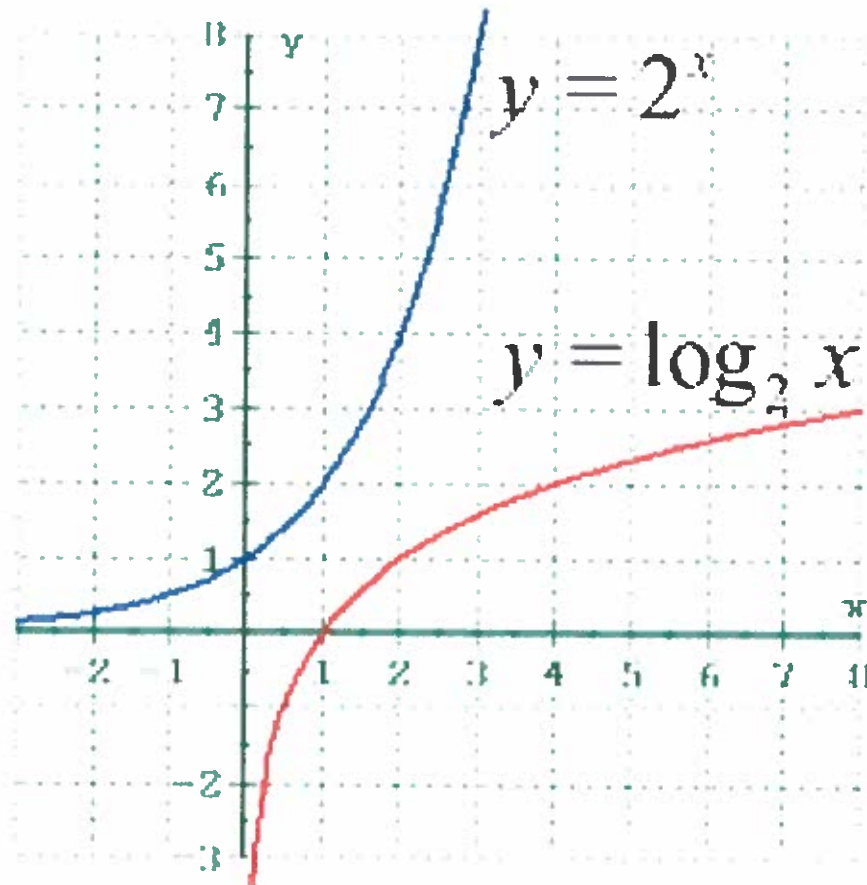


# Logarithms and Logarithmic Functions





## Logarithms ( $\log_b y = x$ )

Logarithms are the inverse of exponents.

Convert  $4=2^2$

- use "log bae"
- b is the log base ex.  $\log_b \rightarrow \log_2$
- a is the answer  $\log_2(4)$
- e is the exponent  $\log_2(4)=2$

You can also use "log bae" to work backwards

Ex.  $\log_2(16)=4$

b: 2

a: 16

e: 4

$16=2^4$

$$2^3 = 8$$
$$\log_2(8) = 3$$

## Common and natural log

- **Natural** logarithms are different than **common** logarithms. While the base of a **common logarithm** is 10, the base of a **natural logarithm** is the special number e. Although this looks like a variable, it represents a fixed irrational number approximately equal to 2.718281828459.
- common log is expressed as  $\log_{10}$  or  $\log$
- Natural log is expressed as  $\ln$  or  $\log_e$

Use calculator to evaluate

1.  $\log_e$
2.  $\ln_{1/3}$
3.  $\ln_{12}$
4.  $\log_{27}$

Answers

1. .77815
2. -1.099
3. 2.485
4. -.5441

## Inverse Properties

- since logarithms are an inverse of exponential functions you can switch them back and forth, and cancel each other out.

Ex.

$$4=10^{\log 4}$$

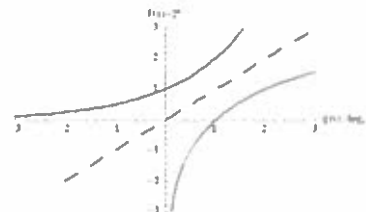
$$\log_5 25 = \log_5 (5)^{2x}$$

2x



## Characteristics of Logarithmic Functions

- passes through (1,0)
- Domain:  $x > 0$  Range: all real numbers
- an asymptote at the y-axis
- Equation:  $y = \log_b x$
- inverse of exponential function  $y = b^x$



## Graphing Log Functions

Graph:  $f(x) = \log_3 x$

Step 1: find inverse of  $f(x)$  which is  $g(x) = 3^x$

Step 2: make a table

x	-2	-1	0	1	2
g(x)	1/9	1/3	1	3	9

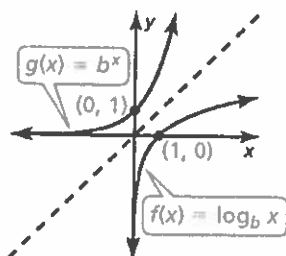
Step 3: plot points for  $g(x)$  and connect points with a smooth curve

Step 4: because  $f(x)$  and  $g(x)$  are inverse functions you can graph  $g(x)$  by reflecting  $f(x)$  over  $y = x$ . Therefore switching the coordinates of  $f(x)$  to make  $g(x)$

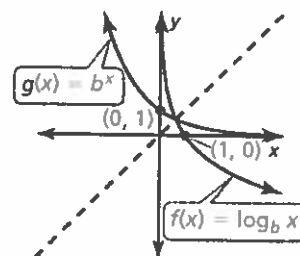
Log functions for when  $b > 1$

Log functions for when  $0 < b < 1$

Graph of  $f(x) = \log_b x$  for  $b > 1$



Graph of  $f(x) = \log_b x$  for  $0 < b < 1$



## Parent Graphs for Logarithmic Functions

The graph of  $f(x) = \log_b x$  is shown below for  $b > 1$  and for  $0 < b < 1$ . Because  $f(x) = \log_b x$  and  $g(x) = b^x$  are inverse functions, the graph of  $f(x) = \log_b x$  is the reflection of the graph of  $g(x) = b^x$  in the line  $y = x$ .

Note that the y-axis is a vertical asymptote of the graph of  $f(x) = \log_b x$ . The domain of  $f(x) = \log_b x$  is  $x > 0$ , and the range is all real numbers.



## Practice Problems

Rewrite equation in exponential form

1.  $\log_3 9 = 2$

2.  $\log_6 1 = 0$

3.  $\log_4 4 = 1$

4.  $\log_7 343 = 3$

Rewrite equation in log form

5.  $6^2 = 36$

6.  $12^0 = 1$

7.  $16^{-1} = 1/16$

8.  $5^{-2} = 1/25$

evaluate log

9.  $\log_3 81$

10.  $\log_3 3$

11.  $\log_7 49$

12.  $\log_{1/2} 1$

Find the inverse

13.  $.3^x$

14.  $11^x$

15.  $\log_2 x$

16.  $\log_{1/5} x$

Graph

17.  $\log_4 x$

18.  $\log_6 x$





## Answer Key

1.  $3^2 = 9$

2.  $6^0 = 1$

3.  $4^1 = 4$

4.  $7^3 = 343$

5.  $\log_6 36 = 2$

6.  $\log_2 2 = 0$

7.  $\log_{16} \frac{1}{16} = -1$

8.  $\log_5 \frac{1}{25} = -2$

9. 4

10. 1

11. 2

12. 6

13.  $\log_3 X = 4$

14.  $\log_4 X = Y$

15.  $4 = 2^x$

16.  $4 = \frac{1}{5}^x$

