Properties of Real Numbers

Warm Up
Directions: Simplify each expression.

1) \[2[5 \div (-6 - 4)]\]

2) \[\frac{5^2 - (5 + 4)}{|4 - 8|}\]

3) \[5 \times 8 - 4 + 16 \div 2^2\]

Commutative Property of Addition and Multiplication

Definition: You can add or multiply numbers in any ________________

- Commutative starts with “CO” which means “C_________ O__________”

- Think of the elements as “commuting” from one location to another.

Examples:
  a) \[4 + 2 + 5 = \]
  b) \[12 \cdot 6 \cdot 8 = \]
  c) \[(4 + 3) + 10 = \]

Associative Property of Addition and Multiplication

Definition: You can add or multiply in different ________________

- Association are ________________

Examples:
  a) \[(3 \cdot 6) \cdot 4 = \]
  b) \[(9 + 5) + 12 = \]
**SWBAT:** identify and apply the commutative, associative, and distributive properties to simplify expressions

**Distributive Property**

- Each element inside the parentheses is __________________ by the element outside the parentheses

  **Examples:**
  
  a) \(4(3 + 6) = \)
  
  b) \(2(7 + 9) = \)
  
  c) \(3(y + 5) = \)

**Identity Property of Addition and Multiplication**

- For both multiplication and addition there is one number that is unique because it doesn’t change another number when operated with. These elements are called **identities**.

  (a) What is the **identity element for addition**? In other words, what number can be added to a given number without changing that given number?

  (b) What is the **identity element for multiplication**? In other words, what number can be multiplied to a given number without changing that given number?

**Inverse Property of Addition and Multiplication**

- Inverse Properties state that when a number is combined with its inverse, it is equal to its identity.

  - \(a\) is said to be additive inverse of \(a\) if \(a + (-a) = 0\).
  
  - \(\frac{1}{a}\) is said to be multiplicative Inverse of \(a\) if \(a \times \frac{1}{a} = 1\)

  **Examples:**
  
  a) \(5 + \_\_\_ = 0\)
  
  b) \(\frac{1}{4} \cdot \_\_\_ = 1\)
**SWBAT:** identify and apply the commutative, associative, and distributive properties to simplify expressions

**Directions:** Name the property that each exercise illustrates.

<table>
<thead>
<tr>
<th>Property</th>
<th>Example</th>
</tr>
</thead>
<tbody>
<tr>
<td>Commutative Property of Addition:</td>
<td>$2 + 3 = 3 + 2$</td>
</tr>
<tr>
<td>Commutative Property of Multiplication:</td>
<td>$2 \times 3 = 3 \times 2$</td>
</tr>
<tr>
<td>Associative Property of Addition:</td>
<td>$(2 + 3) + 4 = 2 + (3 + 4)$</td>
</tr>
<tr>
<td>Associative Property of Multiplication:</td>
<td>$(2 \times 3) \times 4 = 2 \times (3 \times 4)$</td>
</tr>
<tr>
<td>Distributive Property:</td>
<td>$x(a + b) = xa + xb$</td>
</tr>
<tr>
<td>Identity Property of Addition:</td>
<td>$0 + 2 = 2$</td>
</tr>
<tr>
<td>Identity Property of Multiplication:</td>
<td>$1 \times 2 = 2$</td>
</tr>
<tr>
<td>Inverse Property of Addition:</td>
<td>$5 + (-5) = 0$</td>
</tr>
<tr>
<td>Inverse Property of Multiplication:</td>
<td>$2 \times \frac{1}{2} = 1$</td>
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</tbody>
</table>

1) $(-2) + 7 = 7 + (-2)$

2) $3 + 5 + 2 = 5 + 2 + 3$

3) $x = 1x$

4) $xy = yx$

5) $-9 + 9 = 0$

6) $5(10 + 2) = 50 + 10$

7) $x = x + 0$

8) $5 \cdot 3 = 3 \cdot 5$

9) $4(5x - 2) = 20x - 8$

10) $\frac{1}{9} \cdot 9 = 1$

11) $3 + 5 + 7 = 7 + 5 + 3$

12) $7 + (-3) = (-3) + 7$
**Algebra Regents Questions**

1) The statement $2 + 0 = 2$ is an example of the use of which property of real numbers?
   - 1) associative
   - 2) additive identity
   - 3) additive inverse
   - 4) distributive

2) Tori computes the value of $8 \cdot 95$ in her head by thinking $8(100 - 5) = 8 \times 100 - 8 \times 5$. Which number property is she using?
   - 1) associative
   - 2) distributive
   - 3) commutative
   - 4) closure

3) Which property of real numbers is illustrated by the equation $52 + (27 + 36) = (52 + 27) + 36$?
   - 1) commutative property
   - 2) associative property
   - 3) distributive property
   - 4) identity property of addition

4) Which equation is an example of the use of the associative property of addition?
   - 1) $x + 7 = 7 + x$
   - 2) $3(x + y) = 3x + 3y$
   - 3) $(x + y) + 3 = x + (y + 3)$
   - 4) $3 + (x + y) = (x + y) + 3$

5) Which equation illustrates the multiplicative identity element?
   - 1) $x + 0 = x$
   - 2) $x - x = 0$
   - 3) $x \cdot \frac{1}{x} = 1$
   - 4) $x \cdot 1 = x$

6) Which equation illustrates the distributive property?
   - 1) $5(a + b) = 5a + 5b$
   - 2) $a + b = b + a$
   - 3) $a + (b + c) = (a + b) + c$
   - 4) $a + 0 = a$

**Exit Ticket**
Provide an example of each property.
SWBAT: identify and apply the commutative, associative, and distributive properties to simplify expressions

Properties of Operations 1

1. Are the two expressions equal, no matter what values x, y and z might get?

<p>| | | | | |</p>
<table>
<thead>
<tr>
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<tbody>
<tr>
<td>a.</td>
<td>x + y</td>
<td>b.</td>
<td>x - y</td>
<td>c.</td>
</tr>
<tr>
<td>y + x</td>
<td>y - x</td>
<td>xy</td>
<td>x + (y + z)</td>
<td></td>
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<tr>
<td>d.</td>
<td>(x + y) + z</td>
<td>e.</td>
<td>x ÷ y</td>
<td>f.</td>
</tr>
<tr>
<td>y + z</td>
<td>y + x</td>
<td>y + x - x</td>
<td></td>
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<tr>
<td>g.</td>
<td>x (y + z)</td>
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2. For each box above that you answered yes, name the property of the arithmetic that it illustrates.

3. Which properties of the arithmetic let us reason that these expressions are equal?

a. 45y + x = x + 45y - the __________ property of _______________.

b. 2n = n(2) - the __________ property of _______________.

c. (1)w = w - the __________ property of _______________.

d. 5x + 0 = 5x - the __________ property of _______________.

e. 2(s + t) = 2(t + s) - the __________ property of _______________.

f. 2(s + t) = 2s + 2t - the __________ property of _______________.

g. 2(s + t) = (s + t)2 - the __________ property of _______________.

h. \( \frac{1}{4} (12y) = \left( \frac{1}{4} \cdot 12 \right) y \) - the __________ property of _______________.

- the __________ property of _______________.

i. 2(y7) = 2(7y) = (2 \times 7)y - properties of _______________.

SWBAT: identify and apply the commutative, associative, and distributive properties to simplify expressions

Name: Date:

Properties of Operations 2

1. Name the property that justifies each step.
   a. \((5m + 2) + 6m\) = \(5m + (2 + 6m)\)  
      = \(5m + (6m + 2)\)  
      = \((5m + 6m) + 2\)  
      = \((5 + 6)m + 2\)  
      = \(11m + 2\)  
      substitution property

   b. \([(5m)n] \cdot (1/5)\) = \([(m5)n] \cdot (1/5)\)  
      = \([m(5n)] \cdot (1/5)\)  
      = \([m(n5)] \cdot (1/5)\)  
      = \(m[(n5)] \cdot (1/5)\)  
      = \(m[n(5 \cdot 1/5)]\)  
      = \(m[n(1)]\)  
      = \(m[n]\)

2. a. Write down, using variables, the commutative property of addition.
   b. Write down, using variables, the commutative property of multiplication.
   c. Is subtraction commutative? If not, show an example to prove that.
   d. Is division commutative? If not, show an example to prove that.

3. Are the two expressions equal, no matter what values \(x, y\) and \(z\) might get? You can give \(x, y\), and \(z\) some values to test it out.
   a. \(x + y - z\)  
      \(y + x - z\)
   b. \((x + y) - z\)  
      \(x + (y - z)\)
   c. \((x - y) - z\)  
      \(x - (y - z)\)
   d. \(yzx\)  
      \(zxy\)
   e. \((x + y)z\)  
      \((y + x)z\)
   f. \(x + (y + z)\)  
      \((x + y) + z\)

4. Which of the four operations are associative? For those that are not, prove it with an example.