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Solving Exponential & Logarithmic Equations

1. **Core Concept**

Property of Equality for Exponential Equations

Algebra: If *b* is a positive real number other than 1, then $b^x = b^y$ if and only if $x = y$.

Example: If $3^x = 3^5$, than x=5. If x=5 then $3^x = 3^5$.

Solve each equation:

a. $100^x = \left(\frac{1}{10}\right)^{x-3}$

$10^{2x} = 10^{-(x-3)}$

$2x = -x + 3$

$3x = 3$

$x = 1$

b. $2^x = 7$

$\log_2 2^x = \log_2 7$

$x = 2.807$

2. **Core Concept**

Property of Equality for logarithmic Equations

Algebra: If *b, x* and y are positive real numbers with $b \neq 1$, then $\log_b x = \log_b y$.

Example: If $\log_2 x = \log_2 7$, then x=7. If x=7, then $\log_2 x = \log_2 7$.

Solve each equation:

a. $\ln(4x-7) = \ln(x+5)$

$4x - 7 = x + 5$

$3x = 12$

$x = 4$

b. $\log_2(5x-17) = 3$

$2^{\log_2(5x-17)} = 2^3$

$5x - 17 = 2^3$

$5x - 17 = 8$

$5x = 25$

$x = 5$

DA7

Your turn:

3 The temperature of a body at time t after being introduced into a new environment having a constant temperature T_0 is

$A(t) = T_0 + Ce^{-rt}$, where C and r are constants. If C = 100, r = 0.1, and t is time measured in minutes, how long will it take a hot cup of coffee to cool to a temperature of 25°C in a room at 20°C?

Some more problems!

4 Solve the equation in **logarithmic** form

a. $5^x = 18$

b. $2^x = 1560$

c. $4^x = 8$

d. $6^{x+3} = 50$

e. $(1.03)^{\frac{x}{2}-5} = 2$

f. $500(1.02)^{\frac{x}{12}} = 2300$

Condensing

Step 1: Isolate the logarithmic term before you convert the logarithmic equation to an exponential equation. Divide both sides of the original equation by 7:

$$Log(3x) = \frac{15}{7}$$

Step 2: Convert the logarithmic equation to an exponential equation: If no base is indicated, it means the base of the logarithm is 10. Recall also that logarithms are exponents, so the exponent is $\frac{15}{7}$. The equation

$$Log(3x) = \frac{15}{7}$$

can now be written

$$10^{\frac{15}{7}} = 3x$$

Step 3: Divide both sides of the above equation by 3:

$$x = \frac{10^{\frac{15}{7}}}{3} \approx 46.3165164793$$

$x = \frac{10^{\frac{15}{7}}}{3}$ is the exact answer and $x \approx 46.3165164793$ is the approximate answer.

Another example:

Example 3: Solve for x in the equation

$$Ln(x-3) + Ln(x-2) = Ln(2x+24)\,.$$

Solution:

Step 1: Note the first term $Ln(x\text{-}3)$ is valid only when $x>3$; the term $Ln(x\text{-}2)$ is valid only when $x>2$; and the term $Ln(2x+24)$ is valid only when $x>-12$. If we require that x be any real number greater than 3, all three terms will be valid. If all three terms are valid, then the equation is valid.

Your turn!

$\ln(4x - 7) = \ln(x = 11)$

$\log(7x + 3) = \log 38$

$\ln(2x-4) = \ln(x+6)$

$\log(12x - 9) = \log 3x$

$\ln x + \ln(x + 3) = 4$

$\ln x + \ln(x - 2) = 5$

Answer Key

1a. $100^x = \left(\frac{1}{10}\right)^{x-3}$

$10^{2x} = 10^{-1(x-3)}$

$2x = -x + 3$

$3x = 3$

$x = 1$

1b. $2^x = 7$

$\log_2 \cancel{2}^x = \log_2 7$

$x = \log_2 7$

$x = 2.807$

2a. $\cancel{\ln}(4x-7) = \cancel{\ln}(x+5)$

$4x - 7 = x + 5$

$3x = 12$

$x = 4$

2b. $\log_2(5x-17) = 3.$

$\cancel{2}^{\cancel{\log_2}(5x-17)} = 2^3$

$5x - 17 = 8$

$5x = 25$

$x = 5$

3.

$A(t) = T_0 + Ce^{-rt}$

$25 = 20 + 100e^{-(0.1)t}$

$\frac{5}{100} = \frac{100e^{-(0.1)t}}{100}$

$.05 = e^{-0.1t}$

$\ln(.05) = \cancel{\ln e}^{-0.1t}$

$\frac{\ln(.05)}{-0.1} = \frac{-0.1t}{-0.1}$

29.96 minutes $\approx t$

$T_0 = 20$

$A(t) = 25$

$C = 100$

$r = 0.1$

$t = ?$

5a. $4^x = 8$

$2^{2x} = 2^3$

$2x = 3$

$x = \frac{3}{2}$

5b. $49^x = 343$

$7^{2x} = 7^3$

$\frac{2x}{\cancel{2}} = \frac{3}{2}$

$x = \frac{3}{2}$

5c. $3^{4x-2} = 27^{x+2}$

$3^{4x-2} = 3^{3(x+2)}$

$4x - 2 = 3x + 6$

$x = 8$

6a. $140 = 70 + (206-70)e^{-0.283t}$

$70 = 136e^{-0.283t}$

$.515 = e^{-.283t}$

$\ln .515 = \ln e^{-.283t}$

$-.664 = -.283t$

$\boxed{t = 2.3 \text{min}}$

6b. $140 = 86 + (206-86)e^{-.283t}$

$54 = 120e^{-.283t}$

$.45 = e^{-.283t}$

$\ln .45 = \ln e^{-.283t}$

$-.8 = -.283t$

$\boxed{t = 2.8 \text{ min}}$

6c.

$71 = 70 + (206-70)e^{-.283t}$

$1 = 136e^{-.283t}$

$0.007 = e^{-.283t}$

$\boxed{t = 17.5 \text{ min}}$

$\ln .007 = \ln e^{-.283t}$

$-4.961 = -.283t$

$\frac{}{-.283} \quad \frac{}{-.283}$

$17.5 = t$