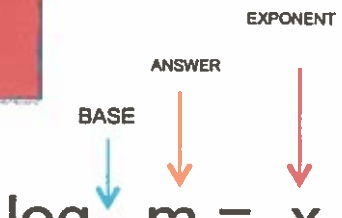
Lesson 6.5 Properties of Logarithms

Remember the structure of logarithmic equations



$b^x = m$

Core Concept

Properties of Logarithms

Let b, m, and n be positive real numbers with $b \neq 1$.

Product Property $\log_b mn = \log_b m + \log_b n$

Quotient Property $\log_b \frac{m}{n} = \log_b m - \log_b n$

Power Property $\log_b m^n = n \log_b m$

Product Property Example 1:

$\log_2 3 \approx 1.585$ $\log_2 7 \approx 2.807$

Find: $\log_2 21$

$\log_2 21 = \log_2 (3 \times 7)$ Write 21 as 3 x 7

$= \log_2 3 + \log_2 4$ Product Property

$= 1.585 + 2.807$ Use values of $\log_2 3$ and $\log_2 7$

$= 4.392$ Add

Quotient Property Example 2:

$\log_2 3 \approx 1.585$ $\log_2 7 \approx 2.807$

Find: $\log_2 \frac{3}{7}$

$\log_2 \frac{3}{7} = \log_2 3 - \log_2 7$ Quotient Property

$= 1.585 - 2.807$ Substitute values

$= -1.222$ Subtract

Power Property Example 3:

$\log_2 3 \approx 1.585$ $\log_2 7 \approx 2.807$

Find $\log_2 49 = \log_2 7^2$ Rewrite 49 as 7

$= 2 \log_2 7$ Power Property

$= 2(2.807)$ Substitute value

$= 5.614$ Multiply

END OF PROPERTIES OF LOGARITHMS

Rewriting Logarithmic Expressions

Expanding Logarithmic Expressions

EXPAND: $\ln \frac{5x^7}{y}$

$\ln \frac{5x^7}{y} = \ln 5x^7 - \ln y$ Quotient Property

$= \ln 5 + \ln x^7 - \ln y$ Product Property

$= \ln 5 + 7\ln x - \ln y$ Power Property

$\ln 5 + 7\ln x - \ln y$

Condensing Logarithmic Expressions

CONDENSE: $\log 9 + 3\log 2 - \log 3$

$\log 9 + 3\log 2 - \log 3 = \log 9 + \log 2^3 - \log 3$ → Power Property

$= \log(9 \cdot 2^3) - \log 3$ → Product Property

$= \log \frac{9 \cdot 2^3}{3}$ → Quotient Property

$= \log 24$ → Simplify

$\log 24$

Change-of-Base Formula

- If a, b, and c are positive real numbers with $b \neq 1$ and $c \neq 1$, then

$$\log_c a = \frac{\log_b a}{\log_b c}$$

In particular:

$\log_c a = \frac{\log a}{\log c}$ and

$\log_c a = \frac{\ln a}{\ln c}$

(used base of 10)

Example: $\log_3 8 = \frac{\log 8}{\log 3}$

$= \frac{.9031}{.4771}$

$= 1.893$

PRACTICE PROBLEMS

In Exercises 3–8, use $\log_7 4 \approx 0.712$ and $\log_7 12 \approx 1.277$ to evaluate the logarithm. *(See Example 1.)*

3. $\log_7 3$

4. $\log_7 48$

In Exercises 13–20, expand the logarithmic expression. *(See Example 2.)*

13. $\log_3 4x$

14. $\log_8 3x$

In Exercises 23–30, condense the logarithmic expression. *(See Example 3.)*

23. $\log_4 7 - \log_4 10$

24. $\ln 12 - \ln 4$

ANSWER KEY

3) $\log_7 3 = \log_7 \frac{12}{4}$

$= \log_7 12 - \log_7 4$

$= 1.277 - 0.712$

$= \boxed{0.565}$

4) $\log_7 48 = \log_7 (12 \cdot 4)$

$= \log_7 12 + \log_7 4$

$= 1.277 + 0.712$

$= \boxed{1.989}$

13) $\log_3 4x = \log_3 (4 \cdot x)$

$= \boxed{\log_3 4 + \log_3 x}$

14) $\log_8 3x = \log_8 (3 \cdot x)$

$= \boxed{\log_8 3 + \log_8 x}$

23) $\log_4 7 - \log_4 10 = \log_4 \frac{7}{10}$

$\boxed{\log_4 \frac{7}{10}}$

24) $\ln 12 - \ln 4 = \ln \frac{12}{4}$

$= \boxed{\ln 3}$

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