

Logarithms and Logarithmic Functions

By Devin Hudayar

What is a Logarithm? - A logarithm is a function that is the inverse on an exponential function, or the "reverse" action of it. The inverse of the function $y=b^x$ is called $y=\log_b x$. For example, $y=\log_3 x$ is the inverse of $y=3^x$. You can use these functions to find the inverse of exponential equations, and it is most clearly shown when using a table to sketch a graph. Like here,

x	-2	-1	0	1	2
$Y=2^x$	1/4	1/2	1	2	4
x	1/4	1/2	1	2	4
$Y=\log_2 x$	-2	-1	0	1	2

**Graph for this is in loose leaf on the back!!

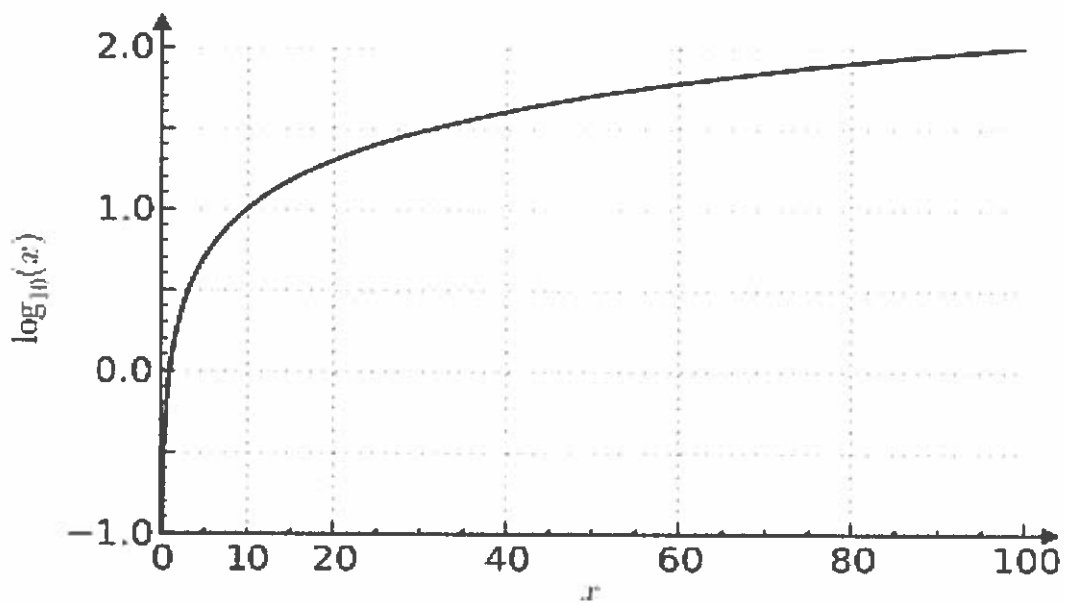
Also, if you were trying to find an equation for the inverse algebraically, here is what the steps would be,

1- Switch the X and the Y. $y=2^x \rightarrow x=2^y$

2- Solve for Y. $x=2^y \rightarrow y=\log_2 x$

Yes, its that simple.

!!!- Finding the logarithm of a non-positive number is not possible in the real number system, so you are able to find the domain of many different types of logarithmic functions.



Determine the domain of the function $y = \log_2(3x-2)$ in set builder notation.

$$3x-2 > 0$$

$$+2 \quad +2$$

$$3x > 2$$

$$/3 \quad /3$$

$$x > 2/3 \rightarrow \blacksquare$$

"Logarithm" means "power", so when you see "log", it means "the power I raise". Ex. - $\log_2 8 = 3$, so $\log = 3$

$$\text{Ex. - } x = \log_{10} 1000 \text{ so } x = 3$$

!!! - $y = \log_b x$ only if $x = b^y$

!!! - $base^{exponent} = answer \longleftrightarrow \log_{base} answer = exponent$

Practice #1: Write each equivalent exponential equation.

1. $\log_2 8 =$ 2. $\log_4 16 =$ 3. $\log_5 625 =$ 4. $\log_{10} 100,000 =$

Lastly, a logarithm with a base of 10 is called a common logarithm. If no base is indicated, then the base 10 is implied.

-Base with no exponent--> x means... x^1

-Radical without an index--> $\sqrt{25}$ means... Square Root

-Logarithms with a base--> $\log_7 x = 2$ means... $7^2 = x$

-Logarithms without a base--> $\log 1000$ means... 3

The Natural Log

First you will be introduced to "e". "e" is like " π " in that it represents an irrational number. It is about equal to 2.72. It is used in exponential modeling. e's inverse is called "ln". ln is equal to \log_e . The natural logarithm gives the power that we must raise e to in order to get the input.

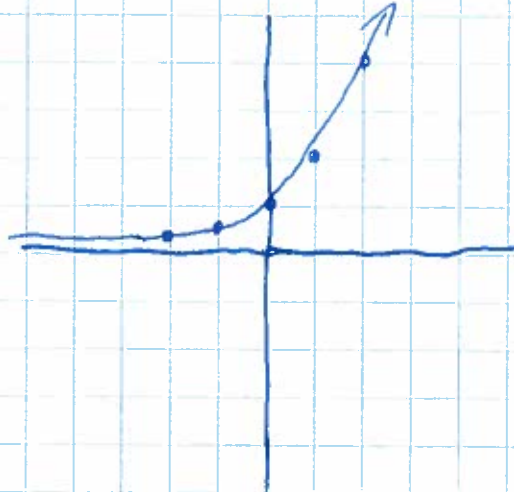
Practice #2: Without use of a calculator, find the values of the following...

1. $\ln(e) =$

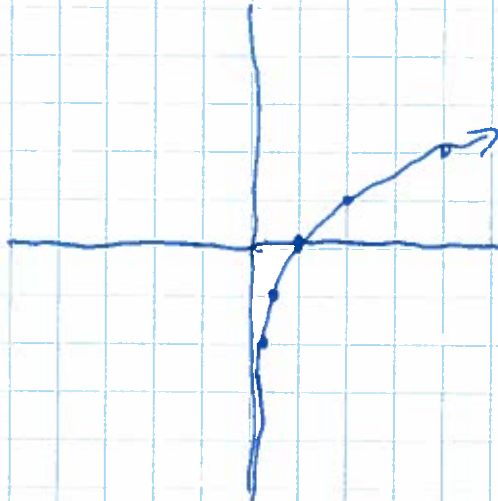
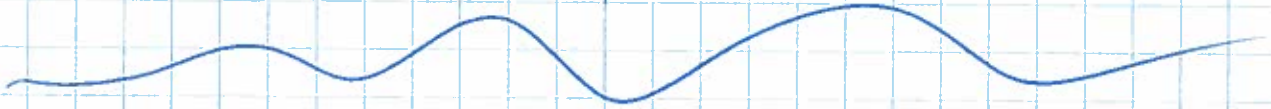
2. $\ln(1) =$

3. $\ln(e^5) =$

Notes



$$y = 2^x$$



$$y = \log_2 x$$

1. Which of the following is equivalent to $y = \log_3 x$?
a. $Y = x^3$ b. $x = 3^y$ c. $x = y^y$ d. $y = x^{1/3}$

2. The value of $\log_6 145$ is closest to which of the following values?
a. 2.78 b. 3.95 c. 8.0 d. 2.64

3. Determine the value for each of the following logarithms.
a. $\log_2 64$ b. $\log_4 64$ c. $\log_3 6561$ d. $\log_5 25$ e. $\log_6 216$

4. The domain of $y = \log(x+5)$ in the real numbers is?
a. $\{x | x > 1+5\}$ b. $\{x | x > -5\}$ c. $\{x > 5\}$ d. $\{x | x < -5\}$

5. Which of the following is true about the function $y = \log_4(x+16) - 1$?
a. It has an x-intercept of -16 and a y-intercept of 1 .
b. It has an x-intercept of -16 and a y-intercept of -1 .
c. It has an x-intercept of 4 and a y-intercept of -1 .
d. It has an x-intercept of -12 and a y-intercept of 1 .

6. Which of the following represents the y-intercept of the function $y = \log_2(32-x) - 2$?
a. 3 b. 4 c. 8 d. 16

7. Which of the following is equivalent to $\ln(x^3/e^2)$?
a. $\ln x + 6$ b. $3 \ln x - 6$ c. $3 \ln x - 2$ d. $\ln x - 9$

1. Which of the following is equivalent to $y = \log_3 x$?
 a. $Y = x^3$ b. $x = 3^y$ c. $x = y^3$ d. $y = x^{1/3}$

$y = \log_b x \rightarrow$ only if $x = b^y$ $b=3$

2. The value of $\log_6 145$ is closest to which of the following values?
 a. 2.78 b. 3.95 c. 8.0 d. 2.64

$\log_6 145 = 2.777568 \approx 2.78$

3. Determine the value for each of the following logarithms.
 a. $\log_2 64$ b. $\log_4 64$ c. $\log_3 6561$ d. $\log_5 25$ e. $\log_6 216$

2^6 4^3 3^8 5^2 6^3

4. The domain of $y = \log(x+5)$ in the real numbers is?
 a. $\{x|x>1+5\}$ b. $\{x|x>-5\}$ c. $\{x>5\}$ d. $\{x|x<-5\}$

$x+5 > 0$
 $-5 \rightarrow -5$
 $x > -5 \rightarrow \{x|x > -5\}$

5. Which of the following is true about the function $y = \log_4(x+16) - 1$?
 a. It has an x-intercept of -16 and a y-intercept of 1.
 b. It has an x-intercept of -16 and a y-intercept of -1.
 c. It has an x-intercept of 4 and a y-intercept of -1.
 d. It has an x-intercept of -12 and a y-intercept of 1.

\leftarrow y-int 1
 $\frac{+16}{+4}$
 $-12 \leftarrow$ x-int.
 Calc. is recommended for this problem.

6. Which of the following represents the y-intercept of the function $y = \log_2(32-x) - 2$?
 a. 3 b. 4 c. 8 d. 16

Calc. is recommended for this problem.

7. Which of the following is equivalent to $\ln(x^3/e^2)$?
 a. $\ln x + 6$ b. $3 \ln x - 6$ c. $3 \ln x - 2$ d. $\ln x - 9$