Geometry Honors
Chapter 1:
Foundations for Geometry
## Unit 1: Vocabulary

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<td>complementary angles</td>
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<td>28</td>
<td>supplementary angles</td>
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<td>29</td>
<td>vertical angles</td>
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</table>
Day 1: Understanding Points, Lines, and Planes

G.CO.1 Know precise definitions of angle, circle, perpendicular line, parallel line, and line segment, based on the undefined notions of point, line, distance along a line, and distance around a circular arc.

Warm-Up  Solve for t:

\[ 5t - 2(t - 5) = 19 \]

The most basic figures in geometry are **undefined terms**, which cannot be defined by using other figures. The undefined terms *point*, *line*, and *plane* are the building blocks of geometry.

<table>
<thead>
<tr>
<th>TERM</th>
<th>NAME</th>
<th>DIAGRAM</th>
</tr>
</thead>
<tbody>
<tr>
<td>A <strong>point</strong> names a location and has no size. It is represented by a dot.</td>
<td>A capital letter point <em>P</em></td>
<td><img src="P" alt="Diagram" /></td>
</tr>
<tr>
<td>A <strong>line</strong> is a straight path that has no thickness and extends forever.</td>
<td>A lowercase letter or two points on the line <em>X</em>, <em>Y</em> or <em>YX</em></td>
<td><img src="Line" alt="Diagram" /></td>
</tr>
<tr>
<td>A <strong>plane</strong> is a flat surface that has no thickness and extends forever.</td>
<td>A script capital letter or three points not on a line plane <em>R</em> or plane <em>ABC</em></td>
<td><img src="Plane" alt="Diagram" /></td>
</tr>
</tbody>
</table>

Points that lie on the same line are **collinear**. *K*, *L*, and *M* are collinear. *K*, *L*, and *N* are **noncollinear**.

Points that lie on the same plane are **coplanar**. Otherwise they are **noncoplanar**.
**Sketches**

A line that is *contained* (lies in) in a plane

A line that intersects a plane in one point

<table>
<thead>
<tr>
<th>Coplanar points</th>
<th>Four non-coplanar points</th>
</tr>
</thead>
</table>

**Segments and Rays**

<table>
<thead>
<tr>
<th>DEFINITION</th>
<th>NAME</th>
<th>DIAGRAM</th>
</tr>
</thead>
<tbody>
<tr>
<td>A segment, or line segment, is the part of a line consisting of two points and all points between them.</td>
<td>The two endpoints $\overline{AB}$ or $\overline{BA}$</td>
<td><img src="image" alt="Diagram" /></td>
</tr>
<tr>
<td>An endpoint is a point at one end of a segment or the starting point of a ray.</td>
<td>A capital letter $C$ and $D$</td>
<td><img src="image" alt="Diagram" /></td>
</tr>
<tr>
<td>A ray is a part of a line that starts at an endpoint and extends forever in one direction.</td>
<td>Its endpoint and any other point on the ray $RS$</td>
<td><img src="image" alt="Diagram" /></td>
</tr>
<tr>
<td>Opposite rays are two rays that have a common endpoint and form a line.</td>
<td>The common endpoint and any other point on each ray $\overline{EF}$ and $\overline{EG}$</td>
<td><img src="image" alt="Diagram" /></td>
</tr>
</tbody>
</table>

**Model Problems**  
Use the diagram at right.

1) Name a point.

2) Name the line that goes through point E in two ways.

3) Name a segment.

4) Name three collinear points.

5) Name three non-collinear points.

6) Name the intersection of $\overline{EC}$ and the segment not on $\overline{EC}$.

7) Name the plane shown in the diagram.
Exercise

1) Name a point.

2) Name the line that goes through point Z in three ways.

3) Name a segment.

4) Name three coplanar points.

5) Name three non-collinear points.

6) Name the intersection of line $m$ and $\overline{YZ}$.

7) Name the plane shown in the diagram.

8) Name the points that determine this plane.

9) Name two lines that intersect line $m$.

10) Name a line that does not intersect line $m$.

Postulates about Lines and Points

A postulate, or axiom, is a statement that is accepted as true without proof. Postulates about points, lines, and planes help describe geometric properties.

<table>
<thead>
<tr>
<th>Postulate</th>
<th>Sketch</th>
<th>Illustration</th>
</tr>
</thead>
<tbody>
<tr>
<td>Two points determine a line.</td>
<td></td>
<td>Any two points are collinear.</td>
</tr>
<tr>
<td>Three points determine a plane.</td>
<td></td>
<td>Any three points are coplanar. Think of a wobbly chair. It will be stable if any three legs are touching the ground.</td>
</tr>
</tbody>
</table>
If two points lie in a plane, then the line containing those points will lie in that plane too.

If you draw two points on a piece of paper, the line that connects them is on the paper too.

The intersection of two lines is a point.

- Street intersection
- Pivot of scissors
- The letter “X”
- A plus sign

The intersection of two planes is a line.

- The crease of a book
- The edge of a door
- A river valley
- The corner where two walls meet

Check for Understanding

Two flat walls meet in the corner of a classroom. Which postulate best describes this situation?

A. Through any three noncollinear points there is exactly one plane.
B. If two points lie in a plane, then the line containing them lies in the plane.
C. If two lines intersect, then they intersect in exactly one point.
D. If two planes intersect, then they intersect in exactly one line.

Model Problems  Draw and label each of the following.

A. Plane $\mathcal{H}$ that contains two lines that intersect at M

B. $\overline{ST}$ intersecting plane $\mathcal{M}$ at R
Check for Understanding

Sketch a figure that shows two lines intersect in one point in a plane, but only one of the lines lies in the plane.

Lesson Quiz

1. Two opposite rays

2. A point on $\overrightarrow{BC}$.

3. The intersection of plane $N$ and plane $T$

4. A plane containing E, D, and B.

Draw each of the following.

5. a line intersecting a plane at one point

6. a ray with endpoint $P$ that passes through $Q$
GUIDED PRACTICE

Vocabulary  Apply the vocabulary from this lesson to answer each question.
1. Give an example from your classroom of three collinear points.
2. Make use of the fact that endpoint is a compound of end and point and name the endpoint of $\overline{ST}$.

1 Use the figure to name each of the following.
3. five points
4. two lines
5. two planes
6. point on $\overrightarrow{BD}$

2 Draw and label each of the following.
7. a segment with endpoints $M$ and $N$
8. a ray with endpoint $F$ that passes through $G$

SEE EXAMPLE 3
p. 7

USE THE FIGURE TO NAME EACH OF THE FOLLOWING.
9. a line that contains $A$ and $C$
10. a plane that contains $A$, $D$, and $C$

SEE EXAMPLE 4
p. 8

Sketch a figure that shows each of the following.
11. three coplanar lines that intersect in a common point
12. two lines that do not intersect
G.CO.1 Know precise definitions of angle, circle, perpendicular line, parallel line, and line segment, based on the undefined notions of point, line, distance along a line, and distance around a circular arc.

Warm-Up

a. Draw and label the following:
   i. A line containing points X and Y
   ii. A pair of opposite rays that both contain point R

b. Campers often use a cooking stove with three legs. Why might they prefer this design to a stove that has four legs?

The distance along a line between any two points is the absolute value of the difference of the coordinates. The coordinates can be measured in a variety of units, such as inches or centimeters.

If the coordinates of points A and B are \( a \) and \( b \), then the distance between \( A \) and \( B \) is \( |a - b| \) or \( |b - a| \). The distance between \( A \) and \( B \) is also called the length, or measure of \( AB \), or \( AB \).

Exercise

Find each length.

\[ \begin{align*}
A & \quad B & \quad C \\
\text{A}. & \quad BC & \quad \text{B}. & \quad AC
\end{align*} \]

Symbol Review – it is important not to mix these up!

\[ \begin{align*}
\overline{AB} & \quad \overline{AB} & \quad \overline{AB} & \quad \overline{AB} \\
\overline{AB} & \quad \overline{AB} & \quad \overline{AB} & \quad \overline{AB}
\end{align*} \]

Congruent Segments  Congruent segments are segments that have the same length. In the diagram, \( PQ = RS \), so you can write \( PQ \cong RS \). This is read as “segment \( PQ \) is congruent to segment \( RS \).”

Tick marks are used in a figure to show congruent segments.
Constructing Congruent Segments

A construction is a special drawing that only uses a compass and a straightedge. Constructions can be justified by using geometric principles to create figures.

Model Problem   Construct a segment congruent to \(\overline{AB}\).

Exercise   Construct a segment congruent to \(\overline{AB}\). Then answer the questions below.

Think About It. Why does this construction result in a line segment with the same length as \(\overline{AB}\)?

Betweenness

In order for you to say that a point \(B\) is between two points \(A\) and \(C\), all three points must lie on the same line, and \(AB + BC = AC\).

**Postulate 1-2-2 (Segment Addition Postulate)**

If \(B\) is between \(A\) and \(C\), then \(AB + BC = AC\).

Model Problems

A. \(M\) is between \(N\) and \(O\). Find \(NO\).

B. \(H\) is between \(I\) and \(J\). If \(HI = 3.9\) and \(HJ = 6.2\), find \(IJ\).
Exercise

1. \( E \) is between \( D \) and \( F \). Find \( DF \).

2. \( H \) is between \( I \) and \( J \). If \( IJ = 25 \) and \( HI = 13 \), find \( HJ \).

Midpoint

The midpoint \( M \) of \( \overline{AB} \) is the point that bisects, or divides the segment into two congruent segments. If \( M \) is the midpoint of \( \overline{AB} \), then \( AM = MB \). So if \( AB = 6 \), then \( AM = 3 \) and \( MB = 3 \).

Model Problems

A. \( D \) is the midpoint of \( \overline{EF} \), \( ED = 4x + 6 \), and \( DF = 7x - 9 \). Find \( ED \), \( DF \), and \( EF \).

B. \( B \) is the midpoint of \( \overline{AC} \). \( AB = 8v \), and \( AC = 2v + 42 \). What is \( BC \)?

Exercise

1. \( H \) is the midpoint of \( \overline{IJ} \).
   i. If \( IJ = 18 \), find \( HI \) and \( HJ \).
   ii. If \( IH = 10 \), find \( HJ \) and \( IJ \).
2. E is the midpoint of $\overline{DF}$. $DE = 2x + 4$ and $EF = 3x - 1$. Find $DE$, $EF$, and $DF$.

$DE =$ ____________________

$EF =$ ____________________

$DF =$ ____________________

3. X is the midpoint of $\overline{AT}$. If $AX = 4x$ and $AT = 3x + 25$, find $AX$, $XT$, and $AT$.

4. S is between R and T. Does that mean that S must be a midpoint? Explain and sketch an appropriate diagram of $\overline{RT}$.

**Homework Day 2** Complete in notebook.

Holt - pg: 17 – 19/ #s 12, 14 – 15, 17- 20, 22, 29, 32, 41

McDougal and Littell - pg: 33 – 35/ #11, 21
G.CO.1 Know precise definitions of angle, circle, perpendicular line, parallel line, and line segment, based on the undefined notions of point, line, distance along a line, and distance around a circular arc.

Warm-Up

M is the midpoint of segment $\overline{AD}$. If $AM = x + 3$ and $AD = x^2 + 6$, find:

a) the value of $x$.  
b) $MD$

An angle is a figure formed by two rays, or sides, with a common endpoint called the vertex (plural: vertices).

You can name an angle several ways: by its vertex, by a point on each ray and the vertex, or by a number.

The set of all points between the sides of the angle is the interior of an angle. The exterior of an angle is the set of all points outside the angle.

Model Problem

Name the angle at right in four ways.

Note: You cannot name an angle just by its vertex if the point is the vertex of more than one angle. In this case, you must use all three points to name the angle, and the middle point is always the vertex.

Wrong:  
Right:
Exercise

| 1) Draw and label \(\angle DEF\) below. | 2) Draw and label \(\angle QRS\) with \(\overline{RT}\) in the interior of the angle. |

3) Write the different ways you can name each given angle in the diagram.

1) \(\angle SVT\): ______________________
   \(\angle SVR\): ______________________
   \(\angle RVT\): ______________________

2) \(\angle 3\): ______________________
   \(\angle 4\): ______________________
   \(\angle DVF\): ______________________

Measuring Angles

The measure of an angle is usually given in degrees. Since there are 360° in a circle, one degree is 1/360 of a circle.

When referring to the degree measure of angles, we write:

\[ m\angle ABC \]

which is read, “the measure of angle ABC.”
Types of Angles

**Congruent Angles**

*Congruent angles* are angles that have the same degree measure.

In the diagram, \( \angle ABC = \angle DEF \), so you can write \( \angle ABC \cong \angle DEF \). This is read as “angle ABC is congruent to angle DEF.”

*Arc marks* are used to show that the two angles are congruent.

Check for Understanding

In the diagram, assume \( \angle DEF \) is *congruent* to \( \angle FEG \).

1) Explain what this means in your own words.

2) Mark the diagram at right to show this congruence.

3) Write "\( \angle DEF \) is congruent to \( \angle FEG \)" using symbols.

4) Write "the measure of \( \angle DEF \) equals the measure of \( \angle FEG \)" in symbols.
Angle Addition Postulate

Postulate 1-3-2 Angle Addition Postulate

If \( S \) is in the interior of \( \angle PQR \), then
\[ m\angle PQS + m\angle SQR = m\angle PQR. \]
(\( \angle \) Add. Post.)

Note that this is similar to segment addition:

“PART + PART = WHOLE”

Model Problem  Mark up the diagram appropriately. Then answer the question below.

In the accompanying diagram, \( m\angle DEG = 115^\circ \), \( m\angle DEF = 2x - 1^\circ \), \( m\angle GEF = 3x + 1^\circ \). Find \( m\angle DEF \) and \( m\angle GEF \).

Exercise

Find \( m\angle KLM \) if \( m\angle KLB = 26^\circ \) and \( m\angle BLM = 60^\circ \).

Find \( m\angle WDC \) if \( m\angle EDC = 145^\circ \) and \( m\angle EDW = 61^\circ \). 
The Angle Bisector

An angle bisector is a ray that divides an angle into two congruent angles.

Given: \(JK\) bisects \(\angle LJM\)

Conclusion: \(\angle LJK \cong \angle MJK\)

Model Problems

A. \(KM\) bisects \(\angle JKL\), \(m\angle JKM = (4x + 6)^\circ\), and \(m\angle MKL = (7x - 12)^\circ\). Find \(m\angle JKM\).
B. Given: \( \overrightarrow{QS} \) bisects \( \angle PQR \).

1. Sketch and label \( \angle PQR \) first, then draw ray \( \overrightarrow{QS} \) from point Q.

2. \( m\angle PQS = (5y - 1)\degree \), and \( m\angle PQR = (8y + 12)\degree \). Find \( m\angle PQS \).

Exercise

\( \overrightarrow{BD} \) bisects \( \angle ABC \). Find \( m\angle ABD \) if \( m\angle ABD = (6x + 4)\degree \) and \( m\angle DBC = (8x - 4)\degree \).
Homework  Day 3  Complete in notebook.

Holt: pages 24-27 #’s 8, 10, 11, 18, 19-22 all, 29, 31, 32, 38, 44, 45, 47

McDougal, Littell: pages 16-17 #’s 15, 17, 21 and page 27 # 14
Day 4: Measuring and Constructing Angles

G.CO.1 Know precise definitions of angle, circle, perpendicular line, parallel line, and line segment, based on the undefined notions of point, line, distance along a line, and distance around a circular arc.

G.CO.12 Make formal geometric constructions with a variety of tools and methods (compass and straightedge, string, reflective devices, paper folding, dynamic geometric software, etc.

Warm-Up

\( \overline{NT} \) bisects \( \angle MNS \). Find \( m\angle TNS \) if \( m\angle MNT = (2x - 30)^\circ \) and \( m\angle DBC = (2x)^\circ \).

Independent Practice

Algebraic Review

\( \overline{QS} \) bisects \( \angle PQT \).

1. If \( m\angle PQT = 60 \) and \( m\angle PQS = 4x + 14 \), find the value of \( x \).

2. If \( m\angle PQS = 3x + 13 \) and \( m\angle SQT = 6x - 2 \), find \( m\angle PQT \).
Constructing Congruent Angles

Task: **Construct** an angle with vertex X that has the same measure as S.

A. In the space below, use a straightedge to draw a ray with endpoint X.

B. Place the point of your compass on S and draw an arc that intersects both sides of the angle. Label the points of intersection T and U.

C. Without adjusting the compass, place the point of the compass on X and draw an arc that intersects the ray. Label the intersection Y.

D. Place the point of the compass on U and open it to the distance TU.

E. Without adjusting the compass, place the point of the compass on Y and draw an arc. Label the intersection with the first arc Z.

F. Use a straightedge to draw XZ.

You Try:

\[ \text{X} \bullet \]

Why does this construction work?
**Practice**

**Construct an angle with the same measure as the given angle.**

1. 

2. 

3. 

**Think About It.** If the angle you construct has longer sides than the original angle, can the two angles still have the same measure? Explain.

_____________________________________________________________________________________
Constructing an Angle Bisector

**Task:**
Construct the bisector of $\angle M$. Work directly on the angle at right.

- **A** Place the point of your compass on point $M$. Draw an arc that intersects both sides of the angle. Label the points of intersection $P$ and $Q$.

- **B** Place the point of the compass on $P$ and draw an arc in the interior of the angle.

- **C** Without adjusting the compass, place the point of the compass on $Q$ and draw an arc that intersects the arc from Step B. Label the intersection of the arcs $R$.

- **D** Use a straightedge to draw $\overline{MR}$.

**Why does this construction work?**

**Practice**
Construct the bisector of each angle.

4. 

5. 

6.
7) Explain how you can use a compass and straightedge to construct an angle that has twice measure of $\angle A$. Then do the construction in the space provided.

8) Explain how you can use a compass and straightedge to construct an angle that has $\frac{1}{4}$ the measure of $\angle B$. Then do the construction in the space provided.
1) Given the diagram at right. Write the number that names the same angle. If the angle does not exist in the diagram, write “does not exist.” Some angles may be used more than once.

   a) \( \angle PMO \) ____________________
   b) \( \angle MNO \) ____________________
   c) \( \angle MPO \) ____________________
   d) \( \angle MOP \) ____________________
   e) \( \angle RPQ \) ____________________
   f) \( \angle QRP \) ____________________
   g) \( \angle MPR \) ____________________
   h) \( \angle NMO \) ____________________
   i) \( \angle ONM \) ____________________
   j) \( \angle NOM \) ____________________

**ALGEBRA** In the figure \( \overline{BA} \) and \( \overline{BC} \) are opposite rays. \( \overline{BF} \) bisects \( \angle CBE \).

3. If \( m\angle EBF = 6x + 4 \) and \( m\angle CBF = 7x - 2 \), find \( m\angle EBF \).

4. If \( m\angle 3 = 4x + 10 \) and \( m\angle 4 = 5x \), find \( m\angle 4 \).
5. If $m\angle 3 = 6y + 2$ and $m\angle 4 = 8y - 14$, find $m\angle CBE$.

6. Let $m\angle 1 = m\angle 2$. If $m\angle ABE = 100$ and $m\angle ABD = 2(r + 5)$, find $r$ and $m\angle DBE$.

Draw each of the following.

7. a line that contains $\overline{AB}$ and $\overline{CB}$

8. two different lines that intersect $\overline{MN}$

9. a plane and a ray that intersect only at $Q$

10. **Critical Thinking** Can an obtuse angle be congruent to an acute angle? Why or why not?

11. **Short Response** If an obtuse angle is bisected, are the resulting angles acute or obtuse? Explain.
G.CO.1 Know precise definitions of angle, circle, perpendicular line, parallel line, and line segment, based on the undefined notions of point, line, distance along a line, and distance around a circular arc.

**Warm-Up**

\( \overline{RT} \) bisects \( \angle QRS \). If \( m\angle QRT = x + 15 \), \( m\angle TRS = 2y + 10 \), and \( m\angle QRS = 2x + 2y \), find the value of \( x \) and \( y \).

Many pairs of angles have special relationships. Some relationships are because of the *measurements* of the angles in the pair. Other relationships are because of the *positions* of the angles in the pair.

**Pairs of Angles**

- **Adjacent angles** are two angles in the same plane with a common vertex and a common side, but no common interior points. \( \angle 1 \) and \( \angle 2 \) are adjacent angles.

- A **linear pair** of angles is a pair of adjacent angles whose noncommon sides are opposite rays. \( \angle 3 \) and \( \angle 4 \) form a linear pair.

**Vertical angles** are two nonadjacent angles formed by two intersecting lines.

\( \angle 1 \) and \( \angle 3 \) are vertical angles, as are \( \angle 2 \) and \( \angle 4 \).

Vertical angles are always congruent.
Exercise

For #1-2, tell whether the angles are only adjacent, adjacent and form a linear pair, or not adjacent.

1) $\angle AEB$ and $\angle BED$

2) $\angle AEB$ and $\angle DEC$

3) Using the diagram at right, name:
   
   a) a pair of **vertical angles**
   
   b) two angles that form a **linear pair**
   
   c) two adjacent angles that do not form a **linear pair**
Complementary and Supplementary Angles

**Complementary angles** are two angles whose measures have a sum of $90^\circ$. \(\angle A\) and \(\angle B\) are complementary.

**Supplementary angles** are two angles whose measures have a sum of $180^\circ$. \(\angle A\) and \(\angle C\) are supplementary.

We say that \(\angle A\) is the complement of \(\angle B\) and the supplement of \(\angle C\).

If two angles form a $90^\circ$ angle (or a right angle), we often see this marked with a square corner:

![Diagram showing a right angle](image)

If supplementary angles are also adjacent, they form a linear pair.

![Diagram showing linear pair](image)

**Exercise**

1) Explain the relationship between \(\angle DZQ\) and \(\angle PQZ\).

2) Find \(m\angle DZQ\) and \(m\angle QZP\).
3) Explain the relationship between $\angle STQ$ and $\angle PTQ$.

4) If $m\angle STQ = 2x + 30$ and $m\angle PTQ = 8x$, find $m\angle STQ$.

Algebraic Representations of Complements

<table>
<thead>
<tr>
<th>Given an angle of…</th>
<th>Calculate its complement:</th>
</tr>
</thead>
<tbody>
<tr>
<td>40°</td>
<td></td>
</tr>
<tr>
<td>60°</td>
<td></td>
</tr>
<tr>
<td>10°</td>
<td></td>
</tr>
<tr>
<td>$x^\circ$</td>
<td></td>
</tr>
</tbody>
</table>

Algebraic Representation of Supplements

<table>
<thead>
<tr>
<th>Given an angle of …</th>
<th>Calculate its supplement:</th>
</tr>
</thead>
<tbody>
<tr>
<td>40°</td>
<td></td>
</tr>
<tr>
<td>60°</td>
<td></td>
</tr>
<tr>
<td>10°</td>
<td></td>
</tr>
<tr>
<td>$x^\circ$</td>
<td></td>
</tr>
</tbody>
</table>
Model Problems  Try these first on your own:

1) An angle is 10 more than 3 times the measure of its complement. Find the measure of the complement.

2) Five times the complement of an angle less twice the angle’s supplement is 40. Find the measure of the supplement.

Ratio Problems

3) Two supplementary angles are in the ratio 11:7. Find the measure of each angle.
Independent Practice

For Exercises 1–6, use the figure at the right. Name an angle or angle pair that satisfies each condition.

1. Name two acute vertical angles.
2. Name two obtuse vertical angles.
3. Name a linear pair.
4. Name two acute adjacent angles.
5. Name an angle complementary to $\angle EKH$.
6. Name an angle supplementary to $\angle FKG$.

Name an angle or angle pair that satisfies each condition.
7) Name two obtuse vertical angles.
8) Name a linear pair whose vertex is $B$.
9) Name an angle not adjacent to, but complementary to $\angle FGC$.
10) Name an angle adjacent and supplementary to $\angle DCB$.

11) If $m\angle PTQ = 3y - 10$ and $m\angle QTR = y$, find the value of $y$ so that $\angle PTR$ is a right angle.

Determine whether each statement can be assumed from the figure. Explain.
12) $\angle NQO$ and $\angle OQP$ are complementary.
13) $\angle SRQ$ and $\angle QRP$ is a linear pair.
14) $\angle MQN$ and $\angle MQR$ are vertical angles.
Homework Day 5 Complete in notebook.

Holt: pages 31-33 #9, 10, 11, 22, 27, 31, 32, 44, 45, 46

McDougal, Littell page 71 #’s 18, 21, 25 page 103 # 15
Day 6: The Distance Formula

G.GPE.4 Use coordinates to prove simple geometric theorems algebraically.

Warm-Up

Which pair of angles are supplementary?

F \( \angle USV, \angle VSW \)  G \( \angle VSW, \angle WSR \)
H \( \angle TSV, \angle VSW \)  J \( \angle TSR, \angle USW \)

The Distance Formula

\( \overline{AB} \) has endpoints \( A(2, 5) \) and \( B(-4, -3) \).
How can we find the length of this line segment?

Draw a right triangle and label the third point C.

Using the Pythagorean Theorem:

(1) \( AB^2 = AC^2 + BC^2 \)
(2) \( AB^2 = _____^2 + _____^2 \)
(3) \( AB^2 = ________ \)
(4) \( AB = ________ \)

How did you find the lengths of \( \overline{AC} \) and \( \overline{BC} \) in step (2)? ________________________________

How can you find these lengths using the coordinates? ________________________________

How did you solve for \( AB \) in step (4)? ________________________________

The length of a line segment with endpoints \((x_1, y_1)\) and \((x_2, y_2)\) is given by:

\[
\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}
\]
Model Problem

A. Use the distance formula to find the length of $AB$:

1. Label the points: (-2, -3) (4, 5)

2. Plug into the formula and simplify:

$$\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Guided Practice

B. $CD$ has coordinates (-1, -2) and (2, 6).

Use the distance formula to find the length of $CD$ to the nearest tenth.

1. Label the points: (-1, -2) (2, 6)

2. Plug into the formula and simplify:

$$\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$
Write the distance formula in this box:

C. $\overline{CD}$ has endpoints C (-2, 4) and D (6, 0).

Plot $\overline{CD}$ on the axes at right and find CD.
Express your answer in simplest radical form.

D. What is the distance between points (-1, -2) and (5, 0)?

Independent Practice

1) Find the distance between the points (-1, -1) and (2, -5).
Write the distance formula in this box:

2) Find, in radical form, the distance between points (-1, -2) and (5, 0).

3) Find the length of \( \overline{PQ} \).

4) Find, in simplest radical form, the length of the line segment joining points (1, 5) and (3, 9).

5) Express, in radical form, the distance between the points (2, 4) and (0, -5).
Challenge!

The vertices of \( \triangle ABC \) are \( A(2, 3) \), \( B(5, 7) \), and \( C(1, 4) \).

Show that \( \triangle ABC \) is an isosceles triangle.
Find the distance between each pair of points. Express in simplest radical form if necessary.

\((-2, 3), (-7, -7)\)

\((2, -9), (-1, 4)\)

\((5, 9), (-7, -7)\)

\((8, 5), (-1, 3)\)

\((-10, -7), (-8, 1)\)

\((-6, -10), (-2, -10)\)

Find the length of each line segment. Round to the nearest tenth if necessary.
Day 7: The Midpoint Formula

G.GPE.4 Use coordinates to prove simple geometric theorems algebraically.

Warm-Up

Ben and Kate are making a map of their neighborhood. They decide to make one unit on the graph paper correspond to 100 yards. They put their homes on the map as shown below.

How many yards apart are Kate and Ben’s homes?

The Midpoint Formula

\( \overline{AB} \) has coordinates \( A(2, 5) \) and \( B(-4, -3) \).

How can we find the midpoint of this segment?

1) Plot \( \overline{AB} \) on the axes at right.

2) Find and plot the midpoint of \( \overline{AB} \) by eye. Label this point \( M \).

3) Explain how you know that \( M \) is the midpoint of \( \overline{AB} \). Justify your answer mathematically.
A. Finding the Midpoint Given the Endpoints

Model Problem

Find the **midpoint** of the segment whose endpoints are A (-2, 6) and B (6, -4).

Exercise

1) Plot and find the coordinates of the **midpoint** of the segment whose endpoints are (-5, 1) and (0, -5).

2) What are the coordinates of the **midpoint** of the segment joining (5, -3) and (6, 3)?

B. Finding the Missing Endpoint Given the Midpoint

Model Problem

The **midpoint** M of \( \overline{AB} \) is (-1, 1). If the coordinates of A are (2, -1), find the coordinates of **endpoint** B.

*Method #1: Graphic Solution*

Plot the known **endpoint** and the **midpoint** on the graph.

Extend the segment to the other **endpoint**.

Answer: ___________
**Method #2: Use a Number Line**

The **midpoint** \( M \) of \( AB \) is \((-1, 1)\). If the coordinates of \( A \) are \((2, -1)\), find the coordinates of **endpoint** \( B \).

**Method #3: Algebraically**

The **midpoint** \( M \) of \( AB \) is \((-1, 1)\). If the coordinates of \( A \) are \((2, -1)\), find the coordinates of **endpoint** \( B \).

**Exercise**

Use any correct method. (The use of the graphs is optional.)

a) \( M \) is the **midpoint** of \( CD \). If the coordinates of \( C \) are \((6, 4)\) and the coordinates of \( M \) are \((0, 6)\), find the coordinates of point \( D \).
b) The coordinates of the **midpoint** of a segment are (-4, 1) and the coordinates of one **endpoint** are (-6, -5). Find the coordinates of the other **endpoint**.

c) The coordinates of the **center** of a circle are (0, 0). If one **endpoint** of the diameter is (-3, 4), find the coordinates of the other endpoint of the diameter.1 (Hint: SKETCH IT!!)

**Independent Practice** You may use graph paper if you wish.

1) What are the coordinates of the **midpoint** of the line segment that connects the points (1,2) and (6, 7)?

2) In a circle, the coordinates of the **endpoints** of the diameter are (4,5) and (10,1). What are the coordinates of the **center** of the circle?

3) What are the coordinates of the **midpoint** of the segment whose **endpoints** are (-4, 6) and (-8, -2)?

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1 BONUS #1: Find the length of the radius of this circle.
BONUS #2: Find the area of this circle in terms of π.
4) The coordinates of the **midpoint** of line segment AB are (1,2). If the coordinates of A are (1,0), find the coordinates of point B.

5) The **midpoint** of $\overline{AB}$ is M. If the coordinates of A are (2, -6), and the coordinates of M are (5, -1), find the coordinates of B.
Homework There are optional graphs at the end of this homework if you need them.

Find the midpoint of the line segment with the given endpoints.

\((-4, 4), (5, -1)\)
\((-1, -6), (-6, 5)\)

\((2, 4), (1, -3)\)
\((-4, 4), (-2, 2)\)

Find the midpoint of each line segment.

1)

2)

3)

4)
Find the other endpoint of the line segment with the given endpoint and midpoint.

Endpoint: (−1, 9), midpoint: (−9, −10)  
Endpoint: (2, 5), midpoint: (5, 1)

Endpoint: (5, 2), midpoint: (−10, −2)  
Endpoint: (9, −10), midpoint: (4, 8)

Endpoint: (−9, 7), midpoint: (10, −3)  
Endpoint: (−6, 4), midpoint: (4, 8)
Day 8: Partitioning a Line Segment

G.GPE.6. Find the point on a directed line segment between two given points that partitions the segment in a given ratio.

Warm-Up

The midpoint of line segment AB is (-2, 3). If point A has coordinates (4, 2), find the coordinates of point B.

Understanding Ratios

Let’s say that instead of dividing a line segment in half, we divide it into a ratio of 2:3. That means there will be five equal parts, because $2 + 3 = 5$:

![Diagram showing five equal parts]

We can use this idea to partition line segments into any ratio we choose.

Model Problem

The endpoints of $\overline{DEF}$ are D(1,4) and F(16,14). Determine and state the coordinates of point E, if $DE:EF = 2:3$.

Exercise

Directed line segment $\overline{PT}$ has endpoints whose coordinates are P(-2,1) and T(4,7). Determine the coordinates of point J that divides the segment in the ratio 2 to 1.
Independent Practice/Homework

1) The coordinates of the endpoints of $\overline{AB}$ are $A(−6,−5)$ and $B(4,0)$. Point $P$ is on $\overline{AB}$. Determine and state the coordinates of point $P$, such that $AP:PB$ is 2:3.

2) What are the coordinates of the point on the directed line segment from $K(−5,−4)$ to $L(5,1)$ that partitions the segment into a ratio of 3 to 2?

3) Point $B$ is between $A(2, 5)$ and $C(10,1)$. Find the coordinates of $B$, if $AB:BC = 1:3$. 
4) A line segment has endpoints (-6, 7) and (9, 2). What are the coordinates of the point on this line segment that divides it into a ratio of 2:3?

5) Point X divides \( \overline{MN} \) into a ratio of 3:5. If the coordinates of M are (4, 3) and the coordinates of N are (20,11), find the coordinates of X.

6) Directed line segment DEG has endpoints D(0, 8) and G(-24, -16). Find the coordinates of point E such that DE:EG = 5:7.
7) \( SP \) has endpoints \( S(-11, 6) \) and \( P(10, -1) \). Point \( U \) is on \( SP \). Determine and state the coordinates of \( U \) such that \( SU:UP = 3:4 \).

8) Two points are located on a coordinate grid at \((8, 1)\) and \((-2, 16)\). Find the coordinates of the point that is \(1/5\) of the directed distance from \((8, 1)\) to \((-2, 16)\).

9) Line segment \( AB \) has endpoints \( A(4, 9) \) and \( B(9, 19) \). Is the point that divides \( AB \) into a ratio of \(2:3\) the same point that divides it into a ratio of \(3:2\)? Explain.

10) **Challenge**
Point \( B \) on line segment \( AC \) divides \( AC \) into a ratio of \(2:3\). If the coordinates of \( A \) are \((4, -1)\) and the coordinates of \( B \) are \((10, 3)\), find the coordinates of point \( C \).
Day 9: Review

As you read, underline all important terms from this unit. Remember to draw diagrams!

1. Draw and label plane \( \mathcal{N} \) containing two lines that intersect at \( B \).

Use the figure to name each of the following.

2. four noncoplanar points
3. line containing \( B \) and \( E \)

4. The coordinate of \( A \) is \(-3\), and the coordinate of \( B \) is \(0.5\). Find \( AB \).

5. \( E, F, \) and \( G \) represent mile markers along a straight highway. Find \( EF \).

6. \( I \) is the midpoint of \( \overline{HK} \). Find \( HI, JK, \) and \( HK \).

7. For (a)-(d), use the diagram at right.

   a) Name the vertex to all the angles in the diagram. ________

   b) Name the angle vertical to \( \angle CBA \). ________________

   c) If \( \angle FBE \) is a right angle, name two complementary angles.

   d) Name the angle supplementary to \( \angle DBC \). ______________

9) \( \overline{TV} \) bisects \( \angle RTS \). If \( m\angle RTV = (16x - 6)^\circ \) and \( m\angle VTS = (13x + 9)^\circ \), what is \( m\angle RTV \)?
10) Find the distance between $A(-12, 13)$ and $B(-2, -11)$.

11) Find the midpoint of the segment with endpoints $(-4, 6)$ and $(3, 2)$.

12) $M$ is the midpoint of $LN$. $M$ has coordinates $(-5, 1)$ and $L$ has coordinates $(2, 4)$. Find the coordinates of $N$.

13) Directed line segment $GH$ is divided by point $I$ into a ratio of 4:5. If point $G$ has coordinates $(-3, 5)$ and point $H$ has coordinates $(6, -13)$, determine and state the coordinates of point $I$. 
Constructions

14) Construct a segment congruent to $\overline{AB}$.

15) Copy angle $S$ at vertex $X$.

16) Construct the bisector of the angle below. Label it $\overline{QT}$. Name two congruent angles.