruent.
- Opposite sides are congruent.
- Consecutive angles are supplementary.
- The diagonals bisect each other.

Also:
- All four angles are right angles.
- The diagonals are congruent.

\[ \angle UTS, \angle TSR, \angle SRU, \text{ and } \angle RUT \text{ are right angles.} \]

\[ TR \cong US \]

**Example 1** Quadrilateral RUTS
above is a rectangle. If \( US = 6x + 3 \) and \( RT = 7x - 2 \), find \( x \).

The diagonals of a rectangle are congruent, so \( US = RT \).

\[
6x + 3 = 7x - 2 \\
3 = x - 2 \\
5 = x
\]

**Example 2** Quadrilateral RUTS
above is a rectangle. If \( m\angle STR = 8x + 3 \) and \( m\angle UTR = 16x - 9 \), find \( m\angle STR \).

\( \angle UTS \) is a right angle, so

\[
m\angle STR + m\angle UTR = 90. \\
8x + 3 + 16x - 9 = 90 \\
24x - 6 = 90 \\
24x = 96 \\
x = 4
\]

\[
m\angle STR = 8x + 3 = 8(4) + 3 \text{ or } 35
\]

**Exit Ticket**
In rectangle \( ABGD \), \( \overline{AC} \) and \( \overline{BD} \) are diagonals. If \( m\angle 1 = 55 \), find \( m\angle ABD \).

1. 20
2. 35
3. 55
4. 65
Chapter 5 (Section 5) – Day 8
Rhombi and Squares

Warm-Up
1. Quadrilateral $DEFG$ is a rectangle.
   If $FD = 3x - 7$ and $EG = x + 5$, find $EG$.

2. Quadrilateral $ABCD$ is a rectangle. Find each measure if $m\angle 2 = 40$.

Rhombus

Definition: A rhombus is a parallelogram with 2 congruent consecutive sides.

Properties of a Rhombus
1. A rhombus has all the properties of a parallelogram.
2. A rhombus is equilateral.
3. The diagonals of a rhombus are perpendicular to each other.
4. The diagonals of a rhombus bisect its angles.

Square

Definition: A square is a rectangle with 2 congruent consecutive sides.

Properties of a Square
1. A square has all the properties of a rectangle.
2. A square has all the properties of a rhombus.
Problems Involving the Squares

- If \( AB = 2x + 4 \) and \( CD = 3x - 5 \),
  Find \( BC \) and \( BD \).

- If \( m\angle AEB = (3x)^\circ \), find ‘x’.

- If \( m\angle BAC = (9x)^\circ \), find ‘x’.

- The perimeter of the square is 32 cm.
  Find the length of diagonal \( DB \).
Problems Involving the Rhombus

- If DM = 6y + 4 and ML = 5y + 8, find the length of KL.

- If \( m\angle D = 7x - 28 \) and \( m\angle K = 10x - 13 \), find the angles in the rhombus.

- Find the measures of the numbered angles in each rhombus.
  
  \[ m\angle 1 = \ldots^\circ \]
  
  \[ m\angle 2 = \ldots^\circ \]
  
  \[ m\angle 3 = \ldots^\circ \]
  
  \[ m\angle 4 = \ldots^\circ \]

- The diagonals of a Rhombus are 10, and 24 cm. Find the length of the side of the rhombus.
**Given:** Rhombus $ABCE$, $FEC$, $AED$, $\angle FAB = \angle DCB$.

**Prove:** $FE = DE$

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**Given:** Rhombus $ABCD$

\[
\overline{BL} = \overline{CM} , \overline{AL} = \overline{BM}
\]

**Prove:** $ABCD$ is a square

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Challenge

In the diagram: quadrilateral $MATH$ is a square, $G$ is the midpoint of $MA$, $F$ is the midpoint of $TH$, and $AT = 6$. Find the area of quadrilateral $MGTE$.

1. 9
2. 12
3. 18
4. 36

SUMMARY

Squares:

1) Opposite sides are congruent (they equal each other).
2) Opposite angles are congruent (they equal each other).
3) Consecutive angles are supplementary (they add up to 180).
4) Diagonals bisect each other (the parts are equal).
5) Diagonals are congruent (they equal each other).
6) All four corner angles are 90°.
7) Diagonals perpendicular (the form right angles in the middle).
8) Diagonals bisect angles (the angles equal to each other).

Rhombi:

1) Opposite sides are congruent (they equal each other).
2) Opposite angles are congruent (they equal each other).
3) Consecutive angles are supplementary (they add up to 180).
4) Diagonals bisect each other (the parts are equal).
5) Diagonals perpendicular (the form right angles in the middle).
6) Diagonals bisect angles (the angles are equal to each other).
7) All four sides are congruent.
8) The diagonals are NOT congruent.

Exit Ticket

<table>
<thead>
<tr>
<th>Property</th>
<th>Rectangle</th>
<th>Rhombus</th>
<th>Square</th>
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<tbody>
<tr>
<td>1. All the properties of a parallelogram?</td>
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<td>2. Equiangular (4 right corner angles?)</td>
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<td>3. Equilateral (4 congruent sides?)</td>
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<td>4. Diagonals bisect angles?</td>
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<td>5. Diagonals congruent?</td>
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<td>6. Diagonals perpendicular?</td>
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Homework

Properties of the Rectangle, Rhombus, and Square

Rectangle
- all properties of parallelograms
  - all diagonals are congruent
  - all angles measure 90°

Rhombus
- all properties of parallelograms
  - all sides are congruent
  - all diagonals are perpendicular
  - all diagonals bisect opposite angles

Square
- all properties of parallelogram
- rectangle
- rhombus

Use the properties to solve for the missing measures in the diagrams.

1. LMNO is a rectangle. If LM = 16, MN = 12, and \( \angle 1 = 60° \), find the following:
   a. ON = ______
   b. OL = ______
   c. LN = ______
   d. LX = ______
   e. \( \angle LON = \) ______
   f. \( \angle 2 = \) ______
   g. OX = ______
   h. \( \angle 3 = \) ______
   i. \( \angle 4 = \) ______

2. WXYZ is a rhombus. If WX = 4 and \( \angle WXY = 60° \), find the following:
   a. XY = ______
   b. \( \angle ZWX = \) ______
   c. \( \angle 1 = \) ______
   d. \( \angle 2 = \) ______
   e. \( \angle 3 = \) ______
   f. \( \angle 4 = \) ______
   g. WO = ______
   h. OX = ______
   i. WY = ______

3. EFGH is a square. If EF = 10, find the following:
   a. FG = ______
   b. \( \angle EFG = \) ______
   c. EG = ______
   d. EI = ______
   e. IF = ______
   f. \( \angle EIF = \) ______
   g. \( \angle 1 = \) ______
   h. \( \angle 3 = \) ______
   i. HF = ______
4. The diagonals of a rhombus are 12 and 16 cm. Find the length of a side of the rhombus.

5. The diagonals of a rhombus are 14 and 48 cm. Find the length of a side of the rhombus.

6. The shorter diagonal of a rhombus measures 18 cm. The side of the rhombus measures 41 cm. Find the length of the longer diagonal.

7. The longer diagonal of a rhombus measures 42 cm. The side of the rhombus measures 29 cm. Find the length of the shorter diagonal.
8. If $\overline{AB} \equiv \overline{DC}$, show that $ABCD$ is not a rhombus

9. Given: $YTWX$ is a parallelogram
$YP \perp TW$
$ZW \perp TY$
$TP \equiv TZ$

Prove: $YTWX$ is a rhombus

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10. Given: \( \frac{ACEF \text{ is a rhombus;}}{AC \cong BC} \)

Prove: \( \angle 1 \cong \angle 2 \)

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11. Given: \( AB \)CD is a rectangle; \( \angle ABX \cong \angle XBC \)

Prove: \( AB \)CD is a square

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Chapter 5 (Section 5) – Day 9
Trapezoids

Warm-Up

1. \(ABCD\) is a rhombus. If \(PB = 12\), \(AB = 15\), and \(m\angle ABD = 24\), find each measure.

23. \(AP\)
24. \(CP\)
25. \(m\angle BDA\)
26. \(m\angle ACB\)

2. \(WXYZ\) is a square. If \(WT = 3\), find each measure.

27. \(ZX\)
28. \(XY\)
29. \(m\angle WTZ\)
30. \(m\angle WYX\)
**Definition:** A trapezoid is a quadrilateral with one pair of parallel sides.

![Diagram of a trapezoid]

**Definition:** An isosceles trapezoid is a trapezoid with non parallel sides congruent.

![Diagram of an isosceles trapezoid]

**Properties of Isosceles Trapezoids**

- If a quadrilateral is an isosceles trapezoid, then each pair of base angles are congruent.

  ![Diagram of an isosceles trapezoid]

  \[
  \angle \text{base 1} \cong \angle \text{base 2} \\
  \angle \text{base 3} \cong \angle \text{base 4}
  \]

  \[
  \angle \text{base 1} + \angle \text{base 3} = 180^\circ \\
  \angle \text{base 2} + \angle \text{base 4} = 180^\circ
  \]

- If a quadrilateral is an isosceles trapezoid, then the diagonals are congruent

  ![Diagram of an isosceles trapezoid]

  \[
  \overline{AC} \cong \overline{BD} \\
  \overline{AD} \cong \overline{BC}
  \]
**Practice Problems**

**ALGEBRA**  Find each measure.

1. $m \angle S$

![Diagram](image1)

2. $m \angle M$

![Diagram](image2)

3. Trapezoid PORS. Find the $m \angle 1$ and $\angle 2$.

![Diagram](image3)

$m \angle 1 = ____^\circ$

$m \angle 2 = ____^\circ$

4. Isosceles Trapezoid ABCD.

![Diagram](image4)

$m \angle 1 = ____^\circ$

$m \angle 2 = ____^\circ$

$m \angle 3 = ____^\circ$

$m \angle 4 = ____^\circ$

5. Find the values of the variables.

$DF = 4x, \ EG = 2x + 16$

![Diagram](image5)

6. $AC = 7x - 15, \ BD = 4x + 15$

![Diagram](image6)
The **midsegment of a trapezoid** is the segment that connects the midpoints of the legs of the trapezoid. The theorem below relates the midsegment and the bases of a trapezoid.

\[
\text{Median} = \frac{1}{2} (\text{base} + \text{base})
\]

\[2m = b_1 + b_2\]

- The midsegment of a trapezoid is parallel to each base. \(AB \parallel MN\) and \(AB \parallel LP\)
- The length of the midsegment is one-half the sum of the length of the bases.
  \[AB = \frac{1}{2} (MN + LP)\]

7. In a trapezoid \(TSRQ\), \(TS \parallel QR\), \(U\) is the midpoint of \(TO\), and \(V\) is the midpoint of \(SR\). If \(TS = 8x + 34\), \(UV = 86\), \(QR = 14x + 92\), find the value of \(x\).

8. In trapezoid \(FGHI\), \(FG \parallel IH\) and \(J\) is the midpoint of \(FI\) and \(K\) is the midpoint of \(GH\). If \(JK = 8\), \(FG = x^2 + x - 2\) and \(IH = x^2 + 3x - 12\), find the value of \(x\).
9. In the accompanying figure, isosceles trapezoid $ABCD$ has bases of lengths 9 and 15 and an altitude of length 4. Find $AB$.

10. In the accompanying diagram, isosceles trapezoid has bases $BC$ and $AB$ with measures of 6 meters and 18 meters respectively. If altitude $CE$ is drawn and $AC$ has a measure of 14 meters, find the length of altitude $CE$ to the nearest meter.
Proofs

1. Given: \(ABCD\) is an isosceles trapezoid.
   Prove: \(\angle DAC \cong \angle CBD\)

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2. Given: \(\overline{WZ} \cong \overline{ZY}, \overline{XY}\) bisects \(\overline{WZ}\)
   and \(\overline{ZY}\), and \(\angle W \cong \angle ZXY\).
   Prove: \(WXYZ\) is an isosceles trapezoid.

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**CHALLENGE**

In the design, eight isosceles trapezoids surround a regular octagon. What is the measure of $\angle B$ in trapezoid $ABCD$?

A 35°  
B 45°  
C 55°  
D 65°

**SUMMARY**

**ISOSCELES TRAPEZOIDs:**

\[ 2m = b_1 + b_2 \]

1. Lower two base angles are congruent (they equal each other).
2. Upper two base angles are congruent (they equal each other).
3. The diagonals are congruent (they equal each other).
4. Opposite angles are supplementary (they add up to 180°).

**Exit Ticket**

CDEF is a trapezoid with $CD \parallel FE$. If $m\angle F$ and $m\angle C$ are in ratio 1:4, find the $m\angle F$. 

---

49
Homework – Trapezoids

Find each measure.

1. $m\angle T$

   \[ \begin{array}{c}
   T \\
   Y \\
   Z
   \end{array} \]

   \[ \quad 60^\circ \]

2. $m\angle Y$

   \[ \begin{array}{c}
   W \\
   X \\
   Y
   \end{array} \]

   \[ \quad 68^\circ \]

3. Trapezoid PQRS. Find the $m\angle 1$ and $\angle 2$.

   \[ \begin{array}{c}
   S \\
   P \\
   Q \\
   R
   \end{array} \]

   \[ \quad 110^\circ, 20^\circ \]

4. ABCD is an isosceles trapezoid. Find the $m\angle 1$ and $\angle 2$.

   \[ \begin{array}{c}
   A \\
   D \\
   C \\
   B
   \end{array} \]

   \[ \quad 55^\circ, 30^\circ \]

5. MATH is an isosceles trapezoid with $\overline{AT} \parallel \overline{MH}$. If $m\angle M = (3x - 9)^\circ$ and $m\angle H = (x + 3)^\circ$, find the value of $x$.

6. Let $AC = 25$ and $DB = 5x$.
7. If $EH = FG$, and $m\angle E = 65^\circ$, then $m\angle G = ?$ and $m\angle GKJ = ?$

8. $EFGH$ is an isosceles trapezoid with legs $\overline{HE}$ and $\overline{GF}$. $HF = 13$.
   Find $EJ$, $JG$, and $HJ$

9. $ABCD$ is an isosceles trapezoid with upper base $\overline{AD}$. $BD = y + 4$.
   Find $AC$
10. In trapezoid $ABCD$, $\overline{AD} \parallel \overline{BC}$, $M$ is the midpoint of $\overline{AB}$ and $N$ is the midpoint of $\overline{DC}$. If $AD = x^2+1$, $MN = 4x+1$, and $BC = x^2+2x+1$, find $AD$, $MN$, and $BC$.

11. The cross section of an attic is in the shape of an isosceles trapezoid, as shown in the accompanying figure. If the height of the attic is 9 feet, $BC = 12$ feet, and $AD = 28$ feet, find the length of $\overline{AB}$ to the nearest foot.

12. In isosceles trapezoid $ABCD$ with $\overline{AB} \cong \overline{CD}$.

a) If $BY = 12$ and $AC = 30$, what is $YD$?

b) If $BD = a^2 + 27$, $AC = 2a^2 - 54$, what is $a$?

c) If $m \angle CBA = 110^\circ$, $m \angle BAD = 4x + 10$, what is $x$?
13. Given trapezoid BARK with midsegment $\overline{NO}$.

a) $BA = 7, KR = 21, NO =$  

b) $BA = 16, NO = 22, KR =$  

c) $BA = n + 2, NO = n + 6, KR = 2n - 5$. Find $n$.  

d) $BA = c^2, NO = 6c, KR = c^2 + 18$. Find $c$.  

14. Given: TRAP is a trapezoid with $TA \cong RP$

Prove: $\triangle RPA \cong \triangle TAP$

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15. Given: Isosceles trapezoid with $BC \parallel AD$, $GP \perp AB$, $EQ \perp CD$, P and Q are midpoints of $AB$ and $CD$ respectively.

Prove: $\triangle APG \cong \triangle EQD$

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Given: Trapezoid ABCD, \( BC \parallel AD \), \( BE \) and \( CF \) are altitudes drawn to \( AD \), \( AE \cong DF \\)
Prove: Trapezoid ABCD is isosceles.

![Diagram of trapezoid ABCD with altitudes BE and CF drawn to AD]

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Warm – Up
Given: Trapezoid ABCD, $BC \parallel AD$, $EB \cong EC$
Prove: ABCD is an isosceles trapezoid

Write the missing terms in the unlabeled sections.
**SUMMARY**

**PARALLELOGRAMS (rectangles, squares, and rhombi):**

1) Opposite sides of a parallelogram are congruent.
2) Opposite angles of a parallelogram are congruent.
3) Consecutive angles in a parallelogram are supplementary.
4) The diagonals of a parallelogram bisect each other.

**RECTANGLES:**

1) Opposite sides are congruent (they equal each other).
2) Opposite angles are congruent (they equal each other).
3) Consecutive angles are supplementary (they add up to 180).
4) Diagonals bisect each other (the parts are equal).
5) Diagonals are congruent (they equal each other).
6) All four corner angles are 90°.

**SQUARES:**

1) Opposite sides are congruent (they equal each other).
2) Opposite angles are congruent (they equal each other).
3) Consecutive angles are supplementary (they add up to 180).
4) Diagonals bisect each other (the parts are equal).
5) Diagonals are congruent (they equal each other).
6) All four corner angles are 90°.
7) Diagonals perpendicular (the form right angles in the middle).
8) Diagonals bisect angles (the angles equal to each other).

**RHOMBI:**

1) Opposite sides are congruent (they equal each other).
2) Opposite angles are congruent (they equal each other).
3) Consecutive angles are supplementary (they add up to 180).
4) Diagonals bisect each other (the parts are equal).
5) Diagonals perpendicular (the form right angles in the middle).
6) Diagonals bisect angles (the angles equal to each other).
7) All four sides are congruent.
8) The diagonals are NOT congruent.

**ISOSCELES TRAPEZIODS:**

\[ \text{Median} = \frac{1}{2} (b_1 + b_2) \]

1) Lower two base angles are congruent (they equal each other).
2) Upper two base angles are congruent (they equal each other).
3) The diagonals are congruent (they equal each other).
4) Opposite angles are supplementary (they add up to 180).
Part I: True/False

1. Opposite sides of a parallelogram are congruent. T F
2. The diagonals of a parallelogram bisect each other. T F
3. Diagonals of a parallelogram are congruent. T F
4. Consecutive sides of a parallelogram are congruent. T F
5. If a parallelogram has congruent diagonals, then it is a square. T F
6. Consecutive angles of a parallelogram are congruent. T F
7. A trapezoid is a parallelogram. T F
8. The diagonals of a rhombus are perpendicular to each other. T F
9. A rectangle is a square. T F
10. A square is a rectangle. T F
11. The diagonals of a rectangle are congruent. T F
12. If the diagonals of a quadrilateral are perpendicular to each other, then the quadrilateral is a rhombus. T F
13. A parallelogram is a rhombus. T F
14. The diagonals of a rectangle bisect opposite angles. T F
15. A square is a rhombus. T F
16. A rectangle is equiangular. T F
17. The diagonals of a trapezoid are congruent. T F
18. The diagonals of a rhombus are perpendicular bisectors of each other. T F
19. A square is both a rectangle and a rhombus. T F
20. In a quadrilateral, if one pair of opposite sides are congruent and parallel, then it is a parallelogram. T F
21. Lower base angles and upper base angles are congruent in an isosceles trapezoid. T F
22. Diagonals of an isosceles trapezoid bisect each other. T F
23. Opposite angles of an isosceles trapezoid are supplementary. T F

Part II: Always, Sometimes, Never – Write A if the statement is always true, S for sometimes true, or N for never true.

24. If a parallelogram is equilateral, then it is equiangular. S
25. A quadrilateral is a parallelogram if each pair of consecutive angles is supplementary. N
26. A quadrilateral is a parallelogram if one pair of opposite sides is congruent and parallel. S
27. A rhombus is a kite. N
28. A kite is a rhombus. N
29. An isosceles trapezoid is a parallelogram. N
30. The diagonals of a kite are perpendicular. A
31. If a quadrilateral has four right angles, then it is a rectangle. A
32. If a parallelogram has one right angle, then the other angles are right angles. N

Part III: Multiple Choice – Circle the most appropriate answer.

33. A quadrilateral that has congruent diagonals is a(n):
   a) square b) rectangle c) rhombus d) isosceles trapezoid
e) all of the above f) a, b, and d are true g) none of the above

34. The best name for a quadrilateral whose diagonals bisect each other, are perpendicular, congruent, is a
   a) rectangle b) rhombus c) square d) parallelogram

35. Which of the following is not a property for a rhombus?
   a) All four sides are congruent.
   b) The diagonals are perpendicular.
   c) The diagonals bisect the opposite angles.
   d) The diagonals bisect each other.
   e) The diagonals are congruent.
36. Which of the following is true for an isosceles trapezoid?
   a) The opposite sides are congruent.  b) Opposite sides are parallel.
   c) The diagonals bisect the opposite angles. d) The diagonals bisect each other.
   e) The diagonals are congruent.

37. Which of the following is true for a kite?
   a) The opposite sides are congruent.  b) Opposite sides are parallel.
   c) The diagonals bisect the opposite angles. d) The diagonals bisect each other.
   e) The diagonals are perpendicular. f) The diagonals are congruent.

38. Which of the following has both pairs of opposite angles supplementary?
   a) rectangle   b) square   c) isosceles trapezoid   d) kite
   e) all of the above   f) both a and b   g) a, b, and c

39. Which of the following has all pairs of consecutive angles supplementary?
   a) rectangle   b) square   c) isosceles trapezoid   d) kite
   e) rhombus   f) parallelogram   g) all of the above
   h) All are true but d   i) a, b, e, and f are true

40. The most descriptive name for the figure at the right is a(n)
   a) square   b) rectangle   c) parallelogram
d) kite   e) rhombus   f) isosceles trapezoid

41. The most descriptive name for the figure at the right is a(n)
   a) square   b) rectangle   c) parallelogram
d) kite   e) rhombus   f) isosceles trapezoid

42. The most descriptive name for the figure at the right is a(n)
   a) square   b) rectangle   c) parallelogram
d) kite   e) rhombus   f) isosceles trapezoid
Part IV: Solve for the variable, sides, and angles as indicated.

43. \(ABCD\) is a rectangle.
\[AB = 2x + 4, \ AD = 4x - 2, \ DC = 3y, \ BC = y.\]
Solve for \(x\) and \(y\).

44. \(ABCD\) is a rhombus.
\[m\angle DAC = 4x + 9, \ m\angle DAB = 11x - 3.\]
Solve for \(x\), \(m\angle BAC\), and \(m\angle ABC\).

45. \(ABCD\) is a square.
\[AE = x^2, \ EB = 8x - 15.\]
Find the value(s) of \(x\), \(AE\), and \(AC\).
46. *PQRS* is a parallelogram.

\[ m\angle PQS = 12x + 7, \quad m\angle RSQ = 15x - 8. \]

Solve for \(x, m\angle PQS\).

47. \[ m\angle G = (x^2 + 40)^\circ, \]

\[ m\angle H = (15x + 4)^\circ, \]

\[ m\angle F = (4y + 1)^\circ. \]

Find the value(s) for \(x\) and \(y\).

48. \(KI = x + 5, \ IT = y, \ TE = 2y, \ KE = 3x - 3.\)

Solve for \(x, y, a,\) and \(b\).

49. \(EF = 2x + 1, AB = 3x + 6, DG = 5x - 2\)

Solve for \(x\).
### SUMMARY CHARTS:

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<tr>
<th>Special Quadrilateral</th>
<th>Diagonals</th>
<th>Diagonals Bisect</th>
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<td>Congruent</td>
<td>Perpendicular</td>
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<tr>
<td>Rectangle</td>
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<tr>
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### Property vs. Rectangle, Rhombus, Square

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<th>Rhombus</th>
<th>Square</th>
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<tbody>
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<td>1. All the properties of a parallelogram?</td>
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<td>2. Equiangular (4 right corner angles?)</td>
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<td>3. Equilateral (4 congruent sides?)</td>
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<td>4. Diagonals bisect angles?</td>
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<td>5. Diagonals congruent?</td>
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<td>6. Diagonals perpendicular?</td>
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**Flowchart**

- **Quadrilateral**
  - Sum of angles is 360
  - 2 pairs of parallel sides
  - One pair of parallel sides
- **Parallelogram**
  - 4 right angles
  - 4 congruent sides
- **Rectangle**
  - 4 right angles and 4 congruent sides
- **Rhombus**
  - 4 congruent sides
- **Trapezoid**
  - 2 congruent legs
- **Isosceles Trapezoid**
  - No parallel sides
  - 2 pair consecutive sides congruent
- **Square**
- **Kite**