Kayla Lambert-Jones

4.7 Transformations of polynomial functions

Describing Transformations of Polynomial Functions

You can transform graphs of polynomial functions in the same way you transformed graphs of linear functions, absolute value functions, and quadratic functions. Examples of transformations of the graph of $f(x) = x^4$ are shown below.

Core Concept

Transformation	f(x) Notation	Examples	
Horizontal Translation Graph shifts left or right.	$f(x-h)$	$g(x) = (x-5)^4$	5 units right
		$g(x) = (x+2)^4$	2 units left
Vertical Translation Graph shifts up or down.	$f(x)+k$	$g(x) = x^4 + 1$	1 unit up
		$g(x) = x^4 - 4$	4 units down
Reflection Graph flips over x- or y-axis.	$f(-x)$	$g(x) = (-x)^4 = x^4$	over y-axis
	$-f(x)$	$g(x) = -x^4$	over x-axis
Horizontal Stretch or Shrink Graph stretches away from or shrinks toward y-axis.	$f(ax)$	$g(x) = (2x)^4$	shrink by a factor of $\frac{1}{2}$
		$g(x) = \left(\frac{1}{2}x\right)^4$	stretch by a factor of 2
Vertical Stretch or Shrink Graph stretches away from or shrinks toward x-axis.	$a \cdot f(x)$	$g(x) = 8x^4$	stretch by a factor of 8
		$g(x) = \frac{1}{4}x^4$	shrink by a factor of $\frac{1}{4}$

EXAMPLE 1 Translating a Polynomial Function

Describe the transformation of $f(x) = x^3$ represented by $g(x) = (x+5)^3 + 2$. Then graph each function.

EXAMPLE 2 Transforming Polynomial Functions

Describe the transformation of f represented by g. Then graph each function.

a. $f(x) = x^4$, $g(x) = -\frac{1}{4}x^4$

b. $f(x) = x^5$, $g(x) = (2x)^5 - 3$

EXAMPLE 3 Writing Transformed Polynomial Functions

Let $f(x) = x^3 + x^2 + 1$. Write a rule for g and then graph each function. Describe the graph of g as a transformation of the graph of f.

a. $g(x) = f(-x)$

b. $g(x) = 3f(x)$

EXAMPLE 4 Writing a Transformed Polynomial Function

Let the graph of g be a vertical stretch by a factor of 2, followed by a translation 3 units up of the graph of $f(x) = x^4 - 2x^2$. Write a rule for g.

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Practice:

Describe the transformation of *f* represented by g.

1. $f(x) = x^4$, $g(x) = (x - 5)^4$

2. $f(x) = x^5$, $g(x) = (x - 2)^5 - 1$

3. $f(x) = x^4$, $g(x) = \frac{1}{2}x^4 + 1$

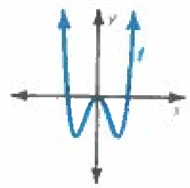
4. $f(x) = x^3$, $g(x) = 5x^3 + 1$

Write a rule for g and sketch each function. Describe the graph of g as a transformation of the graph of *f*.

5. $f(x) = x^4 + 1$, $g(x) = f(x + 2)$

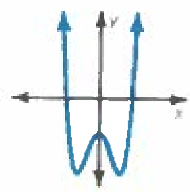
6. $f(x) = 2x^3 - 2x^2 + 6$, $g(x) = -\frac{1}{2}f(x)$

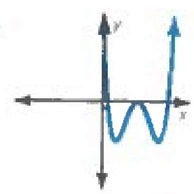
Match each of the functions to the correct graph.

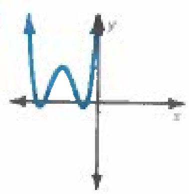
7. 

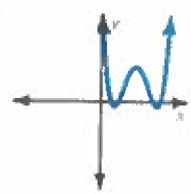
$y = f(x - 2)$ $y = f(x + 2) + 2$

$y = f(x - 2) + 2$ $y = f(x) - 2$

A. 

B. 

C. 

D. 

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Parent Functions and transformations

The absolute value parent function, written as $f(x) = |x|$, is defined as

$$f(x) = \begin{cases} x & \text{if } x > 0 \\ 0 & \text{if } x = 0 \\ -x & \text{if } x < 0 \end{cases}$$

Vertical Shift

To translate the absolute value function $f(x) = |x|$ vertically, you can use the function

$g(x) = f(x) + k$.

When $k > 0$, the graph of $g(x)$ translated k units up.

When $k < 0$, the graph of $g(x)$ translated k units down.

Horizontal Shift

To translate the absolute value function $f(x) = |x|$ horizontally, you can use the function

$g(x) = f(x - h)$.

When $h > 0$, the graph of $f(x)$ is translated h units to the right to get $g(x)$.

When $h < 0$, the graph of $f(x)$ is translated h units to the left to get $g(x)$.

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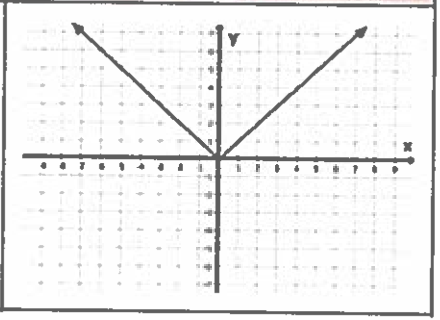
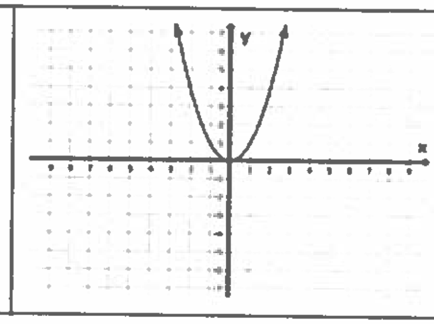
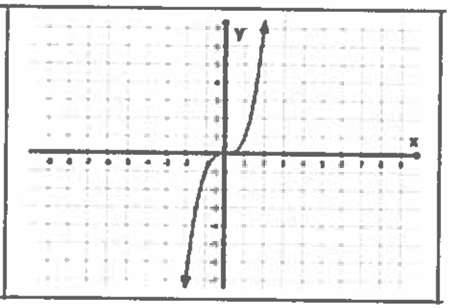
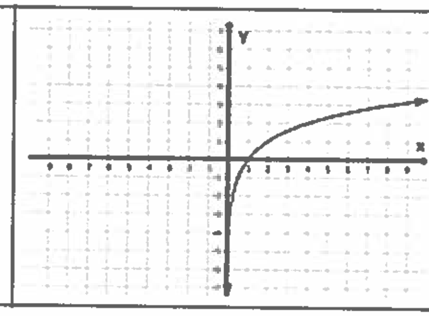
Parent Function	Graph
$y=\lvert x\rvert$ **Absolute Value, Even** Domain: $(-\infty,\infty)$ Range: $[0,\infty)$ End Behavior: $x\to-\infty,\ y\to\infty$ $x\to\infty,\ y\to\infty$	
$y=x^2$ **Quadratic, Even** Domain: $(-\infty,\infty)$ Range: $[0,\infty)$ End Behavior: $x\to-\infty,\ y\to\infty$ $x\to\infty,\ y\to\infty$	
$y=x^3$ **Cubic, Odd** Domain: $(-\infty,\infty)$ Range: $(-\infty,\infty)$ End Behavior: $x\to-\infty,\ y\to-\infty$ $x\to\infty,\ y\to\infty$	
$y=\log_b(x),\ b>1$ **Log, Neither** Domain: $(0,\infty)$ Range: $(-\infty,\infty)$ End Behavior: $x\to0^+,\ y\to-\infty$ $x\to\infty,\ y\to\infty$	

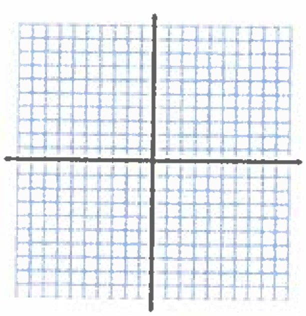
Practice:

Give the name of the parent function and describe the transformation represented.

1. $g(x)=x^2-1$

Parent: ____________________

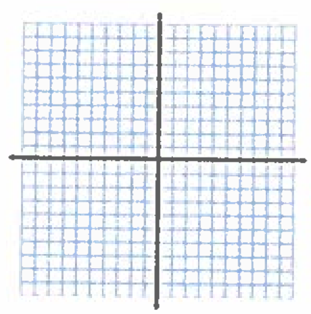
Transformations: ____________________



2. $f(x)=2\lvert x-1\rvert$

Parent: ____________________

Transformations: ____________________



Given the parent function and a description of the transformation, write the equation of the transformedfunction, f(x).

Absolute Value — vertical shift up 5, horizontal shift right 3. ____________________

Logarithmic — flipped over the x axis, vertical shift down 2 ____________________

Quadratic — vertical stretch by 5, horizontal shift left 8. ____________________

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key:

EXAMPLE 1 Translating a Polynomial Function

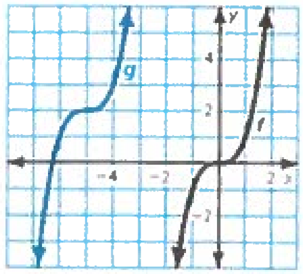
Describe the transformation of $f(x) = x^3$ represented by $g(x) = (x + 5)^3 + 2$. Then graph each function.

SOLUTION

Notice that the function is of the form $g(x) = (x - h)^3 + k$. Rewrite the function to identify h and k.

$$g(x) = (x - (-5))^3 + 2$$

h k



▶ Because $h = -5$ and $k = 2$, the graph of g is a translation 5 units left and 2 units up of the graph of f.

EXAMPLE 2 Transforming Polynomial Functions

Describe the transformation of f represented by g. Then graph each function.

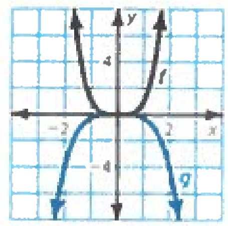
a. $f(x) = x^4$, $g(x) = -\frac{1}{4}x^4$

b. $f(x) = x^5$, $g(x) = (2x)^5 - 3$

SOLUTION

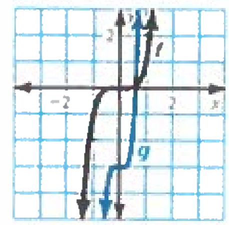
a. Notice that the function is of the form $g(x) = -ax^4$, where $a = \frac{1}{4}$.

▶ So, the graph of g is a reflection in the x-axis and a vertical shrink by a factor of $\frac{1}{4}$ of the graph of f.



b. Notice that the function is of the form $g(x) = (ax)^5 + k$, where $a = 2$ and $k = -3$.

▶ So, the graph of g is a horizontal shrink by a factor of $\frac{1}{2}$ and a translation 3 units down of the graph of f.



EXAMPLE 3 Writing Transformed Polynomial Functions

Let $f(x) = x^3 + x^2 + 1$. Write a rule for g and then graph each function. Describe the graph of g as a transformation of the graph of f.

a. $g(x) = f(-x)$

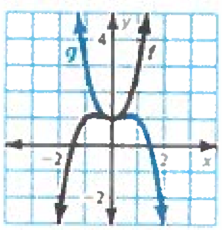
b. $g(x) = 3f(x)$

SOLUTION

a. $g(x) = f(-x)$

$= (-x)^3 + (-x)^2 + 1$

$= -x^3 + x^2 + 1$

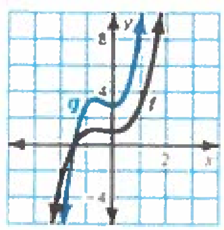


▶ The graph of g is a reflection in the y-axis of the graph of f.

b. $g(x) = 3f(x)$

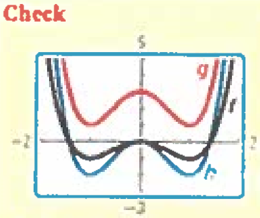
$= 3(x^3 + x^2 + 1)$

$= 3x^3 + 3x^2 + 3$



▶ The graph of g is a vertical stretch by a factor of 3 of the graph of f.

Check



EXAMPLE 4 Writing a Transformed Polynomial Function

Let the graph of g be a vertical stretch by a factor of 2, followed by a translation units up of the graph of $f(x) = x^4 - 2x^2$. Write a rule for g.

SOLUTION

Step 1 First write a function h that represents the vertical stretch of f.

$h(x) = 2 \cdot f(x)$ — Multiply the output by 2.

$= 2(x^4 - 2x^2)$ — Substitute $x^4 - 2x^2$ for $f(x)$.

$= 2x^4 - 4x^2$ — Distributive Property

Step 2 Then write a function g that represents the translation of h.

$g(x) = h(x) + 3$ — Add 3 to the output.

$= 2x^4 - 4x^2 + 3$ — Substitute $2x^4 - 4x^2$ for $h(x)$.

▶ The transformed function is $g(x) = 2x^4 - 4x^2 + 3$.

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Practice:

Describe the transformation of *f* represented by g.

1. $f(x) = x^4$, $g(x) = (x - 5)^4$

The graph of g is a translation 5 units right.

2. $f(x) = x^3$, $g(x) = (x - 2)^3 - 1$

The graph of g is a translation 2 units right and 1 unit down.

3. $f(x) = x^4$, $g(x) = \frac{1}{2}x^4 + 1$

The graph of g is a vertical shrink of $\frac{1}{2}$ and 1 unit up.

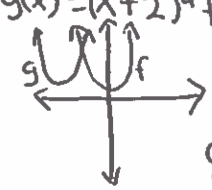
4. $f(x) = x^3$, $g(x) = 5x^3 + 1$

The graph of g is a vertical stretch of 5, translation of 1 unit up.

Write a rule for g and sketch each function. Describe the graph of g as a transformation of the graph of *f*.

5. $f(x) = x^4 + 1$, $g(x) = f(x + 2)$

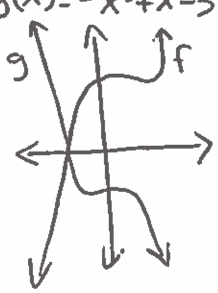
$g(x) = (x+2)^4 + 1$



g is a translation 2 units left.

6. $f(x) = 2x^3 - 2x^2 + 6$, $g(x) = -\frac{1}{2}f(x)$

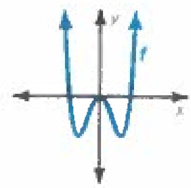
$g(x) = -x^3 + x^2 - 3$



The graph of g is a vertical shrink by a factor of $\frac{1}{2}$, reflection over the x-axis.

Match each of the functions to the correct graph.

7.



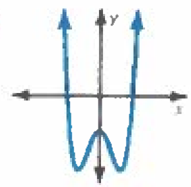
B $y = f(x - 2)$

C $y = f(x + 2) + 2$

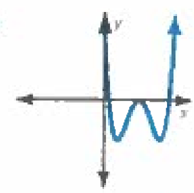
D $y = f(x - 2) + 2$

A $y = f(x) - 2$

A.



B.



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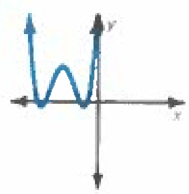
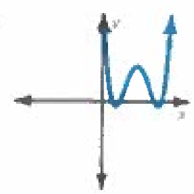


D.



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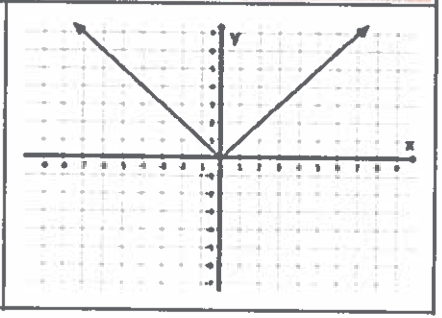
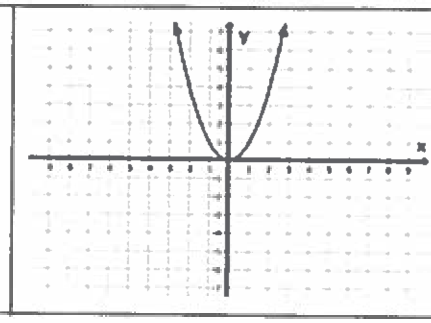
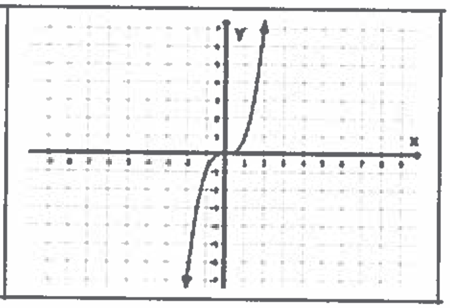
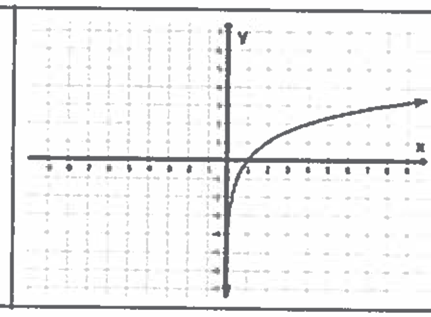
Parent Function	Graph
$y=\|x\|$ Absolute Value, Even Domain: $(-\infty,\infty)$ Range: $[0,\infty)$ End Behavior: $x\to-\infty, y\to\infty$ $x\to\infty, y\to\infty$	

$y=x^2$ Quadratic, Even Domain: $(-\infty,\infty)$ Range: $[0,\infty)$ End Behavior: $x\to-\infty, y\to\infty$ $x\to\infty, y\to\infty$	

$y=x^3$ Cubic, Odd Domain: $(-\infty,\infty)$ Range: $(-\infty,\infty)$ End Behavior: $x\to-\infty, y\to-\infty$ $x\to\infty, y\to\infty$	

$y=\log_b(x),\ b>1$ Log, Neither Domain: $(0,\infty)$ Range: $(-\infty,\infty)$ End Behavior: $x\to0^+, y\to-\infty$ $x\to\infty, y\to\infty$	

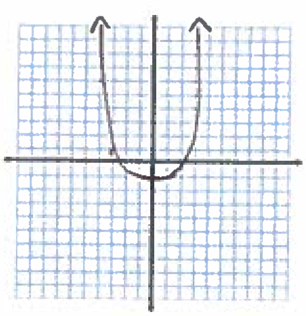
Practice:

Give the name of the parent function and describe the transformation represented.

1. $g(x)=x^2-1$

Parent: $f(x)=x^2$

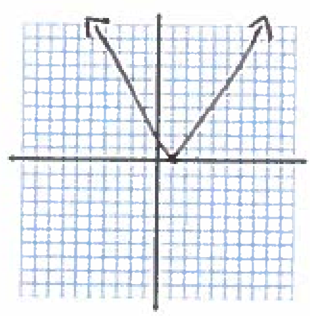
Transformations: 1 unit down



2. $f(x)=2|x-1|$

Parent: $y=|x|$

Transformations: horizontal shrink of $\frac{1}{2}$, 1 unit to the right



Given the parent function and a description of the transformation, write the equation of the transformedfunction, f(x).

Absolute Value — vertical shift up 5, horizontal shift right 3. $f(x)=|x-3|+5$

Logarithmic — flipped over the x axis, vertical shift down 2 $f(x)=-\log_b(x)-2$

Quadratic — vertical stretch by 5, horizontal shift left 8. $f(x)=(5x^2+8)$