Chapter 6 – Discrete random variables & Probability distributions

1. Spell-checking software catches "nonword errors," which result in a string of letters that is not a word, as when "the" is typed as "teh." When undergraduates are asked to write a 250-word essay (without spell-checking), the number X of nonword errors has the following distribution:

<table>
<thead>
<tr>
<th>Value of X</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Probability</td>
<td>0.1</td>
<td>0.2</td>
<td>0.3</td>
<td>0.3</td>
<td>0.1</td>
</tr>
</tbody>
</table>

(a) Write the event "at least one nonword error" in terms of X. What is the probability of this event?

\[ P(X \geq 1) = 0.2 + 0.3 + 0.3 + 0.1 = 0.9 \]

90\% chance of at least one non word error.

(b) Describe the event \( X \leq 2 \) in words. What is its probability? What is the probability that \( X < 2 \)?

\[ P(X \leq 2) = 0.3 + 0.2 + 0.1 = 0.6 \rightarrow 60\% \text{ chance of two or less non word errors} \]

\[ P(X < 2) = 0.2 + 0.1 = 0.3 \rightarrow 30\% \text{ chance of less than two non word errors} \]

2. Choose a person aged 19 to 25 years at random and ask, "In the past seven days, how many times did you go to an exercise or fitness center or work out?" Call the response Y for short. Based on a large sample survey, here is a probability model for the answer you will get:

<table>
<thead>
<tr>
<th>Days: 0 1 2 3 4 5 6 7</th>
</tr>
</thead>
<tbody>
<tr>
<td>Probability: 0.68 0.05 0.07 0.08 0.05 0.04 0.01 0.02</td>
</tr>
</tbody>
</table>

(a) Show that this is a legitimate probability distribution.

\[ \text{All probabilities } = 1 \]

(b) Describe the event \( Y < 7 \) in words. What is \( P(Y < 7) \)?

worked out less than 7 days

\[ P(Y < 7) = 0.98 \]

(c) Express the event "worked out at least once" in terms of Y. What is the probability of this event?

\[ P(Y \geq 1) = 0.32 \]
3. Keno is a favorite game in casinos, and similar games are popular with the states that operate lotteries. Balls numbered 1 to 80 are tumbled in a machine as the bets are placed, then 20 of the balls are chosen at random. Players select numbers by marking a card. The simplest of the many wagers available is "Mark 1 Number." Your payoff is $3 on a $1 bet if the number you select is one of those chosen. Because 20 of 80 numbers are chosen, your probability of winning is 20/80, or 0.25. Let \( X \) = the amount you gain on a single play of the game.

(a) Make a table that shows the probability distribution of \( X \).

\[
\begin{array}{c|c|c}
X & 3 & 0 \\
\hline
P(X) & 0.25 & 0.75 \\
\end{array}
\]

(b) Compute the expected value of \( X \). Explain what this result means for the player.

\[
(3)(0.25) + (0)(0.75) = 0.75 \text{ In the long run the player will gain $0.75 for every dollar bet.}
\]

4. In an experiment on the behavior of young children, each subject is placed in an area with five toys. Past experiments have shown that the probability distribution of the number \( X \) of toys played with by a randomly selected subject is as follows:

<table>
<thead>
<tr>
<th>Number of toys ( x )</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Probability ( p_x )</td>
<td>0.03</td>
<td>0.16</td>
<td>0.30</td>
<td>0.25</td>
<td>0.17</td>
<td>0.11</td>
</tr>
</tbody>
</table>

(a) Write the event "plays with at most two toys" in terms of \( X \). What is the probability of this event?

\( P(X \leq 2) = 0.49 \) \text{ 49\% of children play with at most 2 toys.}

(b) Describe the event \( X > 3 \) in words. What is its probability? What is the probability that \( X \geq 3 \)?

\( P(X > 3) = 0.28 \) \text{ 28\% play with more than 3 toys.}
\( P(X \geq 3) = 0.51 \) \text{ 51\% play with 3 or more toys.}

(c) Calculate the expected mean of the random variable \( X \) and interpret this result in context.

\( \mu_X = 2.06 \) \text{ on average, in the long run, a child will play with 2.06 toys.}

(d) Calculate and interpret the standard deviation of the random variable \( X \). Show your work.

\( \sigma_X = 1.31 \) \text{ Typically, a child will play with 1.31 toys away from the mean.}