Foundations for Geometry

<table>
<thead>
<tr>
<th>Section 1A</th>
<th>Section 1B</th>
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</thead>
<tbody>
<tr>
<td><strong>Euclidean and Construction Tools</strong></td>
<td><strong>Coordinate and Transformation Tools</strong></td>
</tr>
<tr>
<td>1-1 Understanding Points, Lines, and Planes</td>
<td>1-5 Using Formulas in Geometry</td>
</tr>
<tr>
<td>1-2 Technology Lab Explore Properties Associated with Points</td>
<td>Connecting Geometry to Algebra</td>
</tr>
<tr>
<td>1-2 Measuring and Constructing Segments</td>
<td>Graphing in the Coordinate Plane</td>
</tr>
<tr>
<td>1-3 Measuring and Constructing Angles</td>
<td>1-6 Midpoint and Distance in the Coordinate Plane</td>
</tr>
<tr>
<td>1-4 Pairs of Angles</td>
<td>1-7 Transformations in the Coordinate Plane</td>
</tr>
<tr>
<td>1-7 Technology Lab Explore Transformations</td>
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</table>

Pacing Guide for 45-Minute Classes

<table>
<thead>
<tr>
<th>DAY 1</th>
<th>DAY 2</th>
<th>DAY 3</th>
<th>DAY 4</th>
<th>DAY 5</th>
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<tbody>
<tr>
<td>1-1 Lesson</td>
<td>1-2 Technology Lab</td>
<td>1-3 Lesson</td>
<td>1-4 Lesson</td>
<td>Multi-Step Test Prep Ready to Go On?</td>
</tr>
<tr>
<td>DAY 6</td>
<td>DAY 7</td>
<td>DAY 8</td>
<td>DAY 9</td>
<td>DAY 10</td>
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<tr>
<td>1-5 Lesson</td>
<td>Connecting Geometry to Algebra</td>
<td>1-7 Lesson</td>
<td>1-7 Technology Lab</td>
<td>Multi-Step Test Prep Ready to Go On?</td>
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<tr>
<td>DAY 11</td>
<td>DAY 12</td>
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<tr>
<td>Chapter 1 Review</td>
<td>Chapter 1 Test</td>
<td></td>
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</tr>
</tbody>
</table>

Pacing Guide for 90-Minute Classes

<table>
<thead>
<tr>
<th>DAY 1</th>
<th>DAY 2</th>
<th>DAY 3</th>
<th>DAY 4</th>
<th>DAY 5</th>
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</thead>
<tbody>
<tr>
<td>1-1 Lesson</td>
<td>1-3 Lesson</td>
<td>Multi-Step Test Prep Ready to Go On?</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1-2 Technology Lab</td>
<td>1-4 Lesson</td>
<td>Connecting Geometry to Algebra</td>
<td>1-7 Technology Lab Multi-Step Test Prep Ready to Go On?</td>
<td></td>
</tr>
<tr>
<td>DAY 6</td>
<td>Chapter 1 Review</td>
<td></td>
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</tr>
<tr>
<td>Chapter 1 Test</td>
<td></td>
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</tr>
</tbody>
</table>
## Ongoing Assessment and Intervention

### Assess Prior Knowledge

#### Before Chapter 1 Testing
- **Diagnose mastery of concepts in the chapter.**
  - **Ready to Go On?** SE pp. 35, 59
  - **Multi-Step Test Prep** SE pp. 34, 58
  - **Section Quizzes** AR pp. 5–6
  - **Test and Practice Generator**

#### Before Chapter 1
- **Diagnose readiness for the chapter.**
  - **Are You Ready?** SE p. 3
  - **Are You Ready? Intervention** Skills 20, 57, 60, 79

#### Before Every Lesson
- **Diagnose readiness for the lesson.**
  - **Warm Up** TE, every lesson

#### During Every Lesson
- **Diagnose understanding of lesson concepts.**
  - **Check It Out!** SE, every example
  - **Think and Discuss** SE, every lesson
  - **Write About It** SE, every lesson
  - **Journal** TE, every lesson

#### After Every Lesson
- **Diagnose mastery of lesson concepts.**
  - **Lesson Quiz** TE, every lesson
  - **Alternative Assessment** TE, every lesson
  - **Test Prep** SE, every lesson
  - **Test and Practice Generator**

### Summative Assessment

#### Before High Stakes Testing
- **Diagnose mastery of benchmark concepts.**
  - **College Entrance Exam Practice** SE p. 65
  - **New York Test Prep** SE pp. 68–69

#### Before Chapter 1
- **Diagnose readiness for the chapter.**
  - **Are You Ready?** SE p. 3

#### During Every Lesson
- **Diagnose understanding of lesson concepts.**
  - **Check It Out!** SE, every example
  - **Think and Discuss** SE, every lesson
  - **Write About It** SE, every lesson
  - **Journal** TE, every lesson

#### After Every Lesson
- **Diagnose mastery of lesson concepts.**
  - **Lesson Quiz** TE, every lesson
  - **Alternative Assessment** TE, every lesson
  - **Test Prep** SE, every lesson
  - **Test and Practice Generator**

### Formative Assessment

#### Before Chapter 1
- **Diagnose readiness for the chapter.**
  - **Are You Ready?** SE p. 3

#### Before Every Lesson
- **Diagnose readiness for the lesson.**
  - **Warm Up** TE, every lesson

#### During Every Lesson
- **Diagnose understanding of lesson concepts.**
  - **Check It Out!** SE, every example
  - **Think and Discuss** SE, every lesson
  - **Write About It** SE, every lesson
  - **Journal** TE, every lesson

#### After Every Lesson
- **Diagnose mastery of lesson concepts.**
  - **Lesson Quiz** TE, every lesson
  - **Alternative Assessment** TE, every lesson
  - **Test Prep** SE, every lesson
  - **Test and Practice Generator**

### Key:
- **SE** = Student Edition
- **TE** = Teacher’s Edition
- **CRB** = Chapter Resource Book
- **AR** = Assessment Resources
- **Available online**
- **Available on CD-ROM**
## Chapter 1 Resource Book

**Practice A, B, C**  

**Reading Strategies**  
pp. 10, 18, 26, 34, 42, 50, 58

**Reteach**  

**Problem Solving**  
pp. 9, 17, 25, 33, 41, 49, 57

**Challenge**  
pp. 8, 16, 24, 32, 40, 48, 56

**Parent Letter**  
pp. 1–2

## Transparencies

**Lesson Transparencies, Volume 1**  
Chapter 1
- Teaching Tools
- Warm Ups
- Teaching Transparencies
- Additional Examples
- Lesson Quizzes

**Alternate Openers: Explorations**  
pp. 1–7

**Know-It Notebook**  
Chapter 1
- Graphic Organizers

## Workbooks

**New York Homework and Practice Workbook**  
Teacher’s Guide  
pp. 1–7

**Know-It Notebook**  
Teacher’s Guide  
Chapter 1

**New York Problem Solving Workbook**  
Teacher’s Guide  
pp. 1–7

**New York State Test Prep Workbook**  
Teacher’s Guide

## Technology Highlights for the Teacher

### Power Presentations
Dynamic presentations to engage students. Complete PowerPoint® presentations for every lesson in Chapter 1.

### One-Stop Planner
Easy access to Chapter 1 resources and assessments. Includes lesson-planning, test-generation, and puzzle-creation software.

### Premier Online Edition
Chapter 1 includes Tutorial Videos, Lesson Activities, Lesson Quizzes, Homework Help, and Chapter Project.
# Reaching All Learners

## Resources for All Learners

- **Geometry Lab Activities** .......................................................... Chapter 1
- **Technology Lab Activities** ..................................................... Chapter 1
- **New York Homework and Practice Workbook** ....................... pp. 1–7
- **Know-It Notebook** ................................................................. Chapter 1
- **New York Problem Solving Workbook** .................................. pp. 1–7

## DEVELOPING LEARNERS

- **Practice A** .......................................................... CRB, every lesson
- **Reteach** .......................................................... CRB, every lesson
- **Inclusion** .......................................................... TE pp. 18, 25, 30, 36, 42, 45, 51
- **Questioning Strategies** .................................................. TE, every example
- **Modified Chapter 1 Resources** ....................................... IDEA Works!
- **Homework Help Online** ..................................................... IDEA Works!

## ON-LEVEL LEARNERS

- **Practice B** .......................................................... CRB, every lesson
- **Multiple Representations** ................................................ TE p. 18
- **Cognitive Strategies** ...................................................... TE pp. 29, 44

## ADVANCED LEARNERS

- **Practice C** .......................................................... CRB, every lesson
- **Challenge** .......................................................... CRB, every lesson
- **Reading and Writing Math EXTENSION** ........................... TE p. 5
- **Multi-Step Test Prep EXTENSION** .................................. TE pp. 34, 58
- **Critical Thinking** ....................................................... TE p. 14

## English Language Learners

- **Are You Ready? Vocabulary** ........................................ SE p. 3
- **Vocabulary Connections** ................................................ SE p. 4
- **Lesson Vocabulary** ....................................................... SE, every lesson
- **Vocabulary Exercises** ................................................... SE, every exercise set
- **Vocabulary Review** ........................................................ SE p. 60
- **English Language Learners** ........................................... pp. 1–14
- **Multilingual Glossary** ....................................................

## Reaching All Learners Through...

- **Inclusion** .......................................................... TE pp. 18, 25, 30, 36, 42, 45, 51
- **Visual Cues** .......................................................... TE pp. 21, 22, 27, 30
- **Kinesthetic Experience** .................................................... TE pp. 18, 26
- **Concrete Manipulatives** .................................................. TE pp. 14, 51
- **Multiple Representations** ................................................ TE p. 18
- **Cognitive Strategies** ...................................................... TE pp. 29, 44
- **Cooperative Learning** ................................................... TE p. 29
- **Modeling** ............................................................. TE p. 7
- **Critical Thinking** ......................................................... TE p. 14
- **Test Prep Doctor** .......................................................... TE pp. 11, 26, 33, 40, 49, 55, 65, 66, 68
- **Common Error Alerts** ................................................... TE pp. 15, 19, 21, 23, 25, 29, 39, 45, 49
- **Scaffolding Questions** .................................................. TE pp. 34, 58

## Technology Highlights for Reaching All Learners

- **Lesson Tutorial Videos**  
  Starring Holt authors Ed Burger and Freddie Renfro! Live tutorials to support every lesson in Chapter 1.

- **Multilingual Glossary**  
  Searchable glossary includes definitions in English, Spanish, Vietnamese, Chinese, Hmong, Korean, and 4 other languages.

- **Online Interactivities**  
  Interactive tutorials provide visually engaging alternative opportunities to learn concepts and master skills.
Ongoing Assessment

Assessing Prior Knowledge
Determine whether students have the prerequisite concepts and skills for success in Chapter 1.

Are You Ready? ........................................ SE p. 3
Warm Up ............................................... TE, every lesson

Test Preparation
Provide review and practice for Chapter 1 and standardized tests.

Multi-Step Test Prep ..................................... SE pp. 34, 58
Study Guide: Review ...................................... SE pp. 60–63
Test Tacker .................................................. SE pp. 66–67
New York Test Prep ........................................ SE pp. 68–69
College Entrance Exam Practice ......................... SE p. 65
New York State Test Prep Workbook
IDEA Works!

Alternative Assessment
Assess students’ understanding of Chapter 1 concepts and combined problem-solving skills.

Chapter 1 Project ......................................... SE p. 2
Alternative Assessment ................................. TE, every lesson
Performance Assessment ............................... AR pp. 19–20
Portfolio Assessment .................................... AR p. xxxiv

Daily Assessment
Provide formative assessment for each day of Chapter 1.

Questioning Strategies .................................. TE, every example
Think and Discuss ........................................ SE, every lesson
Check It Out! Exercises ............................... SE, every example
Write About It ............................................ SE, every lesson
Journal ..................................................... TE, every lesson
Lesson Quiz .............................................. TE, every lesson
Alternative Assessment ................................. TE, every lesson
Modified Lesson Quizzes .............................. IDEA Works!

Weekly Assessment
Provide formative assessment for each week of Chapter 1.

Multi-Step Test Prep ..................................... SE pp. 34, 58
Ready to Go On? ......................................... SE pp. 35, 59
Cumulative Assessment ................................ SE pp. 68–69
Test and Practice Generator ................................ One-Stop Planner

Formal Assessment
Provide summative assessment of Chapter 1 mastery.

Section Quizzes ........................................ AR pp. 5–6
Chapter 1 Test ............................................ SE p. 64
Chapter Test (Levels A, B, C) ........................ AR pp. 7–18
  • Multiple Choice  • Free Response
Cumulative Test ........................................ AR pp. 21–24
Test and Practice Generator .............................. One-Stop Planner
Modified Chapter 1 Test ................................ IDEA Works!

Technology Highlights for Ongoing Assessment

Are You Ready? .............................................. SPANISH
Automatically assess readiness and prescribe intervention for Chapter 1 prerequisite skills.

Ready to Go On? .............................................. SPANISH
Automatically assess understanding of and prescribe intervention for Sections 1A and 1B.

Test and Practice Generator
Use Chapter 1 problem banks to create assessments and worksheets to print out or deliver online. Includes dynamic problems.


2E Chapter 1
Circle the correct answer.

1. What is the area of a square with a side of 5 units?
   - A. 10
   - B. 25
   - C. 50
   - D. 20

2. Which segment is on line AB?
   - A. AC
   - B. BC
   - C. CD
   - D. DE

3. What is the area of a triangle with a base of 6 units and a height of 3 units?
   - A. 9
   - B. 18
   - C. 36
   - D. 54

4. What is the perimeter of a square with a side of 3 units?
   - A. 6
   - B. 12
   - C. 18
   - D. 24

5. What is the distance from A to B?
   - A. 5 units
   - B. 10 units
   - C. 15 units
   - D. 20 units

Choose the correct answer:

1. What is the area of a triangle with a base of 6 units and a height of 3 units?
   - A. 9
   - B. 18
   - C. 36
   - D. 54

2. What is the perimeter of a square with a side of 3 units?
   - A. 6
   - B. 12
   - C. 18
   - D. 24

3. What is the distance from A to B?
   - A. 5 units
   - B. 10 units
   - C. 15 units
   - D. 20 units

Formal Assessment

Three levels (A, B, C) of multiple-choice and free-response chapter tests are available in the Assessment Resources.

Chapter 1 Test

B Chapter 1 Test

MULTIPLE CHOICE

1. Which angles are adjacent?
   - A. \(\angle 1, \angle 2\)
   - B. \(\angle 2, \angle 3\)
   - C. \(\angle 3, \angle 4\)
   - D. \(\angle 4, \angle 5\)

2. Which segment is on line AB?
   - A. AC
   - B. BC
   - C. CD
   - D. DE

3. What is the area of a triangle with a base of 6 units and a height of 3 units?
   - A. 9
   - B. 18
   - C. 36
   - D. 54

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Choose the correct answer:

1. What is the area of a triangle with a base of 6 units and a height of 3 units?
   - A. 9
   - B. 18
   - C. 36
   - D. 54

2. What is the perimeter of a square with a side of 3 units?
   - A. 6
   - B. 12
   - C. 18
   - D. 24

3. What is the distance from A to B?
   - A. 5 units
   - B. 10 units
   - C. 15 units
   - D. 20 units

Chapter 1 Test

C Chapter 1 Test

FREE RESPONSE

B Chapter 1 Test

FREE RESPONSE

1. Use the diagram for Exercise 1.

2. Name a point on line AB.

3. Name a ray with endpoint C.

4. Name three collinear points.

5. What is the measure of \(\angle A\)?

Use the Distance Formula to find AB:

\[ AB = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \]

Chapter 1 Test

MODIFIED FOR IDEA

B Chapter 1 Test

MODIFIED FOR IDEA

1. Which line describes the transformation?

2. What is the area of a triangle with a base of 6 units and a height of 3 units?

3. What is the perimeter of a square with a side of 3 units?

4. What is the distance from A to B?

Choose the correct answer:

1. Which line describes the transformation?

2. What is the area of a triangle with a base of 6 units and a height of 3 units?

3. What is the perimeter of a square with a side of 3 units?

4. What is the distance from A to B?
**SECTION 1A**

**Euclidean and Construction Tools**

On page 34, students analyze a diagram of an archaeological dig by applying definitions, using the distance formula, and classifying angles. Exercises designed to prepare students for success on the Multi-Step Test Prep can be found on pages 10, 18, 26, and 32.

**SECTION 1B**

**Coordinate and Transformation Tools**

On page 58, students find the area and perimeter of a patio to determine the total cost of the paving stones used. They use distance, midpoint, and transformations to create the construction plans for the patio. Exercises designed to prepare students for success on the Multi-Step Test Prep can be found on pages 39, 48, and 54.

---

**Picture This!**

Many geometric concepts and shapes may be used in creating works of art. Unique designs can be made using only points, lines, planes, or circles.

---

**About the Project**

Students begin by using geoboards to explore designs based on line segments. Then they use a compass and paper folding to make star designs. Finally, students use everything they’ve learned to produce an original piece of string art.

**Project Resources**

All project resources for teachers and students are provided online.

**Materials:**

- Activity 1: geoboard, colored rubber bands or colored string
- Activity 2: compass, straightedge
- Activity 3: straightedge
- Activity 4: wooden board, small nails, hammer, colored string or thread
Vocabulary
Match each term on the left with a definition on the right.

1. coordinate **C**  
   A. a mathematical phrase that contains operations, numbers, and/or variables
2. metric system of measurement **E**  
   B. the measurement system often used in the United States
3. expression **A**  
   C. one of the numbers of an ordered pair that locates a point on a coordinate graph
4. order of operations **D**  
   D. a list of rules for evaluating expressions
5. coordinate  
6. metric system of measurement  
7. expression  
8. order of operations

Measure with Customary and Metric Units
For each object tell which is the better measurement.

5. length of an unsharpened pencil  
   7 1/4 in. or 9 1/2 in.
6. the diameter of a quarter  
   1 m or 2 1/2 cm
7. length of a soccer field  
   100 yd or 40 yd
8. height of a classroom  
   5 ft or 10 ft
9. height of a student’s desk  
   30 in. or 4 ft
10. length of a dollar bill  
    15.6 cm or 35.5 cm

Combine Like Terms
Simplify each expression.

11. \(-y + 3y - 6y + 12y - 8y\)  
12. \(63 + 2x - 7 - 4x - 2x + 56\)
13. \(-5 - 9 - 7x + 6x - x - 14\)  
14. \(24 - 3y + y + 7 - 2y + 31\)

Evaluate Expressions
Evaluate each expression for the given value of the variable.

15. \(x + 3x + 7x\) for \(x = -5\)  
16. \(5p + 10\) for \(p = 78\)
17. \(2a - 8a\) for \(a = 12\)  
18. \(3n - 3\) for \(n = 16\)

Ordered Pairs
Write the ordered pair for each point.

19. \(A (0, 7)\)  
20. \(B (-5, 4)\)
21. \(C (6, 3)\)  
22. \(D (-8, -2)\)
23. \(E (3, -5)\)  
24. \(F (6, -4)\)
Key Vocabulary/Vocabulario

<table>
<thead>
<tr>
<th>Term</th>
<th>English</th>
<th>Spanish</th>
</tr>
</thead>
<tbody>
<tr>
<td>angle</td>
<td>angulo</td>
<td></td>
</tr>
<tr>
<td>area</td>
<td>área</td>
<td></td>
</tr>
<tr>
<td>coordinate plane</td>
<td>plano cartesiano</td>
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</tr>
<tr>
<td>line</td>
<td>linea</td>
<td></td>
</tr>
<tr>
<td>perimeter</td>
<td>perímetro</td>
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<td>plano</td>
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<td>point</td>
<td>punto</td>
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<tr>
<td>transformation</td>
<td>transformación</td>
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<tr>
<td>undefined term</td>
<td>término indefinido</td>
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</tbody>
</table>

Vocabulary Connections

To become familiar with some of the vocabulary terms in the chapter, consider the following. You may refer to the chapter, the glossary, or a dictionary if you like.

1. A **definition** is a statement that gives the meaning of a word or phrase. What do you think the phrase **undefined term** means?

2. **Coordinates** are numbers used to describe a location. A **plane** is a flat surface. How can you use these meanings to understand the term **coordinate plane**?

3. A **point** is often represented by a dot. What real-world items could represent points?

4. **Transformation** means “to change or move a shape.” How can you use these meanings to understand the term **transformation**?

New York Performance Indicators

<table>
<thead>
<tr>
<th>Geometry</th>
<th></th>
<th>Process</th>
</tr>
</thead>
<tbody>
<tr>
<td>G.G.17</td>
<td>Construct a bisector of a given angle, using a straightedge and compass, and justify the construction</td>
<td>G.G.66 Find the midpoint of a line segment, given its endpoints</td>
</tr>
<tr>
<td>G.G.48</td>
<td>Investigate, justify, and apply the Pythagorean Theorem and its converse</td>
<td>G.G.67 Find the length of a line segment, given its endpoints</td>
</tr>
<tr>
<td>G.G.54</td>
<td>Define, investigate, justify, and apply isometries in the plane (rotations, reflections, translations, glide reflections)</td>
<td>G.PS.5 Choose an effective approach to solve a problem from a variety of strategies (numeric, graphic, algebraic)</td>
</tr>
<tr>
<td>G.G.55</td>
<td>Investigate, justify, and apply the properties that remain invariant under translations, rotations, reflections, and glide reflections</td>
<td>G.CM.11 Understand and use appropriate language, representations, and terminology when describing objects, relationships, mathematical solutions, and geometric diagrams</td>
</tr>
<tr>
<td>G.G.56</td>
<td>Identify specific isometries by observing orientation, numbers of invariant points, and/or parallelism</td>
<td>G.CN.6 Recognize and apply mathematics to situations in the outside world</td>
</tr>
</tbody>
</table>
Reading Strategy: Use Your Book for Success

Understanding how your textbook is organized will help you locate and use helpful information.

As you read through an example problem, pay attention to the notes in the margin. These notes highlight key information about the concept and will help you to avoid common mistakes.

Try This

Use your textbook for the following problems.

1. Use the index to find the page where right angle is defined.
2. What formula does the Know-It Note on the first page of Lesson 1-6 refer to?
3. Use the glossary to find the definition of congruent segments.
4. In what part of the textbook can you find help for solving equations?

Glossary

The Glossary is found in the back of your textbook. Use it when you need a definition of an unfamiliar word or phrase.

Index

The Index is located at the end of your textbook. If you need to locate the page where a particular concept is explained, use the Index to find the corresponding page number.

Skills Bank

The Skills Bank is located in the back of your textbook. Look in the Skills Bank for help with math topics that were taught in previous courses, such as the order of operations.

Glossary/Glossary

New York Performance Indicators

<table>
<thead>
<tr>
<th>Performance Indicators</th>
<th>1-1</th>
<th>LAB 1-2</th>
<th>1-3</th>
<th>1-4</th>
<th>1-5</th>
<th>1-6</th>
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<tr>
<td>G.CN.6</td>
<td></td>
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<td></td>
</tr>
</tbody>
</table>

New York Mathematics Performance Indicators are written out completely on pp NY28-NY35.
## Euclidean and Construction Tools

### One-Minute Section Planner

<table>
<thead>
<tr>
<th>Lesson</th>
<th>Lab Resources</th>
<th>Materials</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Lesson 1-1 Understanding Points, Lines, and Planes</strong>&lt;br&gt;• Identify, name, and draw points, lines, segments, rays, and planes.&lt;br&gt;• Apply basic facts about points, lines, and planes.&lt;br&gt;☐ NY Regents Exam ☑ SAT-10 ☑ NAEP ☑ ACT ☑ SAT</td>
<td><strong>Technology Lab Activities</strong>&lt;br&gt;1-2 Lab Recording Sheet</td>
<td><strong>Required</strong>&lt;br&gt;straightedge (MK) <strong>Optional</strong>&lt;br&gt;raw spaghetti, index cards, straw, clay, gumdrops, toothpicks</td>
</tr>
<tr>
<td><strong>1-2 Technology Lab Explore Properties Associated with Points</strong>&lt;br&gt;• Use geometry software to measure distances and explore properties of points on segments.&lt;br&gt;☐ NY Regents Exam ☑ SAT-10 ☑ NAEP ☑ ACT ☐ SAT</td>
<td><strong>Geometry Lab Activities</strong>&lt;br&gt;1-2 Geometry Lab</td>
<td><strong>Required</strong>&lt;br&gt;geometry software <strong>Optional</strong>&lt;br&gt;ruler (MK)</td>
</tr>
<tr>
<td><strong>Lesson 1-2 Measuring and Constructing Segments</strong>&lt;br&gt;• Use length and midpoint of a segment.&lt;br&gt;• Construct midpoints and congruent segments.&lt;br&gt;☐ NY Regents Exam ☑ SAT-10 ☑ NAEP ☑ ACT ☑ SAT</td>
<td><strong>Geometry Lab Activities</strong>&lt;br&gt;1-3 Geometry Lab</td>
<td><strong>Required</strong>&lt;br&gt;compass (MK), straightedge <strong>Optional</strong>&lt;br&gt;road map, masking tape, butcher paper, small plastic disks, meter stick</td>
</tr>
<tr>
<td><strong>Lesson 1-3 Measuring and Constructing Angles</strong>&lt;br&gt;• Name and classify angles.&lt;br&gt;• Measure and construct angles and angle bisectors.&lt;br&gt;☑ NY Regents Exam ☑ SAT-10 ☑ NAEP ☑ ACT ☑ SAT</td>
<td><strong>Geometry Lab Activities</strong>&lt;br&gt;1-3 Geometry Lab</td>
<td><strong>Required</strong>&lt;br&gt;compass (MK), straightedge, protractor (MK), geometry software <strong>Optional</strong>&lt;br&gt;yarn, index cards, pictures of angles, origami paper, sticky notes, acetate or tracing paper, clock (MK), Mira</td>
</tr>
<tr>
<td><strong>Lesson 1-4 Pairs of Angles</strong>&lt;br&gt;• Identify adjacent, vertical, complementary, and supplementary angles.&lt;br&gt;• Find measures of pairs of angles.&lt;br&gt;☐ NY Regents Exam ☐ SAT-10 ☑ NAEP ☑ ACT ☑ SAT</td>
<td></td>
<td><strong>Required</strong>&lt;br&gt;protractor (MK)</td>
</tr>
</tbody>
</table>

*MK = Manipulatives Kit*
Points, Lines, and Planes

**Lesson 1-1**

Understanding points, lines, planes, and segments is fundamental to the entire geometry course.

- The intersection of two lines is a point.
- The intersection of two planes is a line.
- If two points lie in a plane, then the line they determine lies in the plane.

### Segment:

\[ \overline{DC} \]

### Line:

\[ \overrightarrow{DC} \]

### Ray:

\[ \overrightarrow{BA} \]

### Opposite rays:

\[ \overrightarrow{BC}, \overrightarrow{BD} \]

Measuring Segments and Angles

**Lessons 1-2, 1-3**

Distance and angle measure are important in carpentry, engineering, science, and many other areas.

**Congruent Angles**

\[ m\angle G = m\angle H \iff \angle G \cong \angle H \]

**Congruent Segments**

\[ AB = ST \iff \overline{AB} \cong \overline{ST} \]

**Acute Angle**

**Right Angle**

\[ 90^\circ \]

**Obtuse Angle**

Pairs of Angles

**Lesson 1-4**

Properties of angle pairs are used in science and engineering.

**Vertical Angles**

**Linear Pair**

**Adjacent Angles**

**Complementary Angles**

\[ 39^\circ + 51^\circ = 90^\circ \]

**Supplementary Angles**

\[ 39^\circ + 141^\circ = 180^\circ \]
Understanding Points, Lines, and Planes

Objectives
Identify, name, and draw points, lines, segments, and planes.
Apply basic facts about points, lines, and planes.

Vocabulary
undefined term
point
line
plane
collinear
coplanar
segment
epsilon
epsilon endpoints
opposite rays
postulate

Who uses this?
Architects use representations of points, lines, and planes to create models of buildings. Interwoven segments were used to model the beams of Beijing's National Stadium for the 2008 Olympics.

The most basic figures in geometry are undefined terms, which cannot be defined by using other figures. The undefined terms point, line, and plane are the building blocks of geometry.

Undefined Terms

<table>
<thead>
<tr>
<th>TERM</th>
<th>NAME</th>
<th>DIAGRAM</th>
</tr>
</thead>
<tbody>
<tr>
<td>point</td>
<td>capital letter</td>
<td>[P]</td>
</tr>
<tr>
<td>ray</td>
<td>lowercase letter</td>
<td>[x]</td>
</tr>
<tr>
<td>coplanar</td>
<td>script capital</td>
<td>[A]</td>
</tr>
</tbody>
</table>

Points that lie on the same line are collinear. K, L, and M are collinear. K, L, and N are noncollinear. Points that lie in the same plane are coplanar. Otherwise they are noncoplanar.

Naming Points, Lines, and Planes
Refer to the design in the roof of Beijing's National Stadium.

A Name four coplanar points.
K, L, M, and N all lie in plane \( R \).

B Name three lines.
\( AB \), \( BC \), and \( CA \).

1. Use the diagram to name two planes. Possible answer: plane \( R \) and plane \( ABC \).

Introduce

Exploration

The following are some of the terms used as the basic building blocks of geometry. Describe a real-world object that is suggested by each term.

1. point
2. line
3. plane
4. Tell whether each of the following is most like a point, a line, or a plane:
   a. a desktop
   b. a speck of dust
   c. a jet control

Think and Discuss
5. Explain what is represented by the arrowhead at each end of the line in Problem 2.
6. Discuss whether the surface of Earth can be an example of a plane.

Motivate
Point out different objects in the classroom that are representations of points, segments, and planes, such as the tips of pushpins on the bulletin board, rulers, and desktops. Discuss with students what these items have in common.

Ask students to give examples of other objects that have these same characteristics and can be found in the world around them, such as the locations of cities on a map, the lines on a football field, and the bases on a baseball field.
### Postulates

**1-1-1** Through any two points there is exactly one line.

**1-1-2** Through any three noncollinear points there is exactly one plane containing them.

**1-1-3** If two points lie in a plane, then the line containing those points lies in the plane.

### Example 1

**Drawing Segments and Rays**

Draw and label each of the following.

A. a segment with endpoints $U$ and $V$

B. opposite rays with a common endpoint $Q$

2. Draw and label a ray with endpoint $M$ that contains $N$.

A postulate, or axiom, is a statement that is accepted as true without proof. Postulates about points, lines, and planes help describe geometric properties.

### Example 2

**Identifying Points and Lines in a Plane**

Name a line that passes through two points. There is exactly one line $n$ passing through $G$ and $H$.

3. Name a plane that contains three noncollinear points. Possible answer: plane $GHF$
Recall that a system of equations is a set of two or more equations containing two or more of the same variables. The coordinates of the solution of the system satisfy all equations in the system. These coordinates also locate the point where all the graphs of the equations in the system intersect.

An intersection is the set of all points that two or more figures have in common. The next two postulates describe intersections involving lines and planes.

**Postulates**

1-1-4 If two lines intersect, then they intersect in exactly one point.

1-1-5 If two planes intersect, then they intersect in exactly one line.

Use a dashed line to show the hidden parts of any figure that you are drawing. A dashed line will indicate the part of the figure that is not seen.

**Example 4** Representing Intersections

Sketch a figure that shows the following.

A. A line intersects a plane, but does not lie in the plane.

B. Two planes intersect in one line.

4. Sketch a figure that shows two lines intersect in one point in a plane, but only one of the lines lies in the plane.

**Think and Discuss**

1. Explain why any two points are collinear.
2. Which postulate explains the fact that two straight roads cannot cross each other more than once?
3. Explain why points and lines may be coplanar even when the plane containing them is not drawn.
4. Name all the possible lines, segments, and rays for the points A and B. Then give the maximum number of planes that can be determined by these points.
5. GET ORGANIZED Copy and complete the graphic organizer below. In each box, name, describe, and illustrate one of the undefined terms.

**Answers to Think and Discuss**

Possible answers:

1. By Post. 1-1-1, through any 2 pts. there is a line. Therefore any 2 pts. are collinear.
2. Post. 1-1-4
3. Any 3 noncollinear pts. determine a plane.
4. \( AB, AB, BA \); 0 planes
GUIDED PRACTICE

Vocabulary  Apply the vocabulary from this lesson to answer each question.

1. Give an example from your classroom of three collinear points.
   Possible answer: the intersection of 2 floor tiles

2. Make use of the fact that endpoint is a compound of end and point and name the endpoint of \( ST \).

Use the figure to name each of the following.

3. five points \( A, B, C, D, E \)

4. two lines Possible answer: \( \overrightarrow{AB}, \overrightarrow{BD} \)

5. two planes Possible answer: \( ABC \) and \( N \)

6. point on \( \overrightarrow{BD} \) Possible answer: \( B, C, \) or \( D \)

See Example 1 p. 6

Draw and label each of the following.

7. a segment with endpoints \( M \) and \( N \)

8. a ray with endpoint \( F \) that passes through \( G \)

See Example 2 p. 7

Use the figure to name each of the following.

9. a line that contains \( A \) and \( C \) Possible answer: \( \overrightarrow{AB} \)

10. a plane that contains \( A, D, \) and \( C \)
    Possible answer: plane \( ABD \)

See Example 3 p. 7

Sketch a figure that shows each of the following.

11. three coplanar lines that intersect in a common point

12. two lines that do not intersect

See Example 4 p. 8

PRACTICE AND PROBLEM SOLVING

Use the figure to name each of the following.

13. three collinear points \( B, E, A \)

14. four coplanar points Possible answer: \( B, C, D, E \)

15. a plane containing \( E, B, C, D, E \)
    Possible answer: plane \( ABC \)

Draw and label each of the following.

16. a line containing \( X \) and \( Y \)

17. a pair of opposite rays that both contain \( R \)

Use the figure to name each of the following.

18. two points and a line that lie in plane \( \overrightarrow{J} \)
    Possible answer: \( G, J, \) and \( \overrightarrow{J} \)

19. two planes that contain \( \ell \)
    Possible answer: planes \( \overrightarrow{J} \) and \( S \)

Sketch a figure that shows each of the following.

20. a line that intersects two nonintersecting planes

21. three coplanar lines that intersect in three different points

Assignment Guide

Assign Guided Practice exercises as necessary.

If you finished Examples 1–2
Basic 13–17, 33
Average 13–17, 33, 36, 43
Advanced 13–17, 33, 34, 36, 43–45

If you finished Examples 1–4
Basic 13–28, 31–33, 39–42, 47–51
Average 13–28, 30–33, 35–43, 46–51
Advanced 14–28 even, 29–51

Communicating Math In Exercises 3–6, students sometimes forget to place a symbol above the letters that are used to name lines, segments, and rays. Remind students that two letters without a symbol represent a distance.
22. Possible answers:
   a. tip of a stake
   b. string
   c. grid formed by string

28. If 2 pts. lie in a plane, then the line containing those pts. lies in the plane.

30. It is not possible. By Post. 1-1-2, any 3 noncollinear pts. are contained in a unique plane. If the 3 pts. are collinear, they are contained in infinitely many planes. In either case, the 3 pts. will be coplanar.

34. Lines may not intersect: 0 pts.

35. Post. 1-1-3

38. Lines may not intersect: 0 pts. of intersection.

All 3 lines may intersect in 1 pt.

Two of the lines may not intersect, but they might each intersect a third line. Each line may intersect each of the other lines.

Exercise 22 involves identifying geometric shapes at an archaeological site. This exercise prepares students for the Multi-Step Test Prep on page 34.

Visual For Exercise 36, have students make an organized list of all the combinations of three points that could be chosen and then select those that are collinear.

Answers

24.

26. The angle connecting two dots on a sheet of paper lies on the same sheet of paper as the dots.

29. If two lines are walking in straight lines but in different directions, their paths cannot cross more than once. If 2 lines intersect, then they intersect in exactly 1 pt.

30. Critical Thinking Is it possible to draw three points that are noncoplanar? Explain.

31. If two planes intersect, they intersect in a straight line. A

32. If two lines intersect, they intersect at two different points. N

33. \( \overline{AB} \) is another name for \( \overline{BA} \). A

34. If two rays share a common endpoint, then they form a line. S

35. Art Pointillism is a technique in which tiny dots of complementary colors are combined to form a picture. Which postulate ensures that a line connecting two of these points also lies in the plane containing the points?

36. Probability Three of the labeled points are chosen at random. What is the probability that \( \frac{1}{4} \) they are collinear?

37. Campers often use a cooking stove with three legs. Which postulate explains why they might prefer this design to a stove that has four legs? Post. 1-1-2

38. Write About It Explain why three coplanar lines may have zero, one, two, or three points of intersection. Support your answer with a sketch.
39. Which of the following is a set of noncollinear points?
   - A. P, R, T
   - B. Q, T, S
   - C. P, Q, R
   - D. S, T, U

40. What is the greatest number of intersection points four coplanar lines can have?
   - A. 6
   - B. 4
   - C. 0
   - D. 2

41. Two flat walls meet in the corner of a classroom. Which postulate best describes this situation?
   - A. Through any three noncollinear points there is exactly one plane.
   - B. If two points lie in a plane, then the line containing them lies in the plane.
   - C. If two lines intersect, then they intersect in exactly one point.
   - D. If two planes intersect, then they intersect in exactly one line.

42. Gridded Response What is the greatest number of planes determined by four noncollinear points? 4

CHALLENGE AND EXTEND

Use the table for Exercises 43–45.

<table>
<thead>
<tr>
<th>Figure</th>
<th>Number of Points</th>
<th>Maximum Number of Segments</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>2</td>
<td>3</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>4</td>
</tr>
</tbody>
</table>

43. What is the maximum number of segments determined by 4 points? 6

44. Multi-Step Extend the table. What is the maximum number of segments determined by 10 points? 45

\[ \frac{n(n - 1)}{2} \]

45. Write a formula for the maximum number of segments determined by \( n \) points.

46. Critical Thinking Explain how rescue teams could use two of the postulates from this lesson to locate a distress signal.

SPIRAL REVIEW

47. The combined age of a mother and her twin daughters is 58 years. The mother was 25 years old when the twins were born. Write and solve an equation to find the age of each of the three people. (Previous course) Mother is 36; twins are 11.

Determine whether each set of ordered pairs is a function. (Previous course)

48. \{ (0, 1), (1, −1), (5, −1), (−1, 2) \} yes

49. \{ (3, 6), (10, 6), (9, 8), (10, −6) \} no

Find the mean, median, and mode for each set of data. (Previous course)

50. 0, 6, 1, 3, 5, 2, 7, 10  mean: 4.25; median: 4; mode: none

51. 0.47, 0.44, 0.4, 0.46, 0.46  mean: 0.442; median: 0.44; mode: 0.44

1-1 Understanding Points, Lines, and Planes

Answers

46. Rescue teams can use the principles of Post. 1-1-1 and Post. 1-1-4. A distress signal is received by 2 rescue teams. By Post. 1-1-1, 2 pts. determine a line. So 2 lines are created by the 3 pts., the locations of the rescue teams and the distress signal. By Post. 1-1-4, the intersection of the 2 lines will be the location of the distress signal.

6. a ray with endpoint \( P \) that passes through \( Q \)

Also available on transparency
Explore Properties Associated with Points

The two endpoints of a segment determine its length. Other points on the segment are between the endpoints. Only one of these points is the midpoint of the segment. In this lab, you will use geometry software to measure lengths of segments and explore properties of points on segments.

Activity

1. Construct a segment and label its endpoints A and C.
2. Create point B on AC.
3. Measure the distances from A to B and from B to C. Use the Calculate tool to calculate the sum of AB and BC.
4. Measure the length of AC. What do you notice about this length compared with the measurements found in Step 3?
5. Drag point B along AC. Drag one of the endpoints of AC. What relationships do you think are true about the three measurements?
6. Construct the midpoint of AC and label it M.
7. Measure AM and MC. What relationships do you think are true about the lengths of AC, AM, and MC? Use the Calculate tool to confirm your findings.
8. How many midpoints of AC exist?

Try This

1. Repeat the activity with a new segment. Drag each of the points in your figure (the endpoints, the point on the segment, and the midpoint). Write down any relationships you observe about the measurements. Check students’ work.
2. Create a point D not on AC. Measure AD, DC, and AC. Does AD + DC = AC? What do you think has to be true about D for the relationship to always be true? No; D must be between A and C.

Teacher to Teacher

To introduce the concept of midpoint, I like to tell the following riddle to the class.

Q: A hunter walks into the woods and keeps walking in the same direction. At what point does the hunter begin to leave the woods?

A: The midpoint.

Then I can explain to students that once the hunter is halfway through the woods, she is as close to the end as to the beginning. After passing the midpoint, the hunter is going out instead of in. This helps students remember that the midpoint is not just any point between the endpoints, but exactly one point in the middle.

Teresa Salas
Corpus Christi, TX
1-2 Measuring and Constructing Segments

Objectives
Use length and midpoint of a segment.
Construct midpoints and congruent segments.

Vocabulary
coordinate
distance
length
congruent segments
construction
between
midpoint
bisection
segment bisection
bisect
midpoint
between
length
distance
coordinate
Vocabulary
congruent segments.

Why learn this?
You can measure a segment to calculate the distance between two locations. Maps of a race are used to show the distance between stations on the course. (See Example 4.)

A ruler can be used to measure the distance between two points. A point corresponds to one and only one number on the ruler. This number is called a coordinate. The following postulate summarizes this concept.

Postulate 1-2-1  Ruler Postulate

The points on a line can be put into a one-to-one correspondence with the real numbers.

The distance between any two points is the absolute value of the difference of the coordinates. If the coordinates of points A and B are a and b, then the distance between A and B is |b - a|. The distance between A and B is also called the length of AB, or AB.

Finding the Length of a Segment

Find each length.

\[ AB = |a - b| = |b - a| \]

Find each length.

1a. \( XY = \frac{7}{2} \)
1b. \( XZ = \frac{4}{2} \)

Congruent segments are segments that have the same length. In the diagram, PQ = RS, so you can write PQ \( \cong \) RS. This is read as “segment PQ is congruent to segment RS.” Tick marks are used in a figure to show congruent segments.

Example 1

Find each length.

A
\[ DC = |4.5 - 2| = |2.5| = 2.5 \]

B
\[ EF = |-4 - (-1)| = |-4 + 1| = |-3| = 3 \]

Find each length.

1a. \( XY = \frac{7}{2} \)
1b. \( XZ = \frac{4}{2} \)

Caution!
PQ represents a number, while PQ represents a geometric figure. Be sure to use equality for numbers (PQ = RS) and congruence for figures (PQ \( \cong \) RS).

Motivate
Show students a road map. Ask them to locate a place that is midway between two other places. Ask questions about distance between locations on a straight road. “Suppose you are driving to a relative’s house. The only place to stop for food is 75 miles from home, and it is \( \frac{1}{2} \) of the total trip. How far from the food stop does your relative live?”

Explanations and answers are provided in the Explorations binder.

New York Performance Indicators

G.PS.5 Choose an effective approach to solve a problem from a variety of strategies (numeric, graphic, algebraic).

G.RP.2 Recognize and verify, where appropriate, geometric relationships of perpendicularity, parallelism, congruence, and similarity, using algebraic strategies.
You can make a sketch or measure and draw a segment. These may not be exact. A construction is a way of creating a figure that is more precise. One way to make a geometric construction is to use a compass and straightedge.

### Construction  Congruent Segment

Construct a segment congruent to $AB$.

**Example 1**

Find each length.

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>A. $BC$</td>
<td>2</td>
</tr>
<tr>
<td>B. $AC$</td>
<td>5</td>
</tr>
</tbody>
</table>

Also available on transparency

### Copying a Segment

Sketch, draw, and construct a segment congruent to $MN$.

**Step 1** Estimate and sketch.
- Estimate the length of $MN$ and sketch $PQ$ approximately the same length.

**Step 2** Measure and draw.
- Use a ruler to measure $MN$. $MN$ appears to be 3.1 cm. Use a ruler and draw $XY$ to have length 3.1 cm.

**Step 3** Construct and compare.
- Use a compass and straightedge to construct $ST$ congruent to $MN$.

A ruler shows that $PQ$ and $XY$ are approximately the same length. $MN$ but $ST$ is precisely the same length.

2. Sketch, draw, and construct a segment congruent to $JK$.

### Critical Thinking

Point out that just as there are infinitely many real numbers between any two real numbers, there are infinitely many points between any two points on a line.
Using the Segment Addition Postulate

**Example 3**

A. \(B\) is between \(A\) and \(C\), \(AC = 14\), and \(BC = 11.4\). Find \(AB\).

\[ AC = AB + BC \quad \text{Seg. Add. Post.} \]
\[ 14 = AB + 11.4 \]
\[ -11.4 = -11.4 \quad \text{Subtract 11.4 from both sides.} \]
\[ 2.6 = AB \]

B. \(S\) is between \(R\) and \(T\). Find \(RT\).

\[ RT = RS + ST \quad \text{Seg. Add. Post.} \]
\[ 4x = (2x + 7) + 28 \]
\[ 4x = 2x + 35 \]
\[ -2x = 35 \]
\[ 2x = -35 \]
\[ x = -
\]

Substitute the given values.

Simplify.

Divide both sides by 2.

Simplify.

Simplify.

Substitute 17.5 for \(x\).

The midpoint \(M\) of \(\overline{AB}\) is the point that \textit{bisects}, or divides, the segment into two congruent segments. If \(M\) is the midpoint of \(\overline{AB}\), then \(AM = MB\). So if \(AB = 6\), then \(AM = 3\) and \(MB = 3\).

**Example 4**

The map shows the route for a race. You are 365 m from drink station \(R\) and 2 km from drink station \(S\). The first-aid station is located at the midpoint of the two drink stations. How far are you from the first-aid station?

Let your current location be \(X\) and the location of the first-aid station be \(Y\).

\[ XR + RS = XS \quad \text{Seg. Add. Post.} \]
\[ 365 + 365 = 2000 \quad \text{Substitute 365 for } XR \text{ and 2000 for } XS. \]
\[ -365 = -365 \quad \text{Subtract 365 from both sides.} \]
\[ RS = 1635 \quad \text{Simplify.} \]
\[ RY = 817.5 \quad Y \text{ is the mdpt. of } \overline{RS}, \text{ so } RY = \frac{1}{2}RS. \]
\[ XY = XR + RY \]
\[ = 365 + 817.5 = 1182.5 \text{ m} \quad \text{Substitute 365 for } XR \text{ and 817.5 for } RY. \]

You are 1182.5 m from the first-aid station.

4. What is the distance to a drink station located at the midpoint between your current location and the first-aid station? 591.25 m
A segment bisector is any ray, segment, or line that intersects a segment at its midpoint. It divides the segment into two equal parts at its midpoint.

**Construction**

Give each student a ruler. These can be found in the Manipulatives Kit (MK). Use paper other than patty paper and have students bend the line over the ruler. Then fold the paper along the line. Fold the paper end to end to locate the midpoint.

**Power Presentations**

with PowerPoint™

**Example 5**

D is the midpoint of EF, 
$ED = 4x + 6$, and $DF = 7x - 9$. 
Find $ED$, $DF$, and $EF$. 26; 26; 52

Also available on transparency

**INTERVENTION**

Questioning Strategies

**Example 5**

- Is it possible for $x$ to be a negative number in this type of problem? Support your answer with an example.

**Using Midpoints to Find Lengths**

B is the midpoint of $AC$, $AB = 5x$, and $BC = 3x + 4$. Find $AB$, $BC$, and $AC$.

**Step 1** Solve for $x$.

$AB = BC$  
$5x = 3x + 4$  
Substitute $5x$ for $AB$ and $3x + 4$ for $BC$.

$-3x - 3x$  
Subtract $3x$ from both sides.

$2x = 4$  
Simplify.

$\frac{2x}{2} = \frac{4}{2}$  
Divide both sides by 2.

$x = 2$  
Simplify.

**Step 2** Find $AB$, $BC$, and $AC$.

$AB = 5x$  
$BC = 3x + 4$  
$AC = AB + BC$

$= 5(2) = 10$  
$= 3(2) + 4 = 10$  
$= 10 + 10 = 20$

5. $S$ is the midpoint of $RT$, $RS = -2x$, and $ST = -3x - 2$. Find $RS$, $ST$, and $RT$. $RS = 4$; $ST = 4$; $RT = 8$

**THINK AND DISCUSS**

1. Suppose $R$ is the midpoint of $ST$. Explain how $SR$ and $ST$ are related.

2. GET ORGANIZED Copy and complete the graphic organizer. Make a sketch and write an equation to describe each relationship.

<table>
<thead>
<tr>
<th>B is between A and C.</th>
<th>B is the midpoint of AC.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sketch</td>
<td>Equation</td>
</tr>
</tbody>
</table>

**Answers to Think and Discuss**

1. Since $R$ is the mdpt. of $ST$, you know $SR = RT$. Also, $ST = SR + RT$. By subst., $ST = SR + SR = 2SR$. So $ST$ is twice $SR$.


**Close**

Summarize

Review the Segment Addition Postulate, emphasizing that betweenness involves collinearity. Remind students that a segment has exactly one midpoint. Review the steps used to construct a segment congruent to a given segment.
GUIDED PRACTICE

Vocabulary  Apply the vocabulary from this lesson to answer each question.
1. Line \( l \) bisects \( \overline{XY} \) at \( M \) and divides \( \overline{XY} \) into two equal parts. Name a pair of congruent segments. \( XM \) and \( MY \)

2. \( \overline{MN} \) is the amount of space between two points on a line. It is always expressed as a nonnegative number. (distance or midpoint) \( \text{distance} \)

3. Find each length. \( AB \), \( BC \)

4. Sketch, draw, and construct a segment congruent to \( \overline{RS} \). Check students’ work.

5. \( B \) is between \( A \) and \( C \), \( AC = 15.8 \), and \( AB = 9.9 \). Find \( BC \).

6. \( M \) is the midpoint of \( \overline{KL} \), \( JL = 4x - 2 \), and \( JK = 7 \). Find \( x \), \( KL \), and \( JL \). \( x = 4 \); \( KL = 7 \); \( JL = 14 \)

7. Find \( MP \).

8. Travel  If a picnic area is located at the midpoint between Sacramento and Oakland, find the distance to the picnic area from the road sign. \( 66.5 \) mi

9. Multi-Step \( K \) is the midpoint of \( \overline{IL} \). \( JL = 4x - 2 \), and \( JK = 7 \). Find \( x \), \( KL \), and \( JL \). \( x = 4 \); \( KL = 7 \); \( JL = 14 \)

10. \( E \) bisects \( \overline{DF} \). \( DE = 2y \), and \( EF = 8y - 3 \). Find \( DE \), \( EF \), and \( DF \). \( DE = EF = 1 \); \( DF = 2 \)

PRACTICE AND PROBLEM SOLVING

Find each length.

11. \( DB \), \( CD \)

12. Sketch, draw, and construct a segment twice the length of \( \overline{AB} \). Check students’ work.

13. \( D \) is between \( C \) and \( E \). \( CE = 17.1 \), and \( DE = 8 \). Find \( CD \).

14. Find \( MN \).

15. \( A \) is the midpoint of \( \overline{DF} \). \( DE = 2x + 4 \), and \( EF = 3x - 1 \). Find \( DE \), \( EF \), and \( DF \). \( DE = EF = 14 \); \( DF = 28 \)

16. Sports During a football game, a quarterback standing at the 9-yard line passes the ball to a receiver at the 24-yard line. The receiver then runs with the ball halfway to the 50-yard line. How many total yards (passing plus running) did the team gain on the play? \( 28 \) yd

17. \( Q \) bisects \( \overline{PR} \). \( PQ = 3y \), and \( PR = 42 \). Find \( y \) and \( QR \). \( y = 7 \); \( QR = 21 \)
Exercise 19 involves finding distances between points. This exercise prepares students for the Multi-Step Test Prep on page 34.

**Kinesthetic** To demonstrate the difference between equal length and congruence in Exercise 24, have students place one hand over the other to show how they match for congruence. Then, for equality, have them show 5 fingers on each hand for $z = 5$.

**Inclusion** If students are using a ruler for a straightedge in Exercise 35, be sure that they are not using the marks on the ruler to measure distances, and that they show construction arcs.

**Multiple Representations** For Exercise 40, encourage students to draw and label a diagram with all the given information before attempting to solve the problem.

---

**Chapter 1 Foundations for Geometry**

1. This problem will prepare you for the Multi-Step Test Prep on page 34. Archaeologists at Valley Forge were eager to find what remained of the winter camp that soldiers led by George Washington called home for several months. The diagram represents one of the restored log cabins.

   a. How is $C$ related to $AE$? $C$ is the mdpt. of $AE$.
   
   b. If $AC = 7$ ft, $EF = 2(AC) + 2$, and $AB = 2(2EF) - 16$, what are $AB$ and $EF$?

   Use the diagram for Exercises 20–23.

   20. $GD = 4\frac{2}{3}$. Find $GH$, $\frac{9}{3}$
   
   21. $CD \cong DF$, $E$ bisects $DF$, and $CD = 14.2$. Find $EF$. $7.1$
   
   22. $GH = 4x - 1$, and $DH = 8$. Find $x$. $4.25$
   
   23. $\overline{GH}$ bisects $\angle C$, $CF = 2y - 2$, and $CD = 3y - 11$. Find $CD$. $4$

   Tell whether each statement is sometimes, always, or never true. Support each of your answers with a sketch.

   24. Two segments that have the same length must be congruent. $A$
   
   25. If $M$ is between $A$ and $B$, then $M$ bisects $\overline{AB}$. $S$
   
   26. If $Y$ is between $X$ and $Z$, then $X, Y$, and $Z$ are collinear. $A$

27. $\overline{AM} \cong \overline{MB}$ is incorrect. The statement should be written as $\overline{AM} \cong \overline{MB}$, not as 2 distances that are $=$. $A$

28. **Carpentry** A carpenter has a wooden dowel that is 72 cm long. She wants to cut it into two pieces so that one piece is 5 times as long as the other. What are the lengths of the two pieces? 60 cm; 12 cm $6.5; -1.5$

29. The coordinate of $M$ is 2.5, and $MN = 4$. What are the possible coordinates for $N$? $1.5; 5.5$

30. Possible answer: $DE + EF = DF$

31. $RS = 7y - 4$
   
   32. $RS = 3x + 1$
   
   33. $RS = 2x + 6$
   
   ST = $y + 5$

   ST = $\frac{1}{2}x + 3$

   ST = $4z - 3$

   $RT = 28$ $3.375$

   $RT = 18$ $4$

   $RT = 5x + 12$ $9$

34. **Write About It** In the diagram, $B$ is not between $A$ and $C$. Explain. $B$ is not between $A$ and $C$, because $A$, $B$, and $C$ are not collinear.

35. **Construction** Use a compass and straightedge to construct a segment whose length is $\overline{AB} + \overline{CD}$. Check students' constructions.

---

**1-2 RETRACE**

The distance between any two points is the length of the segment between them.

- $\overline{AB}$ and $\overline{CD}$ are the same length.
- $\overline{AB} = \overline{CD}$

Use the figure above to find each length.

<table>
<thead>
<tr>
<th>Example</th>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\overline{AB}$</td>
<td>4 cm</td>
<td>15 cm</td>
<td>3 cm</td>
</tr>
</tbody>
</table>

**1-2 READING STRATEGIES**

Solve for the following terms:

- congruent segments — segments of the same length
- congruent angles — angles of the same measure
- congruent — equal

- segment bisector — line, ray, or point that intersects a segment at its midpoint

- segment extension — the segment that extends from the endpoint of a segment

- equivalent — equal

Use the figure for Exercises 1–6.

1. Name the congruent segments in the figure.

2. Name the endpoint of $\overline{AD}$.

3. Name the endpoint of $\overline{EF}$.

4. Name the congruent segments in the figure.

5. Name the endpoint of $\overline{C}$.

6. Name the endpoint of $\overline{B}$.

---

**1-2 PRACTICE**

**1-2 PRACTICE A**

Find the distance between the following points.

- $A(1, 3)$ and $B(4, 2)$
- $C(1, 5)$ and $D(1, 1)$
- $E(0, 2)$ and $F(3, 5)$
- $G(-2, 3)$ and $H(-5, 1)$

- $J(1, 2)$ and $K(-3, 4)$
- $L(-1, 0)$ and $M(-5, 3)$
- $N(-2, 3)$ and $O(-2, -3)$
- $P(0, 0)$ and $Q(2, 2)$

**1-2 PRACTICE B**

State whether each statement is sometimes, always, or never true.

- $\overline{AB} = \overline{BC}$
- $\angle A = \angle B$
- $\overline{AB} \cong \overline{BC}$
- $\overline{AB} = \overline{BC}$

---

**Chapter 1 Foundations for Geometry**

18
36. \(Q\) is between \(P\) and \(R\). \(S\) is between \(Q\) and \(R\), and \(R\) is between \(Q\) and \(T\). \(PT = 34\), \(QR = 8\), and \(PQ = SQ = SR\). What is the length of \(RT\)?

\(\text{A} \ 9 \quad \text{B} \ 10 \quad \text{C} \ 18 \quad \text{D} \ 22\)

37. \(C\) is the midpoint of \(\overline{AD}\). \(B\) is the midpoint of \(\overline{AC}\). \(BC = 12\). What is the length of \(\overline{AB}\)?

\(\text{A} \ 12 \quad \text{B} \ 24 \quad \text{C} \ 36 \quad \text{D} \ 48\)

38. Which expression correctly states that \(\overline{XY}\) is congruent to \(\overline{VW}\)?

\(\text{A} \ \overline{XY} \equiv \overline{VW} \quad \text{B} \ \overline{XY} \cong \overline{VW} \quad \text{C} \ \overline{XY} \parallel \overline{VW} \quad \text{D} \ \overline{XY} = \overline{VW}\)

39. \(A, B, C, D,\) and \(E\) are collinear points. \(\overline{AE} = 34, \overline{BD} = 16,\) and \(\overline{AB} = \overline{BC} = \overline{CD}\). What is the length of \(\overline{CE}\)?

\(\text{A} \ 10 \quad \text{B} \ 16 \quad \text{C} \ 18 \quad \text{D} \ 24\)

### CHALLENGE AND EXTEND

40. \(HIJ\) is twice \(JK\). \(JK\) is between \(H\) and \(K\). If \(HI = 4x\) and \(HK = 70\), find \(JK\).

\(\text{A} \ 20 \quad \text{B} \ 22 \quad \text{C} \ 24 \quad \text{D} \ 26\)

41. \(A, D, N,\) and \(X\) are collinear points. \(\overline{DN} = \overline{NX}\). Draw a diagram that represents this information.

**Sports**

Use the following information for Exercises 42 and 43.

The table shows regulation distances between hurdles in women’s and men’s races. In both the women’s and men’s events, the race consists of a straight track with 10 equally spaced hurdles.

<table>
<thead>
<tr>
<th>Event</th>
<th>Distance of Race</th>
<th>Distance from Start to First Hurdle</th>
<th>Distance Between Hurdles</th>
<th>Distance from Last Hurdle to Finish</th>
</tr>
</thead>
<tbody>
<tr>
<td>Women’s</td>
<td>100 m</td>
<td>13.00 m</td>
<td>8.50 m</td>
<td></td>
</tr>
<tr>
<td>Men’s</td>
<td>110 m</td>
<td>13.72 m</td>
<td>9.14 m</td>
<td></td>
</tr>
</tbody>
</table>

42. Find the distance from the last hurdle to the finish line for the women’s race. \(10.5\) m

43. Find the distance from the last hurdle to the finish line for the men’s race. \(14.02\) m

44. **Critical Thinking** Given that \(J, K, L,\) and \(M\) are collinear and that \(J\) is between \(F\) and \(K\), is it possible that \(J\) is equal to \(KF\)? If so, draw an example. If not, explain.

**Spiral Review**

Evaluate each expression. (Previous course)

45. \(|20 - 8| = 12\) \quad 46. \(|-9 + 23| = 14\) \quad 47. \(|-4 - 27| = 23\)

Simplify each expression. (Previous course)

48. \(8a - 3(a + 4) - 10 = 5a - 22\) \quad 49. \(x + 2(5 - 2x) - (4 + 5x) = -8x + 6\)

Use the figure to name each of the following. (Lesson 1-1)

50. two lines that contain \(B\) : \(AB, CB\)

51. two segments containing \(D\) : \(AD, BD\)

52. three collinear points \(A, B, D\)

53. a ray with endpoint \(C\) : \(CB\)

### 1-2 Measuring and Constructing Segments

**3. Sketch, draw, and construct a segment congruent to \(\overline{CD}\).**

**Journal**

Have students define the midpoint of a segment in their own words and illustrate what it means for one point to be between two other points.

**Alternative Assessment**

Have students create and solve a real-world problem involving algebraic expressions that illustrate distance and midpoint. Ask them to include a diagram and explain their problem.

**Power Presentations**

With PowerPoint

1. \(M\) is between \(N\) and \(O\), \(MO = 15\), and \(MN = 7.6\). Find \(NO\).

\(\text{A} \ 22.6\)

2. \(S\) is the midpoint of \(\overline{TV}, TS = 4x - 7,\) and \(SV = 5x - 15\). Find \(TS, SV,\) and \(TV\)

\(\text{A} \ 25, 25, 50\)

3. Sketch, draw, and construct a segment congruent to \(\overline{CD}\).

**Check students’ constructions.**

4. \(\overline{LH}\) bisects \(\overline{GK}\) at \(M\). \(GM = 2x + 6\), and \(\overline{GK} = 24\). Find \(x\).

\(\text{A} \ 3\)

5. Tell whether the statement below is sometimes, always, or never true. Support your answer with a sketch.

If \(M\) is the midpoint of \(\overline{KL}\), then \(M, K,\) and \(L\) are collinear.

\(\text{A} \ \text{Sometimes} \quad \text{B} \ \text{Always} \quad \text{C} \ \text{Never}\)

Also available on transparency
1-3 Measuring and Constructing Angles

Objectives
Name and classify angles.
Measure and construct angles and angle bisectors.

Vocabulary
angle vertex
interior of an angle exterior of an angle degree
acute angle
right angle
obtuse angle
straight angle
congruent angles
angle bisector

Who uses this?
Surveys use angles to help them measure and map the earth’s surface.
(See Exercise 27.)

A transit is a tool for measuring angles. It consists of a telescope that swivels horizontally and vertically. Using a transit, a surveyor can measure the angle formed by his or her location and two distant points.

An angle is a figure formed by two rays, or sides, with a common endpoint called the vertex (plural: vertices). You can name an angle several ways: by its vertex, by a point on each ray and the vertex, or by a number.

The set of all points between the sides of the angle is the interior of an angle. The exterior of an angle is the set of all points outside the angle.

Example 1
Naming Angles
A surveyor recorded the angles formed by a transit (point T) and three distant points, Q, R, and S. Name three of the angles.

\[ \angle QTR, \angle QTS, \text{ and } \angle RTS \]

1. Write the different ways you can name the angles in the diagram. \[ \angle RTO, \angle QT, \angle STR, \angle 1, \angle 2 \]

The measure of an angle is usually given in degrees. Since there are 360° in a circle, one degree is \[ \frac{1}{360} \] of a circle. When you use a protractor to measure angles, you are applying the following postulate.

Postulate 1-3-1
Protractor Postulate
Given \( \overrightarrow{AB} \) and a point \( O \) on \( \overrightarrow{AB} \), all rays that can be drawn from \( O \) can be put into a one-to-one correspondence with the real numbers from 0 to 180.

Motivate
Point out examples of different types of angles in the classroom, such as the corner of a piece of paper for a right angle, and open scissors to show an acute angle or an obtuse angle. Have students get in groups of three and use a long strand of yarn and three index cards, each labeled with a different letter, to model for the class the different angles that can be formed.

Explorations and answers are provided in the Explorations binder.
You can use the Protractor Postulate to help you classify angles by their measure. The measure of an angle is the absolute value of the difference of the real numbers that the rays correspond with on a protractor. If \( \overrightarrow{OC} \) corresponds with \( c \) and \( \overrightarrow{OD} \) corresponds with \( d \),

\[ m\angle DOC = |d - c| \text{ or } |c - d| \text{.} \]

### Types of Angles

<table>
<thead>
<tr>
<th>Acute Angle</th>
<th>Right Angle</th>
<th>Obtuse Angle</th>
<th>Straight Angle</th>
</tr>
</thead>
<tbody>
<tr>
<td>Measures greater than 0° and less than 90°</td>
<td>Measures 90°</td>
<td>Measures greater than 90° and less than 180°</td>
<td>Formed by two opposite rays and measures 180°</td>
</tr>
</tbody>
</table>

### Example 2

#### Measuring and Classifying Angles

Find the measure of each angle. Then classify each as acute, right, or obtuse.

**A.** \( \angle AOD \)

\[ m\angle AOD = 165° \]

\( \angle AOD \) is obtuse.

**B.** \( \angle COD \)

\[ m\angle COD = |165 - 75| = 90° \]

\( \angle COD \) is a right angle.

Use the diagram to find the measure of each angle. Then classify each as acute, right, or obtuse.

2a. \( \angle BOA \)

40°; acute

2b. \( \angle DOB \)

125°; obtuse

2c. \( \angle EOC \)

105°; obtuse
Congruent angles are angles that have the same measure. In the diagram, \( \angle ABC \cong \angle DEF \), so you can write \( \angle ABC \equiv \angle DEF \). This is read as ‘angle \( ABC \) is congruent to angle \( DEF \).’ Arc marks are used to show that the two angles are congruent.

### Construction Congruent Angle

Construct an angle congruent to \( \angle A \).

1. Use a straightedge to draw a ray with endpoint \( D \).
2. Place the compass point at \( A \) and draw an arc that intersects both sides of \( \angle A \). Label the intersection points \( B \) and \( C \).
3. Using the same compass setting, place the compass point at \( D \) and draw an arc that intersects the ray. Label the intersection \( E \).
4. Place the compass point at \( B \) and open it to the distance \( BC \). Place the point of the compass at \( E \) and draw an arc. Label its intersection with the first arc \( F \).
5. Use a straightedge to draw \( DF \).

### Postulate 1-3-2 Angle Addition Postulate

If \( S \) is in the interior of \( \angle PQR \), then \( m\angle PQS + m\angle SQR = m\angle PQR \). \( (\angle \text{Add. Post.}) \)

### Example 3

Find \( m\angle FEG \).

\[ m\angle DEG = 115^\circ \text{, and } m\angle DEF = 48^\circ \text{. Find } m\angle FEG. \]

\[ 67^\circ \]

### Example 3

Find \( m\angle YWZ \).

\[ m\angle XWZ = 121^\circ \text{ and } m\angle XWY = 59^\circ. \text{ Find } m\angle YWZ. 62^\circ \]
An **angle bisector** is a ray that divides an angle into two congruent angles. \( JK \) bisects \( \angle LJM \); thus \( \angle LJK \equiv \angle KJM \).

**Construction: Angle Bisector**

Construct the bisector of \( \angle A \).

1. Place the point of the compass at \( A \) and draw an arc. Label its points of intersection with \( \angle A \) as \( B \) and \( C \).
2. Without changing the compass setting, draw intersecting arcs from \( B \) and \( C \). Label the intersection of the arcs as \( D \).
3. Use a straightedge to draw \( \overline{AD} \). \( \overline{AD} \) bisects \( \angle A \).

**Finding the Measure of an Angle**

\( \overline{BD} \) bisects \( \angle ABC \), \( m \angle ABD = (6x + 3)° \), and \( m \angle DBC = (8x - 7)° \). Find \( m \angle ABD \).

**Step 1** Find \( x \).

\[
\begin{align*}
\text{m} \angle ABD &= \text{m} \angle DBC \\
(6x + 3)° &= (8x - 7)° \\
+7 &= +7 \\
6x + 10 &= 8x \\
-6x &= -6x \\
10 &= 2x \\
10 \div 2 &= 2x \div 2 \\
5 &= x
\end{align*}
\]

**Step 2** Find \( m \angle ABD \).

\[
\begin{align*}
m \angle ABD &= 6x + 3 \\
&= 6(5) + 3 \\
&= 33°
\end{align*}
\]

Find the measure of each angle.

4a. \( QS \) bisects \( \angle PQR \), \( m \angle PQS = (5y - 1)° \), and \( m \angle PQR = (8y + 12)° \). Find \( m \angle PQS \). **34°**

4b. \( JK \) bisects \( \angle LJM \), \( m \angle LJK = (10x + 3)° \), and \( m \angle LJM = (-x + 21)° \). Find \( m \angle LJM \). **46°**

**Close**

**Summarize**

Review with students the parts of an angle, the names of the types of angles, and how to measure angles. Review the Angle Addition Postulate and how it relates to the bisector of an angle.
1. Two \( \triangle \) with the same measure are \( \cong \). All rt. \( \triangle \) measure 90°, so any 2 rt. \( \triangle \) are \( \cong \).
2. \( m\angle ABD = m\angle DBC = \frac{1}{2}m\angle ABC \)

**THINK AND DISCUSS**

1. Explain why any two right angles are congruent.
2. \( BD \) bisects \( \angle ABC \). How are \( m\angle ABC, m\angle ABD, \) and \( m\angle DBC \) related?
3. **GET ORGANIZED** Copy and complete the graphic organizer.

<table>
<thead>
<tr>
<th>Angle Type</th>
<th>Diagram</th>
<th>Measure</th>
<th>Name</th>
</tr>
</thead>
<tbody>
<tr>
<td>Acute Angle</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Right Angle</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Obtuse Angle</td>
<td></td>
<td></td>
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</tr>
<tr>
<td>Straight Angle</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**GUIDED PRACTICE**

**Vocabulary**

Apply the vocabulary from this lesson to answer each question.

1. \( \triangle \) A is an acute angle. \( \triangle \) O is an obtuse angle. \( \triangle \) R is a right angle. Put \( \triangle \), \( \angle \), and \( \angle \) in order from least to greatest by measure. \( \angle A, \angle R, \angle O \)
2. Which point is the vertex of \( \angle BCD \)? Which rays form the sides of \( \angle BCD \)?
3. **Music** Musicians use a metronome to keep time as they play. The metronome’s needle swings back and forth in a fixed amount of time. Name all of the angles in the diagram.

Use the protractor to find the measure of each angle. Then classify each as acute, right, or obtuse.

4. \( \angle VXW \) 15°; acute
5. \( \angle TXW \) 105°; obtuse
6. \( \angle RXU \) 110°; obtuse

**SEE EXAMPLE**

p. 20

**SEE EXAMPLE**

p. 21

**SEE EXAMPLE**

p. 22

**SEE EXAMPLE**

p. 23

1. \( I \) is in the interior of \( \angle JKM \). Find each of the following.
2. \( m\angle JKM \) if \( m\angle JKL = 42° \) and \( m\angle KLM = 28° \)
3. \( m\angle KLM \) if \( m\angle JKL = 56.4° \) and \( m\angle JKM = 82.5° \)
4. **Multi-Step** \( BD \) bisects \( \angle ABC \). Find each of the following.
5. \( m\angle ABD \) if \( m\angle ABD = (6x + 4)^\circ \) and \( m\angle DBC = (8x - 4)^\circ \)
6. \( m\angle ABC \) if \( m\angle ABD = (5y - 3)^\circ \) and \( m\angle DBC = (3y + 15)^\circ \)

**Multi-Step**

**SEE EXAMPLE**

p. 21

**SEE EXAMPLE**

p. 22

**SEE EXAMPLE**

p. 23

**SEE EXAMPLE**

p. 24

1. \( \angle \) A is an acute angle. \( \angle \) O is an obtuse angle. \( \angle \) R is a right angle. Put \( \angle \), \( \angle \), and \( \angle \) in order from least to greatest by measure. \( \angle A, \angle R, \angle O \)
2. Which point is the vertex of \( \angle BCD \)? Which rays form the sides of \( \angle BCD \)?
3. **Music** Musicians use a metronome to keep time as they play. The metronome’s needle swings back and forth in a fixed amount of time. Name all of the angles in the diagram.

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**SEE EXAMPLE**

p. 20

**SEE EXAMPLE**

p. 21

**SEE EXAMPLE**

p. 22

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p. 23

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**Multi-Step**

**SEE EXAMPLE**

p. 21

**SEE EXAMPLE**

p. 22

**SEE EXAMPLE**

p. 23

**SEE EXAMPLE**

p. 24
PRACTICE AND PROBLEM SOLVING

11. **Physics** Pendulum clocks have been used since 1656 to keep time. The pendulum swings back and forth once or twice per second. Name all of the angles in the diagram. 

- \( \triangle 1 \) or \( \angle JMK \)
- \( \triangle 2 \) or \( \angle LMK \)
- \( \triangle M \) or \( \angle JML \)

Use the protractor to find the measure of each angle. Then classify each as acute, right, or obtuse.

12. \( \angle CGE \) 13. \( \angle BGD \) 14. \( \angle AGB \)

90°; rt. 93°; obtuse 28°; acute

\( \angle T \) is in the interior of \( \angle RSU \). Find each of the following.

15. \( m\angle RSU \) if \( m\angle RST = 38^\circ \) and \( m\angle TSU = 28.6^\circ \)

16. \( m\angle RST \) if \( m\angle TSU = 46.7^\circ \) and \( m\angle RSU = 83.5^\circ \)

**Multi-Step** 

17. \( m\angle RST \) if \( m\angle RSP = (3x - 2)^\circ \) and \( m\angle PST = (9x - 26)^\circ \)

18. \( m\angle RSP \) if \( m\angle RST = \frac{3}{2}y^\circ \) and \( m\angle PST = (y + 5)^\circ \)

**Estimation**

Use the following information for Exercises 19–22. Assume the corner of a sheet of paper is a right angle. Use the corner to estimate the measure and classify each angle in the diagram.

19. \( \angle BOA \) acute 20. \( \angle COA \) rt.

21. \( \angle EOD \) acute 22. \( \angle EOB \) obtuse

Use a protractor to draw an angle with each of the following measures.

23. 33° 24. 142° 25. 90° 26. 168°

27. **Surveying** A surveyor at point \( S \) discovers that the angle between peaks \( A \) and \( B \) is 3 times as large as the angle between peaks \( B \) and \( C \). The surveyor knows that \( \angle ASC \) is a right angle. Find \( m\angle ASB \) and \( m\angle BSC \).

28. **Math History** As far back as the 5th century B.C., mathematicians have been fascinated by the problem of trisecting an angle. It is possible to construct an angle with \( \frac{1}{3} \) the measure of a given angle. Explain how to do this.

Find the value of \( x \).

29. \( m\angle AOC = 7x - 2 \), \( m\angle DOC = 2x + 8 \), \( m\angle EOD = 27 \)

30. \( m\angle AOB = 4x - 2 \), \( m\angle BOC = 5x + 10 \), \( m\angle COD = 3x - 8 \)

31. \( m\angle AOB = 6x + 5 \), \( m\angle BOC = 4x - 2 \), \( m\angle AOC = 8x + 21 \)

32. **Multi-Step** \( Q \) is in the interior of right \( \angle PRS \). If \( m\angle PRQ \) is 4 times as large as \( m\angle QRS \), what is \( m\angle PRQ \)?

17–18, 4

**Answers**

28. First construct the bisector of the given \( \angle \). Then choose one of the smaller \( \angle \) that was constructed and construct its bisector. The resulting \( \angle \) will have \( \frac{1}{3} \) the measure of the original \( \angle \).
Exercise 33 involves using algebra to find the measures of congruent angles formed by an angle bisector. This exercise prepares students for the Multi-Step Test Prep on page 34.

Kinesthetic For Exercise 30, have students investigate how to find the bisector of the angle using patty paper or a Mira. Draw an angle on a piece of patty paper, and then fold the paper so that the two sides of the angle overlap. The crease created defines a bisector of the angle. To use a Mira, place the Mira on the vertex of the angle so that one side is reflected onto the other side. Then draw the Mira line.

In Exercise 44, if students chose C, they divided 90° by 2 and then added 30° instead of dividing the original angle by 2.

Answers
34. m∠AOB = 90°, rt.; m∠BOC = 126°, obtuse; m∠COD = 36°, acute; m∠DOA = 108°, obtuse

35. m∠COD = 72°; m∠BOC = 90°

36. Suppose a fifth type of music, salsa, is added. What values of x make ∠LOK an acute angle? 0° < x < 15.6

Data Analysis Use the circle graph for Exercises 34–36.

37. Critical Thinking Can an obtuse angle be congruent to an acute angle? Why or why not?

38. The measure of an obtuse angle is (3x + 45)°. What is the largest value for x?

39. Write About It FH bisects ∠EFG. Use the Angle Addition Postulate to explain why m∠EFH = 1/2 m∠EFG.

40. Multi-Step Use a protractor to draw a 70° angle. Then use a compass and straightedge to bisect the angle. What do you think will be the measure of each angle formed? Use a protractor to support your answer.

Check students’ constructions. Each ∠ should be 35°.

41. m∠UOW = 50°, and OV bisects ∠UOW. What is m∠VOY?

42. What is m∠UOX?

43. BD bisects ∠ABC, m∠ABC = (4x + 5)°, and m∠ABD = (3x – 1)°. What is the value of x?

44. If an angle is bisected and then 30° is added to the measure of the bisected angle, the result is the measure of a right angle. What is the measure of the original angle?

45. Short Response If an obtuse angle is bisected, are the resulting angles acute or obtuse? Explain. The ∠ are acute. An obtuse ∠ measures between 90° and 180°. Since 1/2 of 180° is 90, the resulting ∠ must measure less than 90°.

Chapter 1 Foundations for Geometry

1.3 Reading Strategies

The four types of angles are described in this table below.

<table>
<thead>
<tr>
<th>Angle Type</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>Straight</td>
<td>180° angle</td>
</tr>
<tr>
<td>Right</td>
<td>90° angle</td>
</tr>
<tr>
<td>Acute</td>
<td>Angle measures less than 90°</td>
</tr>
<tr>
<td>Obtuse</td>
<td>Angle measures greater than 90°</td>
</tr>
</tbody>
</table>

1.3 Retake

For an angle in a figure made up of two rays, or sides, there are two ways to name the angle.

1. Use two letters: ∠A or ∠B
2. Use three letters: ∠ABC

There are four ways to name this angle.

3. Use three different letters in the figure: ∠F

4. Classify each angle as acute, right, obtuse, or straight.

<table>
<thead>
<tr>
<th>Angle Type</th>
<th>Example</th>
</tr>
</thead>
<tbody>
<tr>
<td>Acute</td>
<td>∠ABC = 45°</td>
</tr>
<tr>
<td>Right</td>
<td>∠DEF = 90°</td>
</tr>
<tr>
<td>Obtuse</td>
<td>∠GHI = 120°</td>
</tr>
<tr>
<td>Straight</td>
<td>∠JKL = 180°</td>
</tr>
</tbody>
</table>

Check students’ drawings.
**CHALLENGE AND EXTEND**

46. Find the measure of the angle formed by the hands of a clock when it is 7:00. **150°**

47. QS bisects \( \angle PQR \), m\( \angle PQR = (x^2)° \), and m\( \angle PQS = (2x + 6)° \). Find all the possible measures for \( \angle PQR \). **36° or 4°**

48. For more precise measurements, a degree can be divided into 60 minutes, and each minute can be divided into 60 seconds. An angle measure of 42 degrees, 30 minutes, and 10 seconds is written as 42°30′10″. Subtract this angle measure from the measure 81°24′15″. **38°54′30″**

49. If 1 degree equals 60 minutes and 1 minute equals 60 seconds, how many seconds are in 2.25 degrees? **8100 seconds**

50. \( \triangle ABC \approx \triangle DBC \). m\( \angle ABC = \left(3x + 4\right)° \) and m\( \angle DBC = \left(2x - 27\right)° \). Is \( \triangle ABD \) a straight angle? Explain. **No; \( x = 62.5 \), and substituting this value into the expressions for the \( \angle \) measures gives a sum of 195.5.**

**SPIRAL REVIEW**

51. What number is 64% of 35? **22.4**

52. What percent of 280 is 33.6? **12%**

53. a line that contains \( A \) and \( B \)

54. two different lines that intersect \( MN \)

55. a plane and a ray that intersect only at \( Q \)

56. \( \overline{JK} \)

57. \( \overline{KL} \)

58. \( \overline{LM} \)

**Find the length of each segment. (Lesson 1-2)**

**Using Technology**

**Segment and Angle Bisectors**

1. Construct the bisector of \( MN \).

2. Construct the bisector of \( \angle BAC \).

a. Draw \( MN \) and construct the midpoint \( B \).

b. Construct a point \( A \) not on the segment.

c. Construct bisector \( AB \) and measure \( MB \) and \( NB \).

d. Drag \( M \) and \( N \) and observe \( MB \) and \( NB \).

**1-3 Measuring and Constructing Angles**

**1-3 PROBLEM SOLVING**

1. Given two different angles that can be formed by a string:

   - Possible string: \( \overline{AB} \), \( \overline{CD} \), \( \overline{EF} \), \( \overline{GH} \), \( \overline{IJ} \)

2. \( \overline{AB} \), \( \overline{CD} \)

3. \( \angle EFG \) and \( \angle HIG \)

4. \( \overline{GH} \), \( \overline{IJ} \)

**Diagram:**

- Properties of Angles:
  - \( \overline{AB} \) is the bisector of \( \angle ABC \), what is the measure of the bisected angle? **90°**

- \( \overline{MN} \) is the angle bisector of \( \angle BAC \), what is the measure of the bisected angles? **90°**

**Choose the best answer:**

1. \( \angle \) vs. \( \triangle \)

2. \( \angle \) vs. \( \overline{AB} \)

3. \( \angle \) vs. \( \overline{CD} \)

4. \( \overline{AB} \), \( \overline{CD} \)

5. \( \overline{AB} \), \( \overline{CD} \)

**Also available on transparency**

**1-3 CHALLENGE**

A circle is the set of all points that are a fixed distance from a single point in a plane. The fixed point is called the center, and the fixed distance is called the radius.

1. A radius of a circle is a segment that connects the center to the edge of the circle. True or False? **True**

2. The diameter of a circle is a segment that connects the center to the center of the circle. True or False? **False**

**Diagram:**

- A circle with center \( O \) and radius \( r \)

**Visual:**

For Exercise 47, students may not try the negative value for \( x \). Remind students to check both values of \( x \) by substituting them back into the expression.

**For Exercise 46, use a representation of a clock, such as a paper plate with a arrow attached to a brad to represent the hands. Given a measurement, students can show the possible time on the clock. Given a time, students can form and then classify the angle.

**Journal:**

Have students name and give the definition of at least 5 other words that use \( bi \) to mean “two.”

**ALTERNATIVE ASSESSMENT**

Have students create a diagram containing each type of angle introduced in this lesson. They should identify the angles in their diagram, measure each with their protractor, choose one to copy, and bisect another.

**Power Presentations**

**1-3 Lesson Quiz**

Classify each angle as acute, right, or obtuse.

1. \( \angle XTS \)

2. \( \angle WTU \)

3. \( K \) is in the interior of \( \angle LMN \), m\( \angle LMK = 52° \), and m\( \angle KMN = 12° \). Find m\( \angle LMKN \). **64°**

4. \( \overline{BD} \) bisects \( \angle ABC \), m\( \angle ABD = \left(\frac{1}{2} y + 10\right) \), and m\( \angle DBC = (y + 4)° \). Find m\( \angle ABC \). **32°**

5. Use a protractor to draw an angle with a measure of 165°. Check students’ work.

6. m\( \angle WYZ = (2x - 5)° \) and m\( \angle ZYW = (3x + 10)° \). Find the value of \( x \). **35°**
1-4 Pairs of Angles

**Objectives**
Identify adjacent, vertical, complementary, and supplementary angles. Find measures of pairs of angles.

**Vocabulary**
- Adjacent angles
- Linear pair
- Complementary angles
- Supplementary angles
- Vertical angles

**Who uses this?**
Scientists use properties of angle pairs to design fiber-optic cables. (See Example 4.)

A fiber-optic cable is a strand of glass as thin as a human hair. Data can be transmitted over long distances by bouncing light off the inner walls of the cable.

Many pairs of angles have special relationships. Some relationships are because of the measurements of the angles in the pair. Other relationships are because of the positions of the angles in the pair.

**Pairs of Angles**
- **Adjacent angles** are two angles in the same plane with a common vertex and a common side, but no common interior points. ∠1 and ∠2 are adjacent angles.
- A **linear pair** of angles is a pair of adjacent angles whose noncommon sides are opposite rays. ∠3 and ∠4 form a linear pair.

**Identifying Angle Pairs**
Tell whether the angles are only adjacent, adjacent and form a linear pair, or not adjacent.

**A** ∠1 and ∠2
∠1 and ∠2 have a common vertex, B, a common side, BC, and no common interior points. Therefore ∠1 and ∠2 are only adjacent angles.

**B** ∠2 and ∠4
∠2 and ∠4 share BC but do not have a common vertex, so ∠2 and ∠4 are not adjacent angles.

**C** ∠1 and ∠3
∠1 and ∠3 are adjacent angles. Their noncommon sides, BC and BA, are opposite rays, so ∠1 and ∠3 also form a linear pair.

**Motivate**
Angles are used in physics, engineering, art and design. Show students a picture of a quilt pattern with angle pairs in it. Discuss the relationships between different pairs of angles. Ask students what they think the angle measures must be so the pieces will fit together.

Explorations and answers are provided in the Explorations binder.
An angle’s measure is 12° more than the supplement of $x$. You can find the supplement of an angle that measures $x$° by subtracting its measure from 180°, or $(180 - x)$°.

### Example 2
#### Finding the Measures of Complements and Supplements

Find the measure of each of the following.

**A.** complement of $\angle M$

$(90 - x)^\circ$

$90° - 26.8° = 63.2°$

**B.** supplement of $\angle N$

$(180 - x)^\circ$

$180° - (2y + 20)^\circ = 180° - 2y - 20$

$= (160 - 2y)^\circ$

#### Finding the measure of each of the following.

2a. complement of $\angle E (102° - 7x)^\circ$

2b. supplement of $\angle F (63° - x)^\circ$

### Example 3
#### Using Complements and Supplements to Solve Problems

An angle measures 3 degrees less than twice the measure of its complement. Find the measure of its complement.

**Step 1**

Let $m\angle A = x$. Then $\angle B$, its complement, measures $(90 - x)^\circ$.

**Step 2**

Write and solve an equation.

$m\angle A = 2m\angle B - 3$

$x = 2(90 - x) - 3$ substitute $x$ for $m\angle A$ and $90 - x$ for $m\angle B$.

$x = 180 - 2x - 3$

$x = 177 - 2x$

$+ 2x + 2x$

$3x = 177$

$\frac{3x}{3} = \frac{177}{3}$

$x = 59$

**Step 3**

The measure of the complement, $\angle B$, is $(90 - 59)^\circ = 31°$.

#### Reaching All Learners

**Through Cooperative Learning**

Each student should create a simple puzzle by dividing a square into six regions using segments. Students should measure all of the angles formed and write several hints for the puzzle solver. For example, a hint might be that one corner of the square is made up of a 15° angle and a 75° angle. Students should then exchange puzzles with a partner and try to reassemble the original square.

### Guided Instruction

Discuss vertical, complementary, supplementary, and adjacent angles and linear pairs. Have students use mental math to find complements and supplements of angles before using variables or expressions.

**Cognitive Strategies**

One way to help students remember that complementary angles add to 90° and that supplementary angles add to 180° is that 90 comes before 180 and C comes before S in the alphabet.

### Additional Examples

#### Example 1

Tell whether the angles are only adjacent, adjacent and form a linear pair, or not adjacent.

A. $\angle AEB$ and $\angle BED$ lin. pair and adjacent

B. $\angle AEB$ and $\angle BEC$ only adj.

C. $\angle DEC$ and $\angle AEB$ not adj.

#### Example 2

Find the measure of each of the following.

A. complement of $\angle F (31°)^\circ$

B. supplement of $\angle G (170° - 7x)^\circ$

#### Example 3

An angle is $10°$ more than 3 times the measure of its complement. Find the measure of the complement. $20°$
Problem-Solving Application

Light passing through a fiber optic cable reflects off the walls in such a way that $\angle 1 \cong \angle 2$, and $\angle 3$ are complementary, and $\angle 2$ and $\angle 4$ are complementary.

If $m\angle 1 = 38^\circ$, find $m\angle 2$, $m\angle 3$, and $m\angle 4$.

Understand the Problem

The answers are the measures of $\angle 2$, $\angle 3$, and $\angle 4$.

List the important information:
- $\angle 1 \cong \angle 2$
- $\angle 1$ and $\angle 3$ are complementary, and $\angle 2$ and $\angle 4$ are complementary.
- $m\angle 1 = 38^\circ$

Make a Plan

If $\angle 1 \cong \angle 2$, then $m\angle 1 = m\angle 2$.

If $\angle 3$ and $\angle 1$ are complementary, then $m\angle 3 = (90 - 38)^\circ$.

If $\angle 4$ and $\angle 2$ are complementary, then $m\angle 4 = (90 - 38)^\circ$.

Solve

By the Transitive Property of Equality, if $m\angle 1 = 38^\circ$ and $m\angle 1 = m\angle 2$, then $m\angle 2 = 38^\circ$. Since $\angle 3$ and $\angle 1$ are complementary, $m\angle 3 = 52^\circ$. Similarly, since $\angle 2$ and $\angle 4$ are complementary, $m\angle 4 = 52^\circ$.

Look Back

The answer makes sense because $38^\circ + 52^\circ = 90^\circ$, so $\angle 1$ and $\angle 3$ are complementary, and $\angle 2$ and $\angle 4$ are complementary. Thus $m\angle 2 = 38^\circ$, $m\angle 3 = 52^\circ$, and $m\angle 4 = 52^\circ$.

What if...? Suppose $m\angle 3 = 27.6^\circ$. Find $m\angle 1$, $m\angle 2$, and $m\angle 4$.

$m\angle 1 = m\angle 2 = 62.4^\circ$; $m\angle 4 = 27.6^\circ$

Identifying Vertical Angles

Another angle pair relationship exists between two angles whose sides form two pairs of opposite rays. **Vertical angles** are two nonadjacent angles formed by two intersecting lines. $\angle 1$ and $\angle 3$ are vertical angles, as are $\angle 2$ and $\angle 4$.

Possible answer: $\angle EDG$ and $\angle FDH$; $m\angle EDG \approx m\angle FDH \approx 45^\circ$.

5. Name another pair of vertical angles. Do they appear to have the same measure? Check by measuring with a protractor.

Close

Summarize

Review the different angle pairs and illustrations in the lesson. Remind students that a linear pair is a pair of adjacent and supplementary angles, but not all complementary and supplementary angles are adjacent. Then have students identify what kind of angle is a complement of an acute angle, a supplement of an obtuse angle, and the supplement of a right angle. acute, acute, right.
THINK AND DISCUSS
1. Explain why any two right angles are supplementary.
2. Is it possible for a pair of vertical angles to also be adjacent? Explain.
3. GET ORGANIZED Copy and complete the graphic organizer below. In each box, draw a diagram and write a definition of the given angle pair.

Guided Practice
Vocabulary Apply the vocabulary from this lesson to answer each question.
1. An angle measures \( x \)°. What is the measure of its complement? What is the measure of its supplement?
2. \( \angle ABC \) and \( \angle CBD \) are adjacent angles. Which side do the angles have in common?
3. Tell whether the angles are only adjacent, adjacent and form a linear pair, or not adjacent.
4. \( \angle 1 \) and \( \angle 3 \) not adj.
5. \( \angle 2 \) and \( \angle 4 \) lin. pair.
6. \( \angle 2 \) and \( \angle 3 \) only adj.

See Example 1 p. 28
Find the measure of each of the following.
7. supplement of \( \angle A \) \( 98.8^\circ \)
8. complement of \( \angle A \) \( 8.8^\circ \)
9. supplement of \( \angle B \) \( (185 - 6x)^\circ \)
10. complement of \( \angle B \) \( (95 - 6x)^\circ \)
11. Multi-Step An angle's measure is 6 degrees more than 3 times the measure of its complement. Find the measure of the angle. \( 69^\circ \)

See Example 2 p. 29
12. Landscaping A sprinkler swings back and forth between \( A \) and \( B \) in such a way that \( \angle 1 \equiv \angle 2 \). \( \angle 1 \) and \( \angle 3 \) are complementary, and \( \angle 2 \) and \( \angle 4 \) are complementary. If \( m\angle 1 = 47.5^\circ \), find \( m\angle 2 \), \( m\angle 3 \), and \( m\angle 4 \).

\( m\angle 2 = 47.5^\circ \); \( m\angle 3 = m\angle 4 = 42.5^\circ \)

See Example 3 p. 29
13. Name each pair of vertical angles. \( \angle ABE \), \( \angle CBD \), \( \angle ABC \), \( \angle EBD \)

Assignment Guide
Assign Guided Practice exercises as necessary.
If you finished Examples 1–3
Basic 14–22, 25, 27–30, 34
Average 14–22, 25, 27–35
Advanced 14–22, 25, 27–35, 37
If you finished Examples 1–5
Average 14–44, 47–55
Advanced 14–55

Homework Quick Check
Quickly check key concepts. Exercises: 14, 18, 22, 24, 28, 30, 34
Exercise 33 involves using algebra to find measures of complementary angles, congruent angles, and linear pairs. This exercise prepares students for the Multi-Step Test Prep on page 34.

Answers

33a.

33b.

33c.

32. The measure of an acute \( \angle \) is less than 90°. Therefore the measure of its supps. must be between 90° and 180°, which means the supps. is an obtuse \( \angle \).

Multi-Step \( \angle ABD \) and \( \angle BDE \) are supplementary. Find the measures of both angles.

26. \( m\angle ABD = 5x^\circ, m\angle BDE = (17x - 18)^\circ \)

27. \( m\angle ABD = (3x + 12)^\circ, m\angle BDE = (7x - 32)^\circ \)

28. \( m\angle ABD = (12x - 12)^\circ, m\angle BDE = (3x + 48)^\circ \)

Multi-Step \( \angle ABD \) and \( \angle BDC \) are complementary. Find the measures of both angles.

29. \( m\angle ABD = (5y + 1)^\circ, m\angle BDC = (3y - 7)^\circ \)

30. \( m\angle ABD = (4y + 5)^\circ, m\angle BDC = (4y + 8)^\circ \)

31. \( m\angle ABD = (y - 30)^\circ, m\angle BDC = 2y^\circ \)

32. Critical Thinking Explain why an angle that is supplementary to an acute angle must be an obtuse angle.

33. This problem will prepare you for the Multi-Step Test Prep on page 34. If \( \angle JAH \) is in the interior of \( \angle AKA \).

a. \( \angle JAH \) and \( \angle KAH \) are complementary angles. \( m\angle JAH = (3x - 8)^\circ \), and \( m\angle KAH = (x + 2)^\circ \). Draw a picture of each relationship. Then find the measure of each angle. 

b. \( \angle JAH \) and \( \angle KAH \) are linear pairs. \( m\angle JAH = 64^\circ \), and \( m\angle KAH = 26^\circ \).

c. \( \angle JAH \) and \( \angle KAH \) are supplementary angles. \( m\angle JAH = 131.5^\circ \), and \( m\angle KAH = 48.5^\circ \).

33. This problem will prepare you for the Multi-Step Test Prep on page 34. If it is in the interior of \( \angle AKA \), find the measures of both angles.

1. \( \angle JAH \) and \( \angle KAH \) are complementary angles. \( m\angle JAH = (3x - 8)^\circ \), and \( m\angle KAH = (x + 2)^\circ \). Draw a picture of each relationship. Then find the measure of each angle.

a. \( \angle JAH \) and \( \angle KAH \) are complementary angles. \( m\angle JAH = 64^\circ \), and \( m\angle KAH = 26^\circ \).

b. \( \angle JAH \) and \( \angle KAH \) are linear pairs. \( m\angle JAH = 131.5^\circ \), and \( m\angle KAH = 48.5^\circ \).

c. \( \angle JAH \) and \( \angle KAH \) are supplementary angles. \( m\angle JAH = 131.5^\circ \), and \( m\angle KAH = 48.5^\circ \).
Determine whether each statement is true or false. If false, explain why.

34. If an angle is acute, then its supplement must be greater than its supplement.

35. A pair of vertical angles may also form a linear pair.

36. If two angles are supplementary and congruent, the measure of each angle is 90°.

37. If a ray divides an angle into two complementary angles, then the original angle is a right angle.

38. Write About It Describe a situation in which two angles are both congruent and complementary. Explain. The 2 ∠s must both measure 45°. 45° + 45° = 90°, so the ∠s are comp. and eq.

39. What is the value of x in the diagram?
   A 15  B 45  C 30  D 90

40. The ratio of the measures of two complementary angles is 1 : 2. What is the measure of the larger angle? (Hint: Let x and 2x represent the angle measures.)
   A 30°  B 45°  C 60°  D 120°

41. m∠A = 3y, and m∠B = 2m∠A. Which value of y makes ∠A supplementary to ∠B?
   A 10  B 18  C 20  D 36

42. The measures of two supplementary angles are in the ratio 7:5. Which value is the measure of the smaller angle? (Hint: Let 7x and 5x represent the angle measures.)
   A 37.5°  B 52.5°  C 75°  D 105°

CHALLENGE AND EXTEND

43. How many pairs of vertical angles are in the diagram?

44. The supplement of an angle is 4 more than twice its complement. Find the measure of the angle.

45. An angle’s measure is twice the measure of its complement. The larger angle is how many degrees greater than the smaller angle?

46. The supplement of an angle is 36° less than twice the supplement of the complement of the angle. Find the measure of the supplement.

Spiral Review

Solve each equation. Check your answer. (Previous course)

47. 4x + 10 = 42
48. 5m − 9 = m + 4
49. 2(y + 3) = 12
50. (d + 4) = 18

Y is between X and Z, XY = 3x + 1, YZ = 2x − 2, and XZ = 84. Find each of the following. (Lesson 1-2)

51. x 17
52. XY 52
53. YZ 32

54. m∠XYZ 26°
55. m∠WZX 52°

1-4 Problem Solving

Use the drawing at the left of the drawing at the right for Exercises 1-6.

1. Name a pair of angles that are adjacent.
   Possible answer: ∠ABE and ∠BCD

2. Name a pair of complementary angles.
   Possible answer: ∠A and ∠B

3. Name an angle that is not labeled.
   Answer: ∠C

4. Name three angles whose measures sum to 180°.
   Possible answer: ∠A, ∠B, and ∠C

5. Name the angle that forms a linear pair with the angle at the top of the right triangle. What is the measure of the angle that is formed by the triangle and the part of the line that is not the hypotenuse? (Lesson 1-3)
   Answer: 110°

6. Add: 8 + 13 + 2 + 14 + 4 = 43, and 6 + 3 + 2 + 1 + 1 = 14, which is a true statement?
   Answer: Yes, it is a true statement.

7. The elbow... 
   a. If x and y are right angles, then x = y.

1-4 Challenge

For greater success in angle measures, students can label the angle parts, including vertical angles and complementary angles, so that their figures do not have space between them?

1. 30°  2. 90°  3. 150°

The angles labeled a, b, and c are angles formed by parallel lines cut by a transversal. What is the relationship between the angles?

a. ∠a and ∠b are corresponding angles.
   Possible answer: ∠a and ∠b

b. ∠b and ∠c are alternate interior angles.
   Possible answer: ∠b and ∠c

2. If two angles are supplementary, then the sum of their measures is 180°. What is the sum of the measures of the angles labeled a, b, and c? (Lesson 1-4)
   Answer: 180°

3. Two angles are complementary. If one angle is 42°, what is the measure of the other angle?
   Answer: 48°

4. Two angles are complementary. If one angle is 57°, what is the measure of the other angle?
   Answer: 33°

5. If two angles are supplementary, find the measure of one angle. 40°; 140°

6. If two angles are supplementary, find the measure of each angle. 36°; 144°

Power Presentations

1-4 Lesson Quiz

m∠A = 64.1°, and m∠B = (4x − 30)°. Find the measure of each of the following.

1. supplement of ∠A 115.9°

2. complement of ∠B (120 − 4x)°

3. Determine whether this statement is true or false. If false, explain why. If two angles are complementary and congruent, then the measure of each is 90°. False; each is 45°.

4. If ∠XYZ and ∠PQR = (8x − 20)°. Find the measure of each angle. 40°; 140°

5. If ∠XYZ and ∠PQR are supplementary, find the measure of each angle. 22°; 68°

Also available on transparency
Chapter 1 Foundations for Geometry

Euclidean and Construction Tools

Can You Dig It? A group of college and high school students participated in an archaeological dig. The team discovered four fossils. To organize their search, Sierra used a protractor and ruler to make a diagram of where different members of the group found fossils. She drew the locations based on the location of the campsite. The campsite is located at $X$ on $XB$. The four fossils were found at $R$, $T$, $W$, and $M$.

1. Are the locations of the campsite at $X$ and the fossils at $R$ and $T$ collinear or noncollinear?

2. How is $X$ related to $−−RT$? If $RX = 10x - 6$ and $XT = 3x + 8$, what is the distance between the locations of the fossils at $R$ and $T$?

3. $∠RXB$ and $∠BXT$ are right angles. Find the measure of each angle formed by the locations of the fossils and the campsite. Then classify each angle by its measure.

4. Identify the special angle pairs shown in the diagram of the archaeological dig.

SECTION 1A

Organizer

Objective: Assess students’ ability to apply concepts and skills in Lessons 1-1 through 1-4 in a real-world format.

Resources

Geometry Assessments
www.mathtekstoolkit.org

<table>
<thead>
<tr>
<th>Problem</th>
<th>Text Reference</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Lesson 1-1</td>
</tr>
<tr>
<td>2</td>
<td>Lesson 1-2</td>
</tr>
<tr>
<td>3</td>
<td>Lesson 1-3</td>
</tr>
<tr>
<td>4</td>
<td>Lesson 1-4</td>
</tr>
</tbody>
</table>

INTERVENTION

Scaffolding Questions


2. How would you compare the distances between $X$ and $R$ and between $X$ and $T$? The distances are the same.

3. You can assume the measure of which angle? What is the term used to classify the angle? $∠RXT = 180°$; it is a straight $∠$.

4. How are the angle pairs $∠RXM$ and $∠MXB$, and $∠RXM$ and $∠MXT$ alike? How are they different? Both $∠$ pairs are adj. angles. $∠RXM$ and $∠MXB$ are comp. $∠RXM$ and $∠MXT$ are supp. and a lin. pair.

Extension

What occupations can you name that use angles and directions in a similar manner? Possible answer: Draftspersons use compasses for greater accuracy in their drawings.
Quiz for Lessons 1-1 Through 1-4

1-1 Understanding Points, Lines, and Planes
Draw and label each of the following.
1. a segment with endpoints X and Y
2. a ray with endpoint M that passes through P
3. three coplanar lines intersecting at a point
4. two points and a line that lie in a plane
Use the figure to name each of the following.
5. three coplanar points
6. two lines \( \overline{XZ} \) and \( \overline{WY} \)
7. a plane containing T, V, and X
8. a line containing V and Z

1-2 Measuring and Constructing Segments
Find the length of each segment.
9. \( \overline{SV} = 6.5 \)
10. \( \overline{PR} = 6 \)
11. \( \overline{ST} = 3.5 \)
12. The diagram represents a straight highway with three towns, Henri, Joaquin, and Kenard. Find the distance from Henri H to Joaquin J.
13. Sketch, draw, and construct a segment congruent to \( \overline{CD} \).
14. Q is the midpoint of \( \overline{PR} \), \( PQ = 2z \), and \( PR = 8z - 12 \). Find \( z \), \( PQ \), and \( PR \).

1-3 Measuring and Constructing Angles
15. Name all the angles in the diagram.
16. \( \angle PVQ = 21° \)
17. \( \angle RVT = 96° \)
18. \( \angle PVS = 143° \)
19. \( \overline{RS} \) bisects \( \angle QRT \), \( m\angle QRS = (x + 3)^\circ \), and \( m\angle SRT = (9x - 4)^\circ \). Find \( m\angle SRT \).
20. Use a protractor and straightedge to draw a 130° angle. Then bisect the angle.

1-4 Pairs of Angles
Tell whether the angles are only adjacent, adjacent and form a linear pair, or not adjacent.
21. \( \angle 1 \) and \( \angle 2 \), \( \text{adj.; lin. pair} \)
22. \( \angle 4 \) and \( \angle 5 \), \( \text{only adj.} \)
23. \( \angle 3 \) and \( \angle 4 \), \( \text{not adj.} \)
24. supplement of \( \angle T \) \( (190 - 5x)^\circ \)
25. complement of \( \angle T \) \( (100 - 5x)^\circ \)

Intervention

Objective: Assess students’ mastery of concepts and skills in Lessons 1-1 through 1-4.

Resources

Assessment Resources
Section 1A Quiz
Test & Practice Generator
One-Stop Planner

Enrichment, Section 1A

Worksheets
CD-ROM
Online
## Coordinate and Transformation Tools

### One-Minute Section Planner

<table>
<thead>
<tr>
<th>Lesson</th>
<th>Lab Resources</th>
<th>Materials</th>
</tr>
</thead>
</table>
| **Lesson 1-5 Using Formulas in Geometry**  
- Apply formulas for perimeter, area, and circumference.  
  - NY Regents Exam  
  - SAT-10  
  - NAEP  
  - ACT  
  - SAT  
  | Geometry Lab Activities  
1-5 Geometry Labs  
  | Optional  
quilts patterns  
  | |
| **Lesson 1-6 Midpoint and Distance in the Coordinate Plane**  
- Develop and apply the formula for midpoint.  
  - Use the Distance Formula and the Pythagorean Theorem to find the distance between two points.  
  - NY Regents Exam  
  - SAT-10  
  - NAEP  
  - ACT  
  - SAT  
  |  
  |  
  | Optional  
coordinate grid with points representing a house and a school, graph paper  
  | |
| **Lesson 1-7 Transformations in the Coordinate Plane**  
- Identify reflections, rotations, and translations.  
  - Graph transformations in the coordinate plane.  
  - NY Regents Exam  
  - SAT-10  
  - NAEP  
  - ACT  
  - SAT  
  |  
  |  
  | Required  
graph paper, straightedge  
Optional  
mirror (MK); examples of tessellations, translations, rotations, and reflections; coordinate grid and cut-out triangle, geometry software  
  | |
| **1-7 Technology Lab Explore Transformations**  
- Use geometry software to perform transformations and explore their properties.  
  - NY Regents Exam  
  - SAT-10  
  - NAEP  
  - ACT  
  - SAT  
  | Technology Lab Activities  
1-7 Lab Recording Sheet  
  | Required  
geometry software  
  | |

MK = Manipulatives Kit
Formulas in Geometry

Lesson 1-5

Finding area and perimeter of figures is an important skill in a variety of occupations.

\[ P = 2\ell + 2w \]
\[ A = \ell \times w \]
\[ P = a + b + c \]
\[ A = \frac{1}{2}bh \]
\[ C = 2\pi r \]
\[ A = \pi r^2 \]
\[ P = 4s \]
\[ A = s^2 \]

Midpoint and Distance

Lesson 1-6

Some problems are easier to solve when the figure is drawn on a coordinate plane.

Midpoint Formula
The midpoint of \(A(x_1, y_1)\) and \(B(x_2, y_2)\) is

\[ M \left( \frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right) \]

Distance Formula
The distance between \(A(x_1, y_1)\) and \(B(x_2, y_2)\) is

\[ AB = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \]

Given \(A(2, 7)\) and \(B(-4, 1)\), the midpoint is

\[ M \left( \frac{2 + (-4)}{2}, \frac{7 + 1}{2} \right) = \left( \frac{-2}{2}, \frac{8}{2} \right) = (-1, 4) \]

Given \(A(2, 7)\) and \(B(-4, 1)\), the distance is

\[ AB = \sqrt{(-4 - 2)^2 + (1 - 7)^2} = \sqrt{36 + 36} = \sqrt{72} \approx 8.5 \]

Transformations

Lesson 1-7

Patterns are formed by translating, reflecting, and rotating figures.

Reflection
Each point and its image are the same distance from the line of reflection.

Rotation
Each point and its image are the same distance from the center of rotation \(P\).

Translation
All points of a figure move the same distance in the same direction.
Using Formulas in Geometry

Objectives: Apply formulas for perimeter, area, and circumference.

Vocabulary:
- Perimeter
- Area
- Base
- Height
- Diameter
- Radius
- Circumference
- Pi

Why learn this?

Puzzles use geometric-shaped pieces. Formulas help determine the amount of materials needed. (See Exercise 6.)

The perimeter $P$ of a plane figure is the sum of the side lengths of the figure. The area $A$ of a plane figure is the number of nonoverlapping square units of a given size that exactly cover the figure.

Finding Perimeter and Area

Perimeter and Area

<table>
<thead>
<tr>
<th>RECTANGLE</th>
<th>SQUARE</th>
<th>TRIANGLE</th>
</tr>
</thead>
<tbody>
<tr>
<td>$P = 2l + 2w$ or $2(l + w)$</td>
<td>$P = 4s$</td>
<td>$P = a + b + c$</td>
</tr>
<tr>
<td>$A = lw$</td>
<td>$A = s^2$</td>
<td>$A = \frac{1}{2}bh$ or $\frac{1}{2}bh$</td>
</tr>
</tbody>
</table>

The base $b$ can be any side of a triangle. The height $h$ is a segment from a vertex that forms a right angle with a line containing the base. The height may be a side of the triangle or in the interior or the exterior of the triangle.

Example 1

Finding the perimeter and area of each figure.

A rectangle in which $l = 17$ cm and $w = 5$ cm

- $P = 2l + 2w$  
- $= 2(17) + 2(5)$  
- $= 34 + 10 = 44$ cm
- $A = lw$  
- $= (17)(5) = 85$ cm$^2$

B triangle in which $a = 8$, $b = (x + 1)$, $c = 4x$, and $h = 6$

- $P = a + b + c$  
- $= 8 + (x + 1) + 4x$  
- $= 5x + 9$  
- $A = \frac{1}{2}bh$  
- $= \frac{1}{2}(x + 1)(6) = 3x + 3$

1. Find the perimeter and area of a square with $s = 3.5$ in.

- $P = 14$ in.; $A = 12.25$ in$^2$

Explorations and answers are provided in the Explorations binder.

Motivate

Discuss buying tile for a room. Ask students how they would determine how much tile they would need to cover a floor. Solicit from them the idea of finding the number of same-sized squares that are needed. Ask them how they could determine the length of baseboard needed to trim the room.

36 Chapter 1 Foundations for Geometry
**Circumference and Area of a Circle**

The circumference \( C \) of a circle is given by the formula \( C = \pi d \) or \( C = 2\pi r \).

The ratio of a circle's circumference to its diameter is the same for all circles. This ratio is represented by the Greek letter \( \pi \) (pi). The value of \( \pi \) is irrational. \( \pi \) is often approximated as 3.14 or \( \frac{22}{7} \).

**Example 2**

Find the amount of fabric used to make the four rectangles.

Each rectangle has a length of 6\( \frac{1}{2} \) in. and a width of 2\( \frac{1}{2} \). 65 in²

**Example 3**

Find the circumference and area of a circle with radius 8 cm. Use \( \pi \) key on your calculator.

1. Find the area of one triangle is \( A = \frac{1}{2} bh = \frac{1}{2} (3) (3) = 4\frac{1}{2} \) in².
2. Find the total area of the 24 triangles is \( 24 \left( 4\frac{1}{2} \right) = 108 \) in².

**Crafts Application**

The Texas Treasures quilt block includes 24 purple triangles. The base and height of each triangle are about 3 in. Find the approximate amount of fabric used to make the 12 triangles.

Example 1

Find the perimeter and area of each figure.

A. 4 in. 6 in. 20 in.; 24 in²

B. \( x + 4 \) 5x 6x + 10; 3x + 12

Example 2

The Queens Quilt block includes 12 blue triangles. The base and height of each triangle are about 4 in. Find the approximate amount of fabric used to make the 12 triangles. 96 in²

**Additional Examples**

**Thinking Strategies**

1. How does knowing the formula for the area of a rectangle help you find the area of a triangle?

2. Compare the results when you multiply 6 by \( \pi \) and multiply 6 by 3.14.

**Ongoing Assessment**

**Diagnose Before the Lesson**

1-5 Warm Up, TE p. 36

**Monitor During the Lesson**

Check It Out! Exercises, SE pp. 36–37

**Assess After the Lesson**

1-5 Lesson Quiz, TE p. 41

Alternative Assessment, TE p. 41
1. Both terms refer to the distance around a figure.

Vocabulary
Apply the vocabulary from this lesson to answer each question.

1. Explain how the concepts of perimeter and circumference are related.

2. For a rectangle, length and width are sometimes used in place of base and height.

3. Find the perimeter and area of each figure.

4. Find the area of each of the following.

5. Find the circumference and area of each circle with the given radius or diameter. Use the calculator. Round to the nearest tenth.

6. Manufacturing A puzzle contains a triangular piece with a base of 3 in. and a height of 4 in. A manufacturer wants to make 80 puzzles. Find the amount of wood used if each puzzle contains 20 triangular pieces. 9600 in$^2$

7. Find the circumference and area of each circle. Use the $\pi$ key on your calculator. Round to the nearest tenth.

8. Find the area of each of the following.

9. Find the perimeter and area of each figure.

PRACTICE AND PROBLEM SOLVING

10. Find the perimeter and area of each figure.

P = $29.6 \text{ m}$; $A = 54.76 \text{ m}^2$

P = $4x + 12$; $A = x^2 + 6x$

P = $9x + 8$; $A = 12x$

11. Find the area of each of the following.

12. Find the area of each of the following.

13. Crafts The quilt pattern includes 32 small triangles. Each has a base of 3 in. and a height of 1.5 in. Find the amount of fabric used to make the 32 triangles. 72 in$^2$

14. C $\approx$ 75.4 m; $A \approx 452.4 \text{ m}^2$

15. C $\approx$ 39.3 ft; $A \approx 122.7 \text{ ft}^2$

16. C $\approx$ 1.6 mi; $A \approx 0.2 \text{ mi}^2$

17. square whose sides are 9.1 yd in length 82.81 yd$^2$

18. square whose sides are $(x + 1)$ in length $x^2 + 2x + 1$

19. triangle whose base is $5\frac{1}{2}$ in. and whose height is $2\frac{1}{4}$ in. 6.1875 in$^2$
Given the area of each of the following figures, find each unknown measure.

20. The area of a triangle is 6.75 m². If the base of the triangle is 3 m, what is the height of the triangle? 2.25 m

21. A rectangle has an area of 347.13 cm². If the length is 20.3 cm, what is the width of the rectangle? 17.1 cm

22. The area of a circle is 64π. Find the radius of the circle. r = 8

23. **ERROR ANALYSIS** Below are two statements about the area of the circle. Which is incorrect? Explain the error.

<table>
<thead>
<tr>
<th>A</th>
<th>B</th>
</tr>
</thead>
<tbody>
<tr>
<td>A = πr²</td>
<td>A = πr²</td>
</tr>
<tr>
<td>= π(8)²</td>
<td>= π(4)²</td>
</tr>
<tr>
<td>= 64π cm²</td>
<td>= 16π cm²</td>
</tr>
</tbody>
</table>

Find the area of each circle. Leave answers in terms of π.

24. circle with a diameter of 28 m

25. circle with a radius of 3 ft

26. **Geography** The radius r of the earth at the equator is approximately 3964 mi. Find the distance around the earth at the equator. Use the π key on your calculator and round to the nearest mile. 24,907 mi

27. **Critical Thinking** Explain how the formulas for the perimeter and area of a square may be derived from the corresponding formulas for a rectangle.

28. Find the perimeter and area of a rectangle whose length is (x + 1) and whose width is (x – 3).

Express your answer in terms of x.

29. **Multi-Step** If the height h of a triangle is 3 inches less than the length of the base b, and the area A of the triangle is 19 times the length of the base, find b and h.

b = 41 in.; h = 38 in.

30. **Multi-Step** This problem will prepare you for the Multi-Step Test Prep on page 58.

A landscaper is installing edging around a garden. The edging costs $1.39 for each 24-inch-long strip. The landscaper estimates it will take 4 hours to install the edging.

a. If the total cost is $120.30, what is the cost of the material purchased? $20.85

b. What is the charge for labor? $99.45

c. What is the area of the semicircle to the nearest tenth? 25.1 ft²

d. What is the area of each triangle? 6 ft²

e. What is the total area of the garden to the nearest foot? 37 ft²

---

**1-5 Using Formulas in Geometry**

**1-5 Practice B**

1. Find the perimeter of each figure.

2. Find the area of each figure.

3. **Critical Thinking** Explain how the formula for the circumference and area of a semicircle may be derived from the corresponding formulas for a circle.

---

**1-5 Practice C**

1. Find the length of the sides of a square whose area is 64 in.²

2. Find the length of the radius of a circle whose area is 16π cm²

3. Explain why the area and the perimeter of a figure can never be equal.

---

**Common Error Alert**

In Exercises 24 and 25, students may confuse πr² in the area formula and 2πr in the circumference formula. Remind them that area is measured in square units, so they should use the formula in which r is squared.

**Reading Math** In Exercises 24 and 25, remind students that the direction to leave answers in terms of π means that π is part of the answer.

**Exercise 30** involves perimeters of triangles and the circumference of a semicircle. This exercise prepares students for the Multi-Step Test Prep on page 58.

---

**Lesson 1-5**
Diversity Exercise 32 gives dimensions of international soccer fields. The size of the playing field in Paralympics soccer, for athletes with cerebral palsy, is 75 m long and 55 m wide.

Number Sense For Exercises 41–43, explain to students that leaving the answer in terms of $\pi$ gives an exact answer. Point out that the decimal answer of the diameter found by using the $\pi$ key on a calculator is an approximation.

If students chose D for Exercise 47, they divided by 2 instead of taking the square root. If they chose F for Exercise 48, they found the length instead of the width. If they chose G for Exercise 50, they did not divide by 2 in the equation representing the area.

Answers
31. a. This must be equal to $(a + b)(c + d)$ because the sum of the areas of the 4 small rects. equals the area of the large rect.

b. This must be equal to the product $(a + 1)(c + 1)$ because the sum of the areas of the 4 small rects. equals the area of the large rect.

c. This must be equal to the product $(a + 1)^2$ because the sum of the areas of the 4 small rects. equals the area of the large rect.

32. a. The large rectangle has length $a + b$ and width $c + d$. Therefore, its area is $(a + b)(c + d)$.

b. Find the area of each of the four small rectangles in the figure. Then find the sum of these areas.

Explain why this sum must be equal to the product $(a + b)(c + d)$.

b. Suppose $b = d = 1$. Write the area of the large rectangle as a product of its length and width. Then find the sum of the areas of the four small rectangles. Explain why this sum must be equal to the product $(a + 1)(c + 1)$.

c. Suppose $b = d = 1$ and $a = c$. Write the area of the large rectangle as a product of its length and width. Then find the sum of the areas of the four small rectangles. Explain why this sum must be equal to the product $(a + 1)^2$.

33. $b = 2$ ft; $h = 3$ ft; $A = 28 \text{ ft}^2$ 28 ft

34. $b = 2$ ft; $h = 22.6$ yd; $A = 282.5 \text{ yd}^2$ 25 yd

35. 9.8 ft; 2.7 ft 26.46 ft$^2$

36. 4 mi 960 ft; 440 ft

37. 3 yd 12 ft; 11 ft

38. 21.4 in.; 7.8 in. 58.4 in.

39. 4 ft 6 in.; 6 in. 10 ft

40. 2 yd 8 ft; 6 ft 13 yd 1 ft

32. Sports The table shows the minimum and maximum dimensions for rectangular soccer fields used in international matches. Find the difference in area of the largest possible field and the smallest possible field.

<table>
<thead>
<tr>
<th>Minimum</th>
<th>Maximum</th>
</tr>
</thead>
<tbody>
<tr>
<td>Length</td>
<td>Width</td>
</tr>
<tr>
<td>100 m</td>
<td>110 m</td>
</tr>
<tr>
<td>64 m</td>
<td>75 m</td>
</tr>
</tbody>
</table>

Find the value of each missing measure of a triangle.

41. $C = 14$ 21.5 mi

42. $A = 100$ 20

43. $C = 50\pi$ 50

44. A skate park consists of two adjacent rectangular regions as shown. Find the perimeter and area of the park.

$P = 52$ yd; $A = 137$ yd$^2$

45. Critical Thinking Explain how you would measure a triangular piece of paper if you wanted to find its area.

46. Write About It A student wrote in her journal, “To find the perimeter of a rectangle, add the length and width together and then double this value.” Does her method work? Explain. The method works because adding the length and width together and doubling the result is $2(l + w)$, which is equivalent to $2l + 2w$.

1.5 READING STRATEGIES

The perimeter of a figure is the sum of the lengths of all the sides of the figure. The area of a square is the product of the lengths of two adjacent sides. Some squares that have the same perimeter do not have the same area.

The perimeter of the square is 14 units. Thus, its area is also 14 units. Therefore, the area of this square is 196 square units.

The perimeter and area of each figure

<table>
<thead>
<tr>
<th>Figure</th>
<th>Perimeter</th>
<th>Area</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. rectangle with $l = 4$, $w = 3$</td>
<td>14</td>
<td>12 $\text{ in}^2$</td>
</tr>
<tr>
<td>2. square with $s = 8$ in</td>
<td>32</td>
<td>64 $\text{ sq in}$</td>
</tr>
</tbody>
</table>

Find the perimeter and area of each triangle.

<table>
<thead>
<tr>
<th>Figure</th>
<th>Perimeter</th>
<th>Area</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. triangle with $b = 8$, $h = 12$</td>
<td>20</td>
<td>24 $\text{ sq in}$</td>
</tr>
<tr>
<td>2. triangle with $b = 10$, $h = 15$</td>
<td>30</td>
<td>75 $\text{ sq in}$</td>
</tr>
</tbody>
</table>

Find the perimeter and area of each rectangle.

<table>
<thead>
<tr>
<th>Figure</th>
<th>Perimeter</th>
<th>Area</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. rectangle with $l = 10$, $w = 5$</td>
<td>25</td>
<td>50 $\text{ sq in}$</td>
</tr>
<tr>
<td>2. rectangle with $l = 8$, $w = 12$</td>
<td>32</td>
<td>96 $\text{ sq in}$</td>
</tr>
</tbody>
</table>

1. How can you find the perimeter of a square? Find the area of a square.

2. What is the perimeter of a square whose area is $25\text{ sq in}$? Find the area of this square.

Test Prep Doctor For Exercise 47, they divided by 2 instead of taking the square root. If they chose F for Exercise 48, they found the length instead of the width. If they chose G for Exercise 50, they did not divide by 2 in the equation representing the area.

Answers
31. a. This must be equal to $(a + b)(c + d)$ because the sum of the areas of the 4 small rects. equals the area of the large rect.

b. This must be equal to the product $(a + 1)(c + 1)$ because the sum of the areas of the 4 small rects. equals the area of the large rect.

c. This must be equal to the product $(a + 1)^2$ because the sum of the areas of the 4 small rects. equals the area of the large rect.

32. a. The large rectangle has length $a + b$ and width $c + d$. Therefore, its area is $(a + b)(c + d)$.

b. Find the area of each of the four small rectangles in the figure. Then find the sum of these areas.

Explain why this sum must be equal to the product $(a + b)(c + d)$.

b. Suppose $b = d = 1$. Write the area of the large rectangle as a product of its length and width. Then find the sum of the areas of the four small rectangles. Explain why this sum must be equal to the product $(a + 1)(c + 1)$.

c. Suppose $b = d = 1$ and $a = c$. Write the area of the large rectangle as a product of its length and width. Then find the sum of the areas of the four small rectangles. Explain why this sum must be equal to the product $(a + 1)^2$.

33. $b = 2$ ft; $h = 3$ ft; $A = 28 \text{ ft}^2$ 28 ft

34. $b = 2$ ft; $h = 22.6$ yd; $A = 282.5 \text{ yd}^2$ 25 yd

35. 9.8 ft; 2.7 ft 26.46 ft$^2$

36. 4 mi 960 ft; 440 ft

37. 3 yd 12 ft; 11 ft

38. 21.4 in.; 7.8 in. 58.4 in.

39. 4 ft 6 in.; 6 in. 10 ft

40. 2 yd 8 ft; 6 ft 13 yd 1 ft

Find the value of each missing measure of a triangle.

41. $C = 14$ 21.5 mi

42. $A = 100$ 20

43. $C = 50\pi$ 50

44. A skate park consists of two adjacent rectangular regions as shown. Find the perimeter and area of the park.

$P = 52$ yd; $A = 137$ yd$^2$

45. Critical Thinking Explain how you would measure a triangular piece of paper if you wanted to find its area.

46. Write About It A student wrote in her journal, “To find the perimeter of a rectangle, add the length and width together and then double this value.” Does her method work? Explain. The method works because adding the length and width together and doubling the result is $2(l + w)$, which is equivalent to $2l + 2w$.
49. Which equation best represents the area \( A \) of the triangle?

\[ A = 2x^2 + 4x \]

\[ B: A = 4x(x + 2) \]

\[ C: A = 2x^2 + 2 \]

\[ D: A = 4x^2 + 8 \]

50. Ryan has a 30 ft piece of string. He wants to use the string to lay out the boundary of a new flower bed in his garden. Which of these shapes would use all the string?

- A circle with a radius of about 9.2 in.
- A rectangle with a perimeter of 6 ft and a width of 5 ft.
- A triangle with each side 4 ft long.
- A square with each side 10 in. long.

**CHALLENGE AND EXTEND**

51. A circle with a 6 in. diameter is stamped out of a rectangular piece of metal as shown. Find the area of the remaining piece of metal. Use the \( \pi \) key on your calculator and round to the nearest tenth.

\( 83.7 \text{ in}^2 \)

52. a. Solve \( P = 2\pi + 2w \) for \( w \).

\[ w = \frac{P - 2\pi}{2} \]

b. Use your result from part a to find the width of a rectangle that has a perimeter of 9 ft and a length of 3 ft.

\( 1.5 \text{ ft} \)

53. Find all possible areas of a rectangle whose sides are natural numbers and whose perimeter is 12.

\( 5, 8, 9 \)

54. **Estimation** The Ahmes Papyrus dates from approximately 1650 B.C.E. Lacking a precise value for \( \pi \), the author assumed that the area of a circle with a diameter of 9 units had the same area as a square with a side length of 8 units. By what percent did the author overestimate or underestimate the actual area of the circle?

overestimated by about 0.6%

55. **Multi-Step** The width of a painting is \( \frac{3}{4} \) the measure of the length of the painting. If the area is 320 in\(^2\), what are the length and width of the painting?

width = 16 in.; length = 20 in.

**Math History**

The Ahmes Papyrus is an ancient Egyptian source of information about mathematics. A page of the Ahmes Papyrus is about 1 foot wide and 18 feet long. Source: scholastic.com

**SPiral Review**

Determine the domain and range of each function. (Previous course)

56. \( \{2, 4\}, (-5, 8), (-3, 4) \}

57. \( \{4, -2\}, (-2, 8), (16, 0) \}

58. the wall of a classroom

59. the place where two walls meet

60. Marion has a piece of fabric that is 10 yd long. She wants to cut it into 2 pieces so that one piece is 4 times as long as the other. Find the lengths of the two pieces.

61. Suppose that \( A, B, \) and \( C \) are collinear points. \( B \) is the midpoint of \( AC \). The coordinate of \( A \) is \(-8\), and the coordinate of \( B \) is \(-2.5\). What is the coordinate of \( C \)?

\( \text{Lesson 1-2} \)

62. An angle's measure is 9 degrees more than 2 times the measure of its supplement. Find the measure of the angle. (Lesson 1-4)

\( 12^\circ \)

**Power Presentations**

1-5 Using Formulas in Geometry

**1-5 PROBLEM SOLVING**

**1-5 CHALLENGE**

A regular polygon is a polygon in which all the sides are congruent and all the angles are congruent. The apothem is the distance from the center of the polygon to a side. The area \( A \) of a regular polygon with a perimeter \( P \) and apothem \( a \) is given by the formula

\[ A = \frac{1}{2} P a \]

Use the regular polygon for Exercises 1-5:

1. Find the perimeter and area.

2. If the side length and apothem are doubled, how will the perimeter and area change?

3. The perimeter will double. The area will be 4 times greater.

4. Find the area.

5. The area will be 4 times greater.

Choose the best answers.

- A rectangle measures 7 ft wide and 11 ft long. If a circle is inscribed in a rectangle, in order to have a circle inscribed, the circle must be a diameter of 12 units. The circle has a diameter of 10 units. What is the approximate area of the remaining surface? (Round to the nearest tenth.)

- 11.3 ft

- 12.0 ft

- 12.3 ft

- 12.6 ft

6. If the area of a regular polygon measures 66.2 in\(^2\), its height is 5 in. If the height of a regular polygon measures 8.3 in, its base is how many units? (Round to the nearest hundredth.)

- 26.6 in.

- 25.6 in.

- 22.6 in.

- 18.6 in.

**1-5 USING FORMULAS IN GEOMETRY**

1. Find the area and perimeter of each figure. Leave answers in terms of \( \pi \).

2. Find the circumference and area of each circle. Leave answers in terms of \( \pi \).

3. Find the area of a triangle with each side 2 cm. Find the area of this triangle with each side 4 cm.

4. 4\( \pi \) cm; 4\( \pi \) cm

5. \( 36\pi \) ft; 12\( \pi \) ft

6. The area of a rectangle is 74.82 in\(^2\), and the length is 12.9 in. Find the width. 5.8 in.
Teach
Remember
Students review the parts of the coordinate plane and the associated terms and locate and plot points.

**INTERVENTION** For additional review and practice on plotting points in the coordinate plane, see Skills Bank page S56.

Inclusion
Have students associate the x-coordinate with right/east and left/west direction and the y-coordinate with up/north and down/south direction from the origin. Point out that both of the ordered pairs (x, y) and (horizontal, vertical) are in alphabetical order.

Close
Assess
Have students draw and label a coordinate plane. Then have them make a design. The students can exchange designs and label each others’ points.

New York Performance Indicators

**Process**
G.CN.6 Recognize and apply mathematics to situations in the outside world
G.R.1 Use physical objects, diagrams, charts, tables, graphs, symbols, equations, or objects created using technology as representations of mathematical concepts

Graphing in the Coordinate Plane

The coordinate plane is used to name and locate points. Points in the coordinate plane are named by ordered pairs of the form (x, y). The first number is the x-coordinate. The second number is the y-coordinate. The x-axis and y-axis intersect at the origin, forming right angles. The axes separate the coordinate plane into four regions, called quadrants, numbered with Roman numerals placed counterclockwise.

**Examples**

1. Name the coordinates of P.
   Starting at the origin (0, 0), you count 1 unit to the right. Then count 3 units up. So the coordinates of P are (1, 3).

2. Plot and label H(−2, −4) on a coordinate plane.
   Name the quadrant in which it is located.
   Start at the origin (0, 0) and move 2 units left. Then move 4 units down. Draw a dot and label it H. H is in Quadrant III.
   You can also use a coordinate plane to locate places on a map.

**Try This**

Name the coordinates of the point where the following streets intersect.
1. Chestnut and Plum (0, 0)
2. Magnolia and Chestnut (0, 4)
3. Oak and Hawthorn (3, 2)
4. Plum and Cedar (−1, 0)

Name the streets that intersect at the given points.
5. (−3, −1) 6. (4, −1)
7. (1, 3) 8. (−2, 1)
9. Spruce and Beech
10. Spruce and Hickory
11. Maple and Elm
12. Pine and Birch
1-6 Midpoint and Distance in the Coordinate Plane

**Objectives**
- Develop and apply the formula for midpoint.
- Use the Distance Formula and the Pythagorean Theorem to find the distance between two points.

**Vocabulary**
- coordinate plane
- leg
- hypotenuse

**NY Performance Indicators**
- G.G.67 Find the length of a line segment, given its endpoints.
- Also, G.G.48, G.G.66.

**Why learn this?**
You can use a coordinate plane to help you calculate distances. (See Example 5.)

Major League baseball fields are laid out according to strict guidelines. Once you know the dimensions of a field, you can use a coordinate plane to find the distance between two of the bases.

A **coordinate plane** is a plane that is divided into four regions by a horizontal line (x-axis) and a vertical line (y-axis). The location, or coordinates, of a point are given by an ordered pair \((x, y)\). You can find the midpoint of a segment by using the coordinates of its endpoints. Calculate the average of the \(x\)-coordinates and the average of the \(y\)-coordinates of the endpoints.

**Midpoint Formula**

The midpoint \(M\) of \(AB\) with endpoints \(A(x_1, y_1)\) and \(B(x_2, y_2)\) is found by:

\[
M \left( \frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)
\]

**Example 1**

Finding the Coordinates of a Midpoint

Find the coordinates of the midpoint of \(CD\) with endpoints \(C(-2, -1)\) and \(D(4, 2)\).

\[
M \left( \frac{-2 + 4}{2}, \frac{-1 + 2}{2} \right) = \left( \frac{2}{2}, \frac{1}{2} \right) = (1, \frac{1}{2})
\]

1. Find the coordinates of the midpoint of \(EF\) with endpoints \(E(-2, 3)\) and \(F(5, -3)\).

**Motivate**

Give students a coordinate graph on which a house and a school have been graphed. Ask them to estimate the distance between the school and the house and to estimate the point that is halfway between these two locations.

Explorations and answers are provided in the Explorations binder.
Example 1
Find the coordinates of the midpoint of \( PQ \) with endpoints \( P(-8, 3) \) and \( Q(-2, 7) \). \((-5, 5)\)

Example 2
\( M \) is the midpoint of \( XY \). \( X \) has coordinates \((2, 7)\), and \( M \) has coordinates \((6, 1)\). Find the coordinates of \( Y \). \((10, -5)\)

Example 3
Find \( FG \) and \( JK \). Then determine whether \( FG \equiv JK \). \( 5; 3\sqrt{2}; \text{no} \)

**INTERVENTION**

**Questioning Strategies**

**EXAMPLE 1**
- Does it matter which point is represented by \((x_1, y_1)\)? Explain.

**EXAMPLE 2**
- Given the midpoint and one endpoint of a segment, how is the Midpoint Formula used to find the missing endpoint?

**EXAMPLE 3**
- How do you substitute the coordinates of two points into the Distance Formula?

**Guided Instruction**

Before teaching the Midpoint Formula, remind students how to find the average of two numbers. Connect average, a measure of central tendency, to the idea of midpoint. Review how to subtract positive and negative numbers to avoid subtracting errors in the Distance Formula. When finding midpoints and distances on a coordinate plane, relate your instruction to finding distances and midpoints on a number line.

**Finding the Coordinates of an Endpoint**

\( M \) is the midpoint of \( AB \). \( A \) has coordinates \((2, 2)\), and \( M \) has coordinates \((4, -3)\). Find the coordinates of \( B \).

**Step 1** Let the coordinates of \( B \) equal \((x, y)\).

**Step 2** Use the Midpoint Formula:
\[
\left( \frac{x + 2}{2}, \frac{y + 2}{2} \right)
\]

**Step 3** Find the \( x \)-coordinate.

<table>
<thead>
<tr>
<th>( x )</th>
<th>( y )</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>(2 + x)</td>
</tr>
<tr>
<td>2((4))</td>
<td>(\frac{2 + x}{2})</td>
</tr>
<tr>
<td>8</td>
<td>(2 + x)</td>
</tr>
<tr>
<td>8</td>
<td>(2)</td>
</tr>
<tr>
<td>(-2)</td>
<td>(-2)</td>
</tr>
<tr>
<td>6</td>
<td>(x)</td>
</tr>
<tr>
<td>(-8)</td>
<td>(y)</td>
</tr>
</tbody>
</table>

The coordinates of \( B \) are \((6, -8)\).

**2.** \( S \) is the midpoint of \( TR \). \( R \) has coordinates \((-6, -1)\), and \( S \) has coordinates \((-1, 1)\). Find the coordinates of \( T \). \((4, 3)\)

The Ruler Postulate can be used to find the distance between two points on a number line. The Distance Formula is used to calculate the distance between two points in a coordinate plane.

**Distance Formula**

In a coordinate plane, the distance \( d \) between two points \((x_1, y_1)\) and \((x_2, y_2)\) is

\[
d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}
\]

**Example 3**
Find \( AB \) and \( CD \). Then determine if \( AB \equiv CD \).

**Step 1** Find the coordinates of each point.

\( A(0, 3), B(5, 1), C(-1, 1), \) and \( D(-3, -4) \)

**Step 2** Use the Distance Formula.

\[
AB = \sqrt{(5 - 0)^2 + (1 - 3)^2}
\]

\[
= \sqrt{25 + 4}
\]

\[
= \sqrt{29}
\]

\[
CD = \sqrt{(-3 - (-1))^2 + (-4 - 1)^2}
\]

\[
= \sqrt{4 + 25}
\]

\[
= \sqrt{29}
\]

Since \( AB = CD \), \( AB \equiv CD \).

**3.** Find \( EF \) and \( GH \). Then determine if \( EF \equiv GH \).

\( EF = 5; GH = 5; EF \equiv GH \)

**Teach**

**Guiding Instruction Through Cognitive Strategies**

Have students use mental math to estimate the midpoint or distance. Then have them use a calculator to support their answer.
Finding Distances in the Coordinate Plane

Use the Distance Formula and the Pythagorean Theorem to find the distance, to the nearest tenth, from $A$ to $B$.

Method 1
Use the Distance Formula. Substitute the values for the coordinates of $A$ and $B$ into the Distance Formula.

$$AB = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$= \sqrt{(2 - (-2))^2 + (-3 - 3)^2}$$

$$= \sqrt{4^2 + (-6)^2}$$

$$= \sqrt{16 + 36}$$

$$= \sqrt{52}$$

$$\approx 7.2$$

Method 2
Use the Pythagorean Theorem. Count the units for sides $a$ and $b$.

$$a = 4$$

$$b = 5$$

$$c^2 = a^2 + b^2$$

$$= 4^2 + 5^2$$

$$= 16 + 25$$

$$= 41$$

$$c = \sqrt{41}$$

$$c \approx 6.4$$

Use the Distance Formula and the Pythagorean Theorem to find the distance, to the nearest tenth, from $R$ to $S$.

4a. $R(3, 2)$ and $S(-3, -1)$

4b. $R(-4, 5)$ and $S(2, -1)$

Additional Examples

Example 2
Use the Distance Formula and the Pythagorean Theorem to find the distance, to the nearest tenth, from $D(3, 4)$ to $E(-2, -5)$. 10.3

Also available on transparency

INTERVENTION Questioning Strategies

- Do you think it is easier to use the Pythagorean Theorem or the Distance Formula to find the distance between two points? Explain.

Algebra Review with students that $\sqrt{3^2 + 4^2} \neq 3 + 4$. They must first square each number and then take the square root of the sum. The answer is 5, not 7.

Inclusion Have students label the coordinates of any two points they are given as $(x_1, y_1)$ and $(x_2, y_2)$ before substituting the numbers into the Midpoint Formula or Distance Formula.
Sports Application

The four bases on a baseball field form a square with 90 ft sides. When a player throws the ball from home plate to second base, what is the distance of the throw, to the nearest tenth?

Set up the field on a coordinate plane so that home plate \( H \) is at the origin, first base \( F \) has coordinates \((90, 0)\), second base \( S \) has coordinates \((90, 90)\), and third base \( T \) has coordinates \((0, 90)\).

The distance \( HS \) from home plate to second base is the length of the hypotenuse of a right triangle.

\[
HS = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}
\]

\[
= \sqrt{(90 - 0)^2 + (90 - 0)^2}
\]

\[
= \sqrt{90^2 + 90^2}
\]

\[
= \sqrt{8100 + 8100}
\]

\[
= \sqrt{16200}
\]

\[
= 127.3 \text{ ft}
\]

5. The center of the pitching mound has coordinates \((42.8, 42.8)\). When a pitcher throws the ball from the center of the mound to home plate, what is the distance of the throw, to the nearest tenth? \textbf{60.5 ft}

THINK AND DISCUSS

1. Can you exchange the coordinates \((x_1, y_1)\) and \((x_2, y_2)\) in the Midpoint Formula and still find the correct midpoint? Explain.

2. A right triangle has sides lengths of \(r\), \(s\), and \(t\). Given that \(s^2 + t^2 = r^2\), which variables represent the lengths of the legs and which variable represents the length of the hypotenuse?

3. Do you always get the same result using the Distance Formula to find distance as you do when using the Pythagorean Theorem? Explain your answer.

4. Why do you think that most cities are laid out in a rectangular grid instead of a triangular or circular grid?

5. GET ORGANIZED Copy and complete the graphic organizer below. In each box, write a formula. Then make a sketch that will illustrate the formula.

**Formulas**

- Midpoint Formula
- Distance Formula
- Pythagorean Theorem

Answers to Think and Discuss

1. yes; \( \frac{x_1 + x_2}{2} = \frac{x_2 + x_1}{2} \) and \( \frac{y_1 + y_2}{2} = \frac{y_2 + y_1}{2} \)

2. \( s \) and \( t \) represent the lengths of the legs, \( r \) represents the length of the hyp.

3. Yes; you can use either method to find the dist. between 2 pts.

4. Possible answer: to make locating addresses easier

**Guided Practice**

1. **Vocabulary** The ____ ? is the side of a right triangle that is directly across from the right angle. (hypotenuse or leg) hypotenuse

2. Find the coordinates of the midpoint of each segment.
   - $AB$ with endpoints $A(4, -6)$ and $B(-4, 2)$
   - $CD$ with endpoints $C(0, -8)$ and $D(3, 0)$

3. $M$ is the midpoint of $LN$. $L$ has coordinates $(3, 3)$.
   - Find the coordinates of $N$.

4. $B$ is the midpoint of $AC$. $A$ has coordinates $(3, 4)$, and $B$ has coordinates $(-1\frac{1}{2}, 1)$. Find the coordinates of $C$.

**Multi-Step** Find the length of the given segments and determine if they are congruent.

5. $JK$ and $FG$ \( \sqrt{29}; \sqrt{29} \); yes

6. $RS$ and $FG$ \( \sqrt{29}; \sqrt{29}; \sqrt{3\sqrt{5}} \); no

**Independent Practice**

Find the coordinates of the midpoint of each segment.

7. $XY$ with endpoints $X(-3, -7)$ and $Y(-1, 1)$

8. $MN$ with endpoints $M(12, -7)$ and $N(-5, -2)$

9. $QM$ with endpoints $Q(3, -3)$ and $M(7, -9)$

10. $DF$ with endpoints $D(-3, -2)$ and $F(1, 1)$

**Extra Practice**

Find the coordinates of the midpoint of each segment.

11. $EY$ with endpoints $E(1, 1)$ and $Y(2, 2)$

12. $XY$ with endpoints $X(2, 3)$ and $Y(4, 3)$

13. $MN$ with endpoints $M(0, 0)$ and $N(4, 0)$

14. $QK$ with endpoints $Q(3, 3)$ and $K(-3, 3)$

15. $DF$ with endpoints $D(-2, -2)$ and $F(2, 2)$

16. $DE$ and $FG$ \( 2\sqrt{5}; 2\sqrt{5} \); yes

17. $DE$ and $RS$ \( 2\sqrt{5}; \sqrt{29} \); no

**Assignment Guide**

Assign Guided Practice exercises as necessary.

If you finished Examples 1–3
   - Basic 12–17, 22, 26, 27, 33
   - Average 12–17, 22, 24–27, 33, 38
   - Advanced 12–17, 22, 24–27, 33, 36, 38

If you finished Examples 1–5
   - Basic 12–23, 26–28, 30
   - Average 12–39, 42–50
   - Advanced 12–50

**Homework Quick Check**

Quickly check key concepts. Exercises: 12, 14, 16, 18, 21, 26, 30
**Exercise 33** involves finding the distance between points on a coordinate plane. This exercise prepares students for the Multi-Step Test Prep on page 58.

### Answers

**32.** When 2 pts. lie on a horiz. or vert. line, they share a common x-coordinate or y-coordinate. To find the dist. between the pts., find the difference of the other coordinates.

**33.** This problem will prepare you for the Multi-Step Test Prep on page 58. Tania uses a coordinate plane to map out plans for landscaping a rectangular patio area. On the plan, one square represents 2 feet. She plans to plant a tree at the midpoint of AC. How far from each corner of the patio does she plant the tree? Round to the nearest tenth. Let M be the mdpt. of AC. AM = MC = 5.0 ft; MB = MD = 6.4 ft.  

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**Chapter 1 Foundations for Geometry**

1. **Reading Strategies**
   - Find the coordinates of the endpoint of each segment.
   - Use the Distance Formula to find the distance between endpoints of a horizontal or vertical line. (0, 1) and (2, 7)  
   - Use the Distance Formula to find the distance between endpoints of a diagonal segment. (2, 4) and (6, 8)  
   - Use the Distance Formula to find the distance between endpoints of a diagonal segment. (−2, 3) and (4, −1)

2. **Multi-Step Problems**
   - Use the Distance Formula to find the distance between two points that lie on a horizontal or a vertical line.
   - Use the Distance Formula to find the distance between two points that lie on a horizontal or a vertical line.
   - Use the Distance Formula to find the distance between two points that lie on a horizontal or a vertical line.

3. **Critical Thinking**
   - The Forbidden City was the world's largest palace complex. Surrounded by a wall and a moat, the rectangular complex is 960 m long and 750 m wide. Find the distance, to the nearest meter, from one corner of the complex to the opposite corner. 1218 m

4. **Write About It**
   - Explain why the Distance Formula is not needed to find the distance between two points that lie on a horizontal or a vertical line.
1. Find the coordinates of the midpoint of \( MN \) with endpoints \( M(\text{-}2, 6) \) and \( N(8, 0) \).

(3, 3)

2. \( K \) is the midpoint of \( \overline{HL} \). \( H \) has coordinates \( (1, \text{-}7) \), and \( K \) has coordinates \( (9, 3) \). Find the coordinates of \( L \).

(17, 13)

3. Find the distance, to the nearest tenth, between \( S(6, 5) \) and \( T(\text{-}3, \text{-}4) \).

12.7

4. The coordinates of the vertices of \( \triangle ABC \) are \( A(2, 5) \), \( B(6, -1) \), and \( C(-\text{-}4, -2) \). Find the perimeter of \( \triangle ABC \), to the nearest tenth.

26.5

5. Find the lengths of \( \overline{AB} \) and \( \overline{CD} \) and determine whether they are congruent. \( \sqrt{10} \); \( \sqrt{10} \); yes

34. Which segment has a length closest to 4 units?

- \( \overline{BC} \)
- \( \overline{LM} \)
- \( \overline{GH} \)
- \( \overline{BC} \)

35. Find the distance, to the nearest tenth, between the midpoints of \( \overline{LM} \) and \( \overline{JK} \).

- \( 1.8 \)
- \( 4.0 \)
- \( 3.6 \)
- \( 5.3 \)

36. What are the coordinates of the midpoint of a line segment that connects the points \( (7, \text{-}3) \) and \( (\text{-}5, 6) \)?

- \( (\text{-}2, 2) \)
- \( (\text{-}2, 6) \)
- \( (2, 2) \)
- \( (2, 6) \)

37. A coordinate plane is placed over the map of a town. A library is located at \( (\text{-}5, 1) \), and a museum is located at \( (3, 5) \). What is the distance, to the nearest tenth, from the library to the museum?

- \( 4.5 \)
- \( 5.7 \)
- \( 6.3 \)
- \( 8.9 \)

38. Use the diagram to find the following.

- \( P \) is the midpoint of \( \overline{AB} \) and \( R \) is the midpoint of \( \overline{BC} \). Find the coordinates of \( Q(\text{-}2.5, 2) \).
- \( B \) is the area of rectangle \( PPBQ \).
- \( DB \) Round to the nearest tenth. \( \sqrt{13} \approx 3.6 \)

39. The coordinates of \( X \) are \( (a + 5, -2a) \), the coordinates of \( Y \) are \( (a + 1, 2a) \). If the distance between \( X \) and \( Y \) is 10, find the value of \( a \).

- \( \pm 2 \)

40. Find two points on the \( x \)-axis that are a distance of 5 units from \( (4, 2) \).

- \( (0, 5) \); \( (0, -5) \)

41. Given \( \angle ABC \) is a right angle of \( \triangle ABC \), \( AC = x \), and \( BC = y \), find \( AB \) in terms of \( x \) and \( y \).

- \( AB = \sqrt{x^2 + y^2} \)

**CHALLENGE AND EXTEND**

42. Find the distance, to the nearest tenth, between the midpoints of \( \overline{LM} \) and \( \overline{JK} \).

- \( 1.8 \)
- \( 4.0 \)
- \( 3.6 \)
- \( 5.3 \)

43. Which segment has a length closest to 4 units?

- \( \overline{GH} \)
- \( \overline{LM} \)
- \( \overline{BC} \)
- \( \overline{LM} \)

44. Find two points on the \( y \)-axis that are a distance of 5 units from \((4, 2)\). \((0, 5)\); \((0, -5)\)

45. Find the distance, to the nearest tenth, from \( (\text{-}1, 0) \) to \( (2, 3) \).

- \( 3.6 \)

46. Given \( \angle ABC \) is a right angle of \( \triangle ABC \), \( AC = x \), and \( BC = y \), find \( AB \) in terms of \( x \) and \( y \).

- \( AB = \sqrt{x^2 + y^2} \)

**SPIRAL REVIEW**

Determine if the ordered pair \((-1, 4)\) satisfies each function. (Previous course)

42. \( y = 3x - 1 \)

- no

43. \( f(x) = 5 - x^2 \)

- yes

44. \( g(x) = x^2 - x + 2 \)

- yes

\( \overline{BB} \) bisects straight angle \( \angle ABC \), and \( \overline{BB} \) bisects \( \angle CBD \).

Find the measure of each angle and classify it as acute, right, or obtuse. (Lesson 1-3)

45. \( \angle ABD \)

- 90°; rt.

46. \( \angle CBE \)

- 45°; acute

47. \( \angle ABE \)

- 135°; obtuse

Find the area of each of the following. (Lesson 1-5)

48. Square whose perimeter is 20 in.

- 25 in²

49. Triangle whose height is 2 ft and whose base is twice its height

- 4 ft²

50. Rectangle whose length is \( x \) and whose width is \((4x - 5)\)

- 4x² + 5x
**Objective**

Identify reflections, rotations, and translations.

Graph transformations in the coordinate plane.

**Vocabulary**

- **transformation**
- **rotation**
- **reflection**
- **translation**

**Who uses this?**

Artists use transformations to create decorative patterns. (See Example 4.)

The Alhambra, a 13th-century palace in Granada, Spain, is famous for the geometric patterns that cover its walls and floors. To create a variety of designs, the builders based the patterns on several different transformations.

A **transformation** is a change in the position, size, or shape of a figure. The original figure is called the *preimage*. The resulting figure is called the *image*. A transformation maps the preimage to the image. Arrow notation (→) is used to describe a transformation, and primes (′) are used to label the image.

**New York Performance Indicators**

- **G.G.54** Define, investigate, justify, and apply isometries in the plane (rotations, reflections, translations, glide reflections).
- **G.G.56** Identify specific isometries by observing orientation, numbers of invariant points, and/or parallelism.

**Example 1**

**Identifying Transformations**

Identify the transformation. Then use arrow notation to describe the transformation.

The transformation cannot be a translation because each point and its image are not in the same position. The transformation is a reflection. \( \triangle EFG \rightarrow \triangle E'F'G' \)

**Motivate**

Show students examples of tessellations, translations, rotations, and reflections in the real world, from magazines or advertisements. Ask students to demonstrate transformations by standing and sliding two steps right and then rotating 90° clock-wise.

Explorations and answers are provided in the Explorations binder.
Identify the transformation. Then use arrow notation to describe the transformation.

The transformation is a 90° rotation. RSTU \rightarrow R'S'T'U'

Identify each transformation. Then use arrow notation to describe the transformation.

Example 1a. M'N'O'P' \rightarrow MNOP

Example 1b. \triangle XYZ \rightarrow \triangle X'Y'Z'

Example 2

A figure has vertices at A(−1, 4), B(−1, 1), and C(3, 1). After a transformation, the image of the figure has vertices at A'(−1, −4), B'(−1, −1), and C'(3, −1). Draw the preimage and image. Then identify the transformation.

Plot the points. Then use a straightedge to connect the vertices.

The transformation is a reflection across the x-axis because each point and its image are the same distance from the x-axis.

Example 3

A figure has vertices at E(2, 0), F(2, −1), G(5, −1), and H(5, 0). After a transformation, the image of the figure has vertices at E'(0, 2), F'(1, 2), G'(1, 5), and H'(0, 5). Draw the preimage and image. Then identify the transformation.

To find coordinates for the image of a figure in a translation, add a to the x-coordinates of the preimage and add b to the y-coordinates of the preimage.

Translations in the Coordinate Plane

Find the coordinates for the image of \triangle ABC after the translation (x, y) \rightarrow (x + 3, y − 4).

Draw the image.

Step 1 Find the coordinates of \triangle ABC.

The vertices of \triangle ABC are A(−1, 1), B(−3, 3), and C(−4, 0).

INTERVENTION

Questioning Strategies

Example 1

How would each image in Example 1A and Example 1B differ if the transformation were a translation?

Example 2

How did you decide what type of transformation this was?
**Example 3**

Find the coordinates for the image of \( \triangle ABC \) after the translation \((x, y) \to (x + 2, y - 1)\).

Draw the image. \( A'(-2, 1), B'(-1, 3), C'(1, 0) \)

**Example 4**

The figure shows part of a tile floor. Write a rule for the translation of hexagon 1 to hexagon 2.

\((x, y) \to (x - 3, y + 1.5)\)

Also available on transparency

**Art History Application**

The pattern shown is similar to a pattern on a wall of the Alhambra. Write a rule for the translation of square 1 to square 2.

**Step 1** Choose 2 points

Choose a point \( A \) on the preimage and a corresponding point \( A' \) on the image. \( A \) has coordinates \((3, 1)\), and \( A' \) has coordinates \((1, 3)\).

**Step 2** Translate

To translate \( A \) to \( A' \), 2 units are subtracted from the \( x \)-coordinate and 2 units are added to the \( y \)-coordinate. Therefore, the translation rule is \((x, y) \to (x - 2, y + 2)\).

**Step 3** Plot the points. Then finish drawing the image by using a straightedge to connect the vertices.

3. Find the coordinates for the image of \(JKLM\) after the translation \((x, y) \to (x - 2, y + 4)\). Draw the image. \( J'(-1, 5); K'(1, 5); L'(1, 0); M'(-1, 0) \)

**Check It Out!**

4. Use the diagram to write a rule for the translation of square 1 to square 3. \((x, y) \to (x - 4, y - 4)\)

**THINK AND DISCUSS**

1. Explain how to recognize a reflection when given a figure and its image.
2. GET ORGANIZED Copy and complete the graphic organizer. In each box, sketch an example of each transformation.

**Answers to Think and Discuss**

Possible answers:

1. The preimage and image will be mirror images of each other.
GUIDED PRACTICE

**Vocabulary**  Apply the vocabulary from this lesson to answer each question.

1. Given the transformation \(\triangle XYZ \rightarrow \triangle XYZ'\), name the preimage and image of the transformation. **Preimage is** \(\triangle XYZ\), **image is** \(\triangle XYZ'\).

2. The types of transformations of geometric figures in the coordinate plane can be described as a slide, a flip, or a turn. What are the other names used to identify these transformations? **translation; reflection; rotation**

Identify each transformation. Then use arrow notation to describe the transformation.

3. Identify each transformation. Then use arrow notation to describe the transformation.

4. \(\triangle ABC \rightarrow \triangle A'B'C'\)

5. A figure has vertices at \(A(-3, 2), B(-1, -1),\) and \(C(-4, -2)\). After a transformation, the image of the figure has vertices at \(A'(3, 2), B'(1, -1),\) and \(C'(4, -2)\). Draw the preimage and image. Then identify the transformation. **translation**

6. **Multi-Step** The coordinates of the vertices of \(\triangle DEF\) are \(D(2, 3), E(1, 1),\) and \(F(4, 0)\). Find the coordinates for the image of \(\triangle DEF\) after the translation \((x, y) \rightarrow (x - 3, y - 2)\). Draw the preimage and image.

7. **Animation** In an animated film, a simple scene can be created by translating a figure against a still background. Write a rule for the translation that maps the rocket from position 1 to position 2. \((x, y) \rightarrow (x + 4, y + 4)\)

PRACTICE AND PROBLEM SOLVING

Identify each transformation. Then use arrow notation to describe the transformation.

8. **rotation;** \(DEFG \rightarrow D'E'FG'\)

9. **reflection;** \(WXYZ \rightarrow W'X'Y'Z'\)

10. A figure has vertices at \(J(-2, 3), K(0, 3), L(0, 1),\) and \(M(-2, 1)\). After a transformation, the image of the figure has vertices at \(J'(2, 1), K'(4, 1), L'(4, -1),\) and \(M'(2, -1)\). Draw the preimage and image. Then identify the transformation. **translation**

Answers

10.
Chapter 1 Foundations for Geometry

11. **Multi-Step** The coordinates of the vertices of rectangle $ABCD$ are $A(-4, 1)$, $B(1, 1)$, $C(1, -2)$, and $D(-4, -2)$. Find the coordinates for the image of rectangle $ABCD$ after the translation $(x, y) \rightarrow (x + 3, y - 2)$. Draw the preimage and the image.

12. **Travel** Write a rule for the translation that maps the descent of the hot air balloon, $(x, y) \rightarrow (x + 11, y - 4)$.

Which transformation is suggested by each of the following?

13. mountain range and its image on a lake
14. straight line path of a band
15. wings of a butterfly

A transformation maps $A$ onto $B$ and $C$ onto $D$.

16. the $x$-axis
17. the $y$-axis
18. Find the vertices of one of the triangles on the graph. Then use arrow notation to write a rule for translating the other three triangles.

A transformation maps $A$ onto $B$ and $C$ onto $D$.

19. Name the image of $A$ after the given transformation.
20. Name the preimage of $A$.
21. Name the image of $C$.
22. Name the preimage of $D$.

14. straight line path of a band
15. wings of a butterfly

Graph each figure and its image after the given translation.

23. Find the coordinates for the image of triangle $RST$ with vertices $R'(1, -4)$, $S'(-1, -1)$, and $T'(-5, 1)$ after the translation $(x, y) \rightarrow (x - 2, y - 8)$, $R''(-1, -12)$, $S''(-3, -9)$, $T''(-7, -7)$

24. **Critical Thinking** Consider the translations $(x, y) \rightarrow (x + 5, y + 3)$ and $(x, y) \rightarrow (x + 10, y + 5)$. Compare the two translations.

Given points $F(3, 5)$, $G(-1, 4)$, and $H(5, 0)$, draw triangle $FGH$ and its reflection across each of the following lines.

16. the $x$-axis
17. the $y$-axis
18. $y \rightarrow x$
19. $x \rightarrow y$
20. Find the coordinates for the image of $F$.
21. Name the preimage of $F$.

25. $MN$ with endpoints $M(2, 8)$ and $N(-3, 4)$ after the translation $(x, y) \rightarrow (x + 2, y - 5)$
26. $KL$ with endpoints $K(-1, 1)$ and $L(3, -4)$ after the translation $(x, y) \rightarrow (x - 4, y + 3)$

27. **Write About It** Given a triangle in the coordinate plane, explain how to draw its image after the translation $(x, y) \rightarrow (x + 1, y + 1)$.

28. This problem will prepare you for the Multi-Step Test Prep on page 58. Greg wants to rearrange the triangular pattern of colored stones on his patio. What combination of transformations could he use to transform triangle $CAE$ to the image on the coordinate plane?

Possible answer: 2 reflections (across the $y$-axis and across $EC$)
29. Which type of transformation maps \( \triangle XYZ \) to \( \triangle XYZ' ? \)
- Reflection
- Rotation
- Not here

30. \( \triangle DEF \) has vertices at \( D(-4, 2), E(-3, -3), \) and \( F(1, 4). \)
Which of these points is a vertex of the image of \( \triangle DEF \) after the translation \( (x, y) \rightarrow (x - 2, y + 1)? \)
\[ \begin{align*}
\text{A} & : (-2, 1) \\
\text{B} & : (-5, -2) \\
\text{C} & : (3, 3) \\
\text{D} & : (-6, -1)
\end{align*} \]

31. Consider the translation \((1, 4) \rightarrow (-2, 3). \) What number was added to the x-coordinate?
-3
-1
1
7

32. Consider the translation \((-5, -7) \rightarrow (-2, -1). \) What number was added to the y-coordinate?
-3
3
6
8

33. \( \triangle RST \) with vertices \( R(-2, -2), S(-3, 1), \) and \( T(1, 1) \) is translated by \( (x, y) \rightarrow (x - 1, y + 3). \) Then the image, \( \triangle R'S'T', \) is translated by \( (x, y) \rightarrow (x + 4, y - 1), \) resulting in \( \triangle R''S''T''. \)

a. Find the coordinates for the vertices of \( \triangle R''S''T''. \)
\( R'(1, 0), S'(0, 3), T'(4, 3) \)
b. Write a rule for a single translation that maps \( \triangle RST \) to \( \triangle R''S''T''. \)

34. Find the angle through which the minute hand of a clock rotates per minute.
72°

35. A triangle has vertices \( A(1, 0), B(5, 0), \) and \( C(2, 3). \) The triangle is rotated 90° counterclockwise about the origin. Draw and label the image of the triangle.

Determine the coordinates for the reflection image of any point \( A(x, y) \) across the given line.
-36. \( x \)-axis \((x, -y)\)
-37. \( y \)-axis \((-x, y)\)

38. \( y = x^2 + 12x + 35 \)
-39. \( y = x^2 + 3x - 18 \)
-40. \( y = x^2 - 18x + 81 \)
-41. \( y = x^2 - 3x + 21 \)

42. Given \( m\angle A = 76.1°, \) find the measure of each of the following.
-43. Supplement of \( \angle A = 103.9° \)
-44. Complement of \( \angle A = 13.9° \)

44. \((2, 3) \) and \((4, 6)\)
-45. \((-1, 4) \) and \((0, 8)\)
-46. \((-3, 7) \) and \((-6, -2)\)
-47. \((5, 1) \) and \((-1, 3)\)

1-7 Transformations in the Coordinate Plane
Explore Transformations

A transformation is a movement of a figure from its original position (preimage) to a new position (image). In this lab, you will use geometry software to perform transformations and explore their properties.

**Activity 1**

1. Construct a triangle using the segment tool.
   Use the text tool to label the vertices A, B, and C.
2. Select points A and B in that order. Choose Mark Vector from the Transform menu.
3. Select △ABC by clicking on all three segments of the triangle.
4. Choose Translate from the Transform menu, using Marked as the translation vector. What do you notice about the relationship between your preimage and its image?
   They appear to be $\cong$.
5. What happens when you drag a vertex or a side of △ABC?
   The $\triangle$s move together and stay the same size and shape.

**Try This**

For Problems 1 and 2 choose New Sketch from the File menu.

1. The image of the $\triangle$ moves in the same direction as the endpt. and remains a fixed dist. apart.
2. The $\triangle$s move together and remain a fixed dist. apart.

1. Construct a triangle and a segment outside the triangle. Mark this segment as a translation vector as you did in Step 2 of Activity 1. Use Step 4 of Activity 1 to translate the triangle. What happens when you drag an endpoint of the new segment?
2. Instead of translating by a marked vector, use Rectangular as the translation vector and translate by a horizontal distance of 1 cm and a vertical distance of 2 cm. Compare this method with the marked vector method. What happens when you drag a side or vertex of the triangle?
3. Select the angles and sides of the preimage and image triangles. Use the tools in the Measure menu to measure length, angle measure, perimeter, and area. What do you think is true about these two figures? Each measurement is the same for the preimage and image $\triangle$s. The $\triangle$s appear to be $\cong$.
Activity 2


2. Select point H and choose Mark Center from the Transform menu.

3. Select $\angle GHI$ by selecting points G, H, and I in that order. Choose Mark Angle from the Transform menu.

4. Select the entire triangle $\triangle GHI$ by dragging a selection box around the figure.

5. Choose Rotate from the Transform menu, using Marked Angle as the angle of rotation.

6. What happens when you drag a vertex or a side of $\triangle GHI$? The $\triangle$ and its image rotate by the same $\angle$ measure and remain the same size and shape.

Try This

For Problems 4–6 choose New Sketch from the File menu.

4. Instead of selecting an angle of the triangle as the rotation angle, draw a new angle outside of the triangle. Mark this angle. Mark $\angle GHI$ as Center and rotate the triangle. What happens when you drag one of the points that form the rotation angle? The image rotates by the same $\angle$ measure as the marked $\angle$.

The $\triangle$ rotates by the same $\angle$ measure. When $P$ is inside, the image overlaps the $\triangle$. When $P$ coincides with a vertex, the image also coincides with the vertex.

5. Construct $\triangle QRS$, a new rotation angle, and a point $P$ not on the triangle. Mark $P$ as the center and mark the angle. Rotate the triangle. What happens when you drag $P$ outside, inside, or on the preimage triangle?

6. Instead of rotating by a marked angle, use Fixed Angle as the rotation method and rotate by a fixed angle measure of 30°. Compare this method with the marked angle method.

7. Using the fixed angle method of rotation, can you find an angle measure that will result in an image figure that exactly covers the preimage figure? $360^\circ$

6. The $\triangle$ rotated by an $\angle$ of 30°, not by the measure of the marked $\angle$.

Close

Key Concept
When you translate or rotate a figure, the lengths of sides, angle measures, perimeter, and area do not change. The image is congruent to the preimage.

Assessment

Journal Have students explain how the preimage and image of a figure compare under a translation and a rotation. Have them discuss which properties are preserved.
Chapter 1 Foundations for Geometry

1. **Coordinate and Transformation Tools**

   **Pave the Way** Julia wants to use L-shaped paving stones to pave a patio. Two stones will cover a 12 in. by 18 in. rectangle.

   1. She drew diagram $ABCDEF$ to represent the patio. Find the area and perimeter of the patio. How many paving stones would Julia need to purchase to pave the patio? If each stone costs $2.25, what is the total cost of the stones for the patio? Describe how you calculated your answer.

   2. Julia plans to place a fountain at the midpoint of $AF$. How far is the fountain from $B$, $C$, $E$, and $F$? Round to the nearest tenth.

   3. Julia used a pair of paving stones to create another pattern for the patio. Describe the transformation she used to create the pattern. If she uses just one transformation, how many other patterns can she create using two stones? Draw all the possible combinations. Describe the transformation used to create each pattern.

---

**INTERVENTION**

**Scaffolding Questions**

1. What is the significance of the way the sides of the stones are marked? How can you find the area of this figure? The marks on the sides of the figure mean that these sides are the same length. Right angle marks tell you that $\angle A$ and $\angle F$ measure $90^\circ$. You will use the formulas for the area of a rectangle and the area of a square to find the areas of the two parts of the figure. Then add the two areas together.

2. What formulas will you use to find the location of the fountain? First you will find the midpoint of a segment. Then use the Distance Formula to find the distance between the fountain and each point.

3. How can you decide which transformation to use? What is meant by creating different patterns? You can use a reflection, a rotation, or a translation. You just need to be sure that the sides of the figures will touch when you repeat your pattern. Different patterns can be created using the two tiles and one transformation.

**Extension**

Have students choose a pattern and reproduce it on graph paper. They could also create the tile patterns using geometry software.
Quiz for Lessons 1-5 Through 1-7

1-5 Using Formulas in Geometry
Find the perimeter and area of each figure.

1. \( P = 56 \text{ in.}; \ A = 160 \text{ in}^2 \)

2. \( P = 5x + 22; \ A = 13x + 130 \)

3. \( P = 18x + 4; \ A = 18x^2 + 12x \)

4. \( P = 19x + 22; \ A = 25x + 70 \)

5. Find the circumference and area of a circle with a radius of 6 m. Use the \( \pi \) key on your calculator and round to the nearest tenth. \( C \approx 37.7 \text{ m}; \ A \approx 113.1 \text{ m}^2 \)

1-6 Midpoint and Distance in the Coordinate Plane

6. Find the coordinates for the midpoint of \( XY \) with endpoints \( X(-4, 6) \) and \( Y(3, 8) \). \((-0.5, 7)\)

7. \( J \) is the midpoint of \( HK \). \( H \) has coordinates \((6, -2)\), and \( J \) has coordinates \((9, 3)\). Find the coordinates of \( K \). \((12, 8)\)

8. Using the Distance Formula, find \( QR \) and \( ST \) to the nearest tenth. Then determine if \( QR \parallel ST \). \( QR = 7.3 \); \( ST = 7.3 \); \( QR = ST \)

9. Using the Distance Formula and the Pythagorean Theorem, find the distance, to the nearest tenth, from \( P(4, 3) \) to \( G(-3, -2) \). \( \approx 8.6 \)

1-7 Transformations in the Coordinate Plane

Identify the transformation. Then use arrow notation to describe the transformation.

10. \( \triangle ABC \rightarrow \triangle A'B'C' \)

11. \( PQRS \rightarrow P'Q'R'S' \)

12. A graphic designer used the translation \((x, y) \rightarrow (x - 3, y + 2)\) to transform square \(HIJKL\). Find the coordinates and graph the image of square \(HIJKL\).

13. A figure has vertices at \(X(1, 1)\), \(Y(3, 1)\), and \(Z(3, 4)\). After a transformation, the image of the figure has vertices at \(X'(-1, -1)\), \(Y'(-3, -1)\), and \(Z'(-3, -4)\). Graph the preimage and image. Then identify the transformation.

12. \( H'(-1, 3); J(2, 3); K'(2, 0); L'(-1, 0) \)

Organizer

Objective: Assess students’ mastery of concepts and skills in Lessons 1-5 through 1-7.

Resources

Assessment Resources
Section 1B Quiz

Test & Practice Generator
One-Step Planner

INTERVENTION

Resources

Ready to Go On? Intervention and Enrichment Worksheets

Ready to Go On? CD-ROM

Ready to Go On? Online

my.hrw.com

Answers

Vocabulary

acute angle ................. 21
diameter ..................... 37
adjacent angles ............. 28
distance ....................... 13
angle ......................... 20
endpoint ....................... 7
angle bisector ............. 23
exterior of an angle ....... 20
area ......................... 36
height ......................... 36
base ......................... 36
hypotenuse .................. 45
between ....................... 14
image ......................... 50
bisect ......................... 15
interior of an angle ...... 20
right angle .................. 21
circumference ............. 37
leg ......................... 45
collinear ...................... 6
length ....................... 13
complementary angles ... 29
line .......................... 6
congruent angles ........ 22
linear pair .................. 28
congruent segments ..... 13
measure ..................... 20
construction ............... 14
midpoint ...................... 15
coordinate .................. 13
obtuse angle ................ 21
coordinate plane .......... 43
translation .................. 50
co-planar ..................... 6
opposite rays .............. 7
degree ....................... 20
perimeter ................... 36
pi .......................... 37
vertical angles ............. 30

Complete the sentences below with vocabulary words from the list above.

1. A(n) ______ divides an angle into two congruent angles.
2. ______ are two angles whose measures have a sum of 90°.
3. The length of the longest side of a right triangle is called the ______.

1-1 Understanding Points, Lines, and Planes (pp. 6–11)

**EXAMPLES**

- Name the common endpoint of $\overrightarrow{ST}$ and $\overrightarrow{SR}$.

$\overrightarrow{SR}$ and $\overrightarrow{ST}$ are opposite rays with common endpoint $S$.

**EXERCISES**

Name each of the following.

4. four coplanar points
5. line containing $B$ and $C$
6. plane that contains $A$, $G$, and $E$
Draw and label three coplanar lines intersecting in one point.

1-2 Measuring and Constructing Segments (pp. 13–19)

EXAMPLES

Find the length of $XY$.

$$XY = | -2 - 1 | = |-3| = 3$$

$S$ is between $R$ and $T$. Find $RT$.

$$RT = RS + ST$$

$$3x + 2 = 5x - 6 + 2x$$

$$3x + 2 = 7x - 6$$

$$x = 2$$

$$RT = 3(2) + 2 = 8$$

EXERCISES

Find each length.

10. $JL$

11. $HK$

12. $y$ is between $x$ and $z$, $XY = 13.8$, and $XZ = 21.4$.

Find $YZ$.

13. $Q$ is between $P$ and $R$.

Find $PR$.

14. $U$ is the midpoint of $TV$, $TU = 3x + 4$, and $UV = 5x - 2$.

Find $TU$, $UV$, and $TV$.

15. $E$ is the midpoint of $DF$, $DE = 9x$, and $EF = 4x + 10$.

Find $DE$, $EF$, and $DF$.

1-3 Measuring and Constructing Angles (pp. 20–27)

EXAMPLES

Classify each angle as acute, right, or obtuse.

$\angle ABC$ acute; $\angle CBD$ acute; $\angle ABD$ obtuse; $\angle DBE$ acute; $\angle CBE$ obtuse

$KM$ bisects $\angle JKL$, $m \angle JKM = (3x + 4)^\circ$, and $m \angle MKL = (6x - 5)^\circ$.

Find $m \angle JKL$.

$$3x + 4 = 6x - 5$$

Def. of bisector

$$3x + 9 = 6x$$

Add 5 to both sides.

$$9 = 3x$$

Subtract 3x from both sides.

$$x = 3$$

Divide both sides by 3.

$$m \angle JKL = 3x + 4 + 6x - 5$$

$$= 9x - 1$$

$$= 9(3) - 1 = 26^\circ$$

EXERCISES

Classify each angle as acute, right, or obtuse.

16. $m \angle HJL = 116^\circ$.

Find $m \angle HJK$.

17. $N$ is the midpoint of $MNQ$.

$m \angle MNQ = (6x - 12)^\circ$, and $m \angle PNO = (4x + 8)^\circ$.

Find $m \angle MNQ$.
1-4 Pairs of Angles (pp. 28–33)

**Examples**

- Tell whether the angles are only adjacent, adjacent and form a linear pair, or not adjacent.

  1. $\angle 1$ and $\angle 2$ are only adjacent.
  2. $\angle 2$ and $\angle 4$ are not adjacent.
  3. $\angle 2$ and $\angle 3$ are adjacent and form a linear pair.
  4. $\angle 1$ and $\angle 4$ are adjacent and form a linear pair.

- Find the measure of the complement and supplement of each angle.

  19. $90 - 67.3 = 22.7^\circ$
  20. $180 - 67.3 = 112.7^\circ$
  21. $90 - (3x - 8) = (98 - 3x)^\circ$
  22. $180 - (3x - 8) = (188 - 3x)^\circ$

**Exercises**

Tell whether the angles are only adjacent, adjacent and form a linear pair, or not adjacent.

19. $\angle 1$ and $\angle 2$
20. $\angle 3$ and $\angle 4$
21. $\angle 2$ and $\angle 5$

Find the measure of the complement and supplement of each angle.

22. $\angle 1$ and $\angle 2$
23. $\angle 3$ and $\angle 4$
24. $\angle 2$ and $\angle 5$

25. $90 - 67.3 = 22.7^\circ$
26. $180 - 67.3 = 112.7^\circ$
27. $90 - (3x - 8) = (98 - 3x)^\circ$
28. $180 - (3x - 8) = (188 - 3x)^\circ$

1-5 Using Formulas in Geometry (pp. 36–41)

**Examples**

- Find the perimeter and area of the triangle.

  \[ P = 2x + 3x + 5 + 10 = 5x + 15 \]
  \[ A = \frac{1}{2} (3x + 5)(2x) = 3x^2 + 5x \]

- Find the circumference and area of the circle to the nearest tenth.

  \[ C = 2\pi r = 2\pi (11) = 22\pi \approx 69.1 \text{ cm} \]
  \[ A = \pi r^2 = \pi (11)^2 = 121\pi \approx 380.1 \text{ cm}^2 \]

**Exercises**

Find the perimeter and area of each figure.

25. $4x - 1$
26. $x + 4$
27. $12$
28. $5x + 7$
29. $21$
30. $14$

Find the circumference and area of each circle to the nearest tenth.

29. $21$
30. $14$

31. The area of a triangle is $102 \text{ m}^2$. The base of the triangle is $17 \text{ m}$. What is the height of the triangle?
### EXAMPLES

**EXAMPLES**

- **EXERCISES**

Use the Distance Formula and the Pythagorean Theorem to find the distance, to the nearest tenth, between each pair of points.

**EXERCISES**

1. Identify each transformation. Then use arrow notation to describe the transformation.

2. Find the coordinates of the image of each point.

3. Use the distances to find the distance, to the nearest tenth, from each point.

4. Identify the transformation. Then use arrow notation to describe the transformation.

5. Determine whether each transformation is a reflection, a translation, a rotation, or a dilation.

6. Use the Distance Formula and the Pythagorean Theorem to find the distance, to the nearest tenth, between each pair of points.

7. Find the sum of the lengths of the sides of each triangle.

8. Find the area of each triangle.

9. Determine whether each transformation is a reflection, a translation, a rotation, or a dilation.

10. Use the distances to find the distance, to the nearest tenth, from each point.

11. Identify the transformation. Then use arrow notation to describe the transformation.

12. Determine whether each transformation is a reflection, a translation, a rotation, or a dilation.

13. Use the distances to find the distance, to the nearest tenth, from each point.

14. Identify the transformation. Then use arrow notation to describe the transformation.

15. Determine whether each transformation is a reflection, a translation, a rotation, or a dilation.

16. Use the distances to find the distance, to the nearest tenth, from each point.

17. Identify the transformation. Then use arrow notation to describe the transformation.

18. Determine whether each transformation is a reflection, a translation, a rotation, or a dilation.

19. Use the distances to find the distance, to the nearest tenth, from each point.

20. Identify the transformation. Then use arrow notation to describe the transformation.

21. Determine whether each transformation is a reflection, a translation, a rotation, or a dilation.

22. Use the distances to find the distance, to the nearest tenth, from each point.

23. Identify the transformation. Then use arrow notation to describe the transformation.

24. Determine whether each transformation is a reflection, a translation, a rotation, or a dilation.

25. Use the distances to find the distance, to the nearest tenth, from each point.

26. Identify the transformation. Then use arrow notation to describe the transformation.

27. Determine whether each transformation is a reflection, a translation, a rotation, or a dilation.

28. Use the distances to find the distance, to the nearest tenth, from each point.

29. Identify the transformation. Then use arrow notation to describe the transformation.

30. Determine whether each transformation is a reflection, a translation, a rotation, or a dilation.

31. Use the distances to find the distance, to the nearest tenth, from each point.

32. Identify the transformation. Then use arrow notation to describe the transformation.

33. Determine whether each transformation is a reflection, a translation, a rotation, or a dilation.

34. Use the distances to find the distance, to the nearest tenth, from each point.

35. Identify the transformation. Then use arrow notation to describe the transformation.

36. Determine whether each transformation is a reflection, a translation, a rotation, or a dilation.

37. Use the distances to find the distance, to the nearest tenth, from each point.

38. Identify the transformation. Then use arrow notation to describe the transformation.

39. Determine whether each transformation is a reflection, a translation, a rotation, or a dilation.

40. Use the distances to find the distance, to the nearest tenth, from each point.

41. Identify the transformation. Then use arrow notation to describe the transformation.

42. Determine whether each transformation is a reflection, a translation, a rotation, or a dilation.

43. Use the distances to find the distance, to the nearest tenth, from each point.
1. Draw and label plane \( \mathcal{N} \) containing two lines that intersect at \( B \).

2. Possible answer: \( D, E, C, A \)

Use the figure to name each of the following.

3. Possible Answer: \( \overrightarrow{BE} \)

4. Find \( AB \).

5. \( E, F, \) and \( G \) represent mile markers along a straight highway. Find \( EF \).

6. \( J \) is the midpoint of \( HK \). Find \( HI, JK, \) and \( HK \).

Classify each angle by its measure.

7. \( \angle LMP = 70^\circ \) acute

8. \( \angle QMN = 90^\circ \) right

9. \( \angle PMN = 125^\circ \) obtuse

10. \( \overline{TV} \) bisects \( \angle RTS \). If the \( \angle RTV = (16x - 6)^\circ \) and \( \angle VTS = (13x + 9)^\circ \), what is the m\( \angle RTV \)?

11. An angle’s measure is 5 degrees less than 3 times the measure of its supplement.

Tell whether the angles are only adjacent, adjacent and form a linear pair, or not adjacent.

12. \( \angle 2 \) and \( \angle 3 \) only adj.

13. \( \angle 4 \) and \( \angle 5 \) adj. and form a lin. pair

14. \( \angle 1 \) and \( \angle 4 \) not adj.

15. Find the perimeter and area of a rectangle with \( b = 8 \) ft and \( h = 4 \) ft.

Find the circumference and area of each circle to the nearest tenth.

16. \( r = 15 \) m

17. \( d = 25 \) ft

18. \( d = 2.8 \) cm

19. Find the midpoint of the segment with endpoints \((-4, 6)\) and \((3, 2)\).

20. \( M \) is the midpoint of \( \overline{LN} \). \( M \) has coordinates \((-5, 1)\), and \( L \) has coordinates \((2, 4)\).

Find the coordinates of \( N \).

21. Given \( A(-5, 1), B(-1, 3), C(1, 4), \) and \( D(4, 1)\), is \( \overline{AB} \equiv \overline{CD} \)? Explain. No; \( AB \approx 4.5; CD \approx 4.2 \)

Identify each transformation. Then use arrow notation to describe the transformation.

22. \( \overrightarrow{QR} \rightarrow \overrightarrow{Q'R'R'} \) \( 180^\circ \) rotation

23. \( \overrightarrow{WZ} \rightarrow \overrightarrow{W'Y'Z'} \) reflection

24. A designer used the translation \( (x, y) \rightarrow (x + 3, y - 3) \) to transform a triangular-shaped pin \( \triangle ABC \). Find the coordinates and draw the image of \( \triangle ABC \).

\( A'(-2, -2); B'(1, 1); C'(2, -2) \)
FOCUS ON SAT

The SAT has three sections: Math, Critical Reading, and Writing. Your SAT scores show how you compare with other students. It can be used by colleges to determine admission and to award merit-based financial aid.

You may want to time yourself as you take this practice test. It should take you about 6 minutes to complete.

On SAT multiple-choice questions, you receive one point for each correct answer, but you lose a fraction of a point for each incorrect response. Guess only when you can eliminate at least one of the answer choices.

College Entrance Exam Practice

1. Points D, E, F, and G are on a line, in that order. If DE = 2, FG = 5, and DF = 6, what is the value of EG (DG)?
   (A) 13
   (B) 18
   (C) 19
   (D) 42
   (E) 99

2. QS bisects ∠PQR, m∠PQR = (4x + 2)°, and m∠SQR = (3x - 6)°. What is the value of x?
   (A) 1
   (B) 4
   (C) 7
   (D) 10
   (E) 19

3. A rectangular garden is enclosed by a brick border. The total length of bricks used to enclose the garden is 42 meters. If the length of the garden is twice the width, what is the area of the garden?
   (A) 7 meters
   (B) 14 meters
   (C) 42 meters
   (D) 42 square meters
   (E) 98 square meters

4. What is the area of the square?
   (A) 16
   (B) 25
   (C) 32
   (D) 36
   (E) 41

5. If ∠BFD and ∠AFC are right angles and m∠CFD = 72°, what is the value of x?
   (A) 18
   (B) 36
   (C) 72
   (D) 90
   (E) 108

Note: Figure not drawn to scale.

Test Prep Doctor

1. Students may choose B because they have found the incorrect value of EF, possibly by subtracting 2 from 5. Remind students to sketch a figure of the situation.
2. Students may choose A because they did not do the opposite operation. Also remind students of the definition of bisect and the relationship of bisected angles.
3. Students may choose C or D because they calculated the perimeter of the garden. Remind students to read each test item carefully to determine what question is being asked.
4. Students may choose A or D because they incorrectly calculated the length of a side of the square. Remind students of the Distance Formula.
5. Students may choose E because they assumed the angles were complementary. Remind students that they cannot assume information that is not given in the test item.
Chapter 1 Foundations for Geometry

Multiple Choice: Work Backward

When you do not know how to solve a multiple-choice test item, use the answer choices and work the question backward. Plug in the answer choices to see which choice makes the question true.

**Example 1**

\(T\) is the midpoint of \(RC\), \(RT = 12x - 8\), and \(TC = 28\). What is the value of \(x\)?

\[
\begin{array}{c|c|c}
\text{A} & -4 & \text{D} & 28 \\
\text{B} & 2 & \text{C} & 3 \\
\end{array}
\]

Since \(T\) is the midpoint of \(RC\), then \(RT = RC\), or \(12x - 8 = 28\).

Try choice A: If \(x = -4\), then \(12x - 8 = 12(-4) - 8 = -56\).

This choice is not correct because length is always a positive number.

Try choice B: If \(x = 2\), then \(12x - 8 = 12(2) - 8 = 16\).

Since \(16 \neq 28\), choice B is not the answer.

Try choice C: If \(x = 3\), then \(12x - 8 = 12(3) - 8 = 28\).

Since \(28 = 28\), the correct answer is C, 3.

**Example 2**

Joel used 6400 feet of fencing to make a rectangular horse pen. The width of the pen is 4 times as long as the length. What is the length of the horse pen?

\[
\begin{array}{c|c|c}
\text{F} & 25 \text{ feet} & \text{D} & 640 \text{ feet} \\
\text{G} & 480 \text{ feet} & \text{J} & 1600 \text{ feet} \\
\end{array}
\]

Use the formula \(P = 2l + 2w\). \(P = 6400\) and \(w = 4l\). You can work backward to determine which answer choice is the most reasonable.

Try choice J: Use mental math. If \(l = 1600\), then \(4l = 6400\). This choice is not reasonable because the perimeter of the pen would then be far greater than 6400 feet.

Try choice F: Use mental math. If \(l = 25\), then \(4l = 100\). This choice is incorrect because the perimeter of the pen is 6400 ft, which is far greater than \(2(25) + 2(100)\).

Try choice H: If \(l = 640\), then \(4l = 2560\). When you substitute these values into the perimeter formula, it makes a true statement.

The correct answer is H, 640 ft.
Read each test item and answer the questions that follow.

**Item A**
The measure of an angle is 3 times as great as that of its complement. Which value is the measure of the smaller angle?

- **A.** 22.5°
- **B.** 27.5°
- **C.** 63.5°
- **D.** 67.5°

1. Are there any definitions that you can use to solve this problem? If so, what are they?
2. Describe how to work backward to find the correct answer.

**Item B**
In a town's annual relay marathon race, the second runner of each team starts at mile marker 4 and runs to the halfway point of the 26-mile marathon. At that point the second runner passes the relay baton to the third runner of the team. How many total miles does the second runner of each team run?

- **C.** 4 miles
- **D.** 6.5 miles
- **E.** 9 miles
- **F.** 13 miles

3. Which answer choice should you plug in first? Why?
4. Describe, by working backward, how you know that choices F and G are not correct.

**Item C**
Consider the translation \((-2, 8) \rightarrow (8, -4)\). What number was added to the x-coordinate?

- **A.** -12
- **B.** -6
- **C.** 4
- **D.** 10

5. Which answer choice should you plug in first? Why?
6. Explain how to work the test question backward to determine the correct answer.

**Item D**
\(\triangle QRS\) has vertices at \(Q(3, 5), R(3, 9),\) and \(S(7, 5)\). Which of these points is a vertex of the image of \(\triangle QRS\) after the translation \((x, y) \rightarrow (x - 7, y - 6)\)?

- **A.** (-4, 3)
- **B.** (4, 1)
- **C.** (0, 0)
- **D.** (4, -3)

7. Explain how to use mental math to find an answer that is NOT reasonable.
8. Describe, by working backward, how you can determine the correct answer.

**Item E**
\(TS\) bisects \(\angle P T R\). If \(m\angle P T S = (9x + 2)^\circ\) and \(m\angle S T R = (x + 18)^\circ\), what is the value of \(x\)?

- **A.** -10
- **B.** 0
- **C.** 2
- **D.** 20

9. Explain how to use mental math to find an answer that is NOT reasonable.
10. Describe how to use the answer choices to work backward to find which answer is reasonable.

**Answers to Test Items**

A. A
B. H
C. D
D. F
E. C

**Answers**

Possible answers:

9. If you substitute \(-10\) into \((9x + 2)\), the result is neg. Since an \(\angle\) measure cannot be a neg. value, choice A is not reasonable.

10. Substitute the value of \(x\) given in choice B into both \(\angle\) measures, and simplify. If the resulting values are equivalent, then that value of \(x\) is the correct answer. Repeat this process with choices C and D, or until the correct answer is found.
CUMULATIVE ASSESSMENT, CHAPTER 1

Multiple Choice

Use the diagram for Items 1–3.

1. Which points are collinear?
   - A, B, and C
   - B, C, and D
   - C, D, and E
   - A, B, and E

2. What is another name for plane \( \overline{CD} \)?
   - Plane \( \overline{CD} \)
   - Plane \( \overline{AB} \)
   - Plane \( \overline{BD} \)
   - Plane \( \overline{AC} \)

3. Use your protractor to find the approximate measure of \( \angle ABD \).
   - 123°
   - 117°
   - 53°

4. \( S \) is between \( R \) and \( T \). The distance between \( R \) and \( T \) is 4 times the distance between \( S \) and \( T \). If \( RS = 18 \), what is \( RT \)?
   - 24
   - 14.4
   - 22.5

5. A ray bisects a straight angle into two congruent angles. Which term describes each of the congruent angles that are formed?
   - Acute
   - Obtuse
   - Right
   - Straight

6. Which expression states that \( AB \) is congruent to \( CD \)?
   - \( AB = CD \)
   - \( AB \neq CD \)
   - \( AB = CD \)

7. The measure of an angle is 35°. What is the measure of its complement?
   - 35°
   - 45°
   - 55°
   - 145°

8. Which of these angles is adjacent to \( \angle MQN \)?
   - \( \angle QMN \)
   - \( \angle QNP \)
   - \( \angle NPQ \)
   - \( \angle PQN \)

9. What is the area of \( \triangle NQP \)?
   - 3.7 square meters
   - 7.4 square meters
   - 6.8 square meters
   - 13.6 square meters

10. Which of the following pairs of angles are complementary?
    - \( \angle MNQ \) and \( \angle QNP \)
    - \( \angle NQP \) and \( \angle QPN \)
    - \( \angle MNP \) and \( \angle QNP \)
    - \( \angle QMN \) and \( \angle NPQ \)

11. \( K \) is the midpoint of \( \overline{JK} \). \( J \) has coordinates \( (2, -1) \), and \( K \) has coordinates \( (-4, 3) \).
    What are the coordinates of \( L \)?
    - \( (3, -2) \)
    - \( (-1, 1) \)
    - \( (-1, -1) \)
    - \( (-10, 7) \)

12. A circle with a diameter of 10 inches has a circumference equal to the perimeter of a square.
    To the nearest tenth, what is the length of each side of the square?
    - 2.5 inches
    - 3.9 inches
    - 7.9 inches

13. The map coordinates of a campground are \( (1, 4) \), and the coordinates of a fishing pier are \( (4, 7) \). Each unit on the map represents 1 kilometer. If Alejandro walks in a straight line from the campground to the pier, how many kilometers, to the nearest tenth, will he walk?
    - 3.5 kilometers
    - 6.0 kilometers
    - 7.9 kilometers
    - 12.1 kilometers

For Item 11, encourage students to draw a diagram. Students who do not draw a diagram may be more likely to choose answer \( A \), which gives the coordinates of the midpoint of \( \overline{JK} \), not the coordinates of \( L \).
14. m\angle R is 57°. What is the measure of its supplement?
   \begin{align*}
   \text{F} & \quad 33° \\
   \text{G} & \quad 43° \\
   \text{H} & \quad 123° \\
   \text{I} & \quad 133°
   \end{align*}

15. What rule would you use to translate a triangle 4 units to the right?
   \begin{align*}
   \text{A} & \quad (x, y) \rightarrow (x + 4, y) \\
   \text{B} & \quad (x, y) \rightarrow (x - 4, y) \\
   \text{C} & \quad (x, y) \rightarrow (x, y + 4) \\
   \text{D} & \quad (x, y) \rightarrow (x, y - 4)
   \end{align*}

16. If \( WZ \) bisects \( \angle XWY \), which of the following statements is true?
   \begin{align*}
   \text{A} & \quad m\angle XWZ > m\angle YWZ \\
   \text{B} & \quad m\angle XWZ < m\angle YWZ \\
   \text{C} & \quad m\angle XWZ = m\angle YWZ \\
   \text{D} & \quad m\angle XWZ \approx m\angle YWZ
   \end{align*}

17. The \( x \)- and \( y \)-axes separate the coordinate plane into four regions, called quadrants. If \( (c, d) \) is a point that is not on the axes, such that \( c < 0 \) and \( d < 0 \), which quadrant would contain point \( (c, d) \)?
   \begin{align*}
   \text{A} & \quad I \\
   \text{B} & \quad II \\
   \text{C} & \quad III \\
   \text{D} & \quad IV
   \end{align*}

**Gridded Response**

18. The measure of \( \angle 1 \) is 4 times the measure of its supplement. What is the measure, in degrees, of \( \angle 1 ? \) \( 144 \)

19. The exits for Market St. and Finch St. are 3.5 miles apart on a straight highway. The exit for King St. is at the midpoint between these two exits. How many miles apart are the King St. and Finch St. exits? \( 1.75 \)

20. \( R \) has coordinates \((-4, 9)\). \( S \) has coordinates \((4, -6)\). What is \( RS ? \) \( 17 \)

21. If \( \triangle A \) is a supplement of \( \angle B \) and is a right angle, then what is \( m\angle B \) in degrees? \( 90 \)

22. \( \angle C \) and \( \angle D \) are complementary. \( m\angle C \) is 4 times \( m\angle D \). What is \( m\angle C ? \) \( 72 \)

**Answers**

23a. 

24. Possible answer: \( \angle 1 \) is supp. to \( \angle 4 \), so \( m\angle 1 = 180° - m\angle 4 \). You continue to subtract from 180° to find the measure of each \( \angle \). 

25. \( 4\pi \text{yd}^2 \); possible answer: The largest circular piece can have a diam. no larger than the width of the fabric. The width of the fabric is 4 yd. If the diam. of the circular piece is 4 yd, then its radius is 2 yd and its area is \( \pi(2^2) = 4\pi \text{yd}^2 \).

26a. 

26b. Possible answer: She could have reflected rect. \( PQRS \) across the \( y \)-axis and then translated it 6 units down.

26c. No; possible answer: a figure and its reflected image should be the same size and shape. Rect. \( PQRS \) has a length of 2 units and a width of 1 unit. The image Demara drew is a square with a side length of 2 units.

Because the 2 figures have different shapes, Demara did not perform the reflection correctly.