Core Concept

The Fundamental Theorem of Algebra

Theorem If $f(x)$ is a polynomial of degree n where $n > 0$, then the equation $f(x) = 0$ has at least one solution in the set of complex numbers.

Corollary If $f(x)$ is a polynomial of degree n where $n > 0$, then the equation $f(x) = 0$ has exactly n solutions provided each solution repeated twice is counted as two solutions, each solution repeated three times is counted as three solutions, and so on.

Short Summary:

The highest degree is a polynomial is the number of zeroes/ solutions the polynomial has

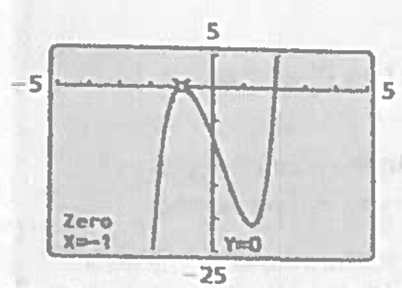
Ex: $X^4 + 6x^3 + 12x^2 + 8x$

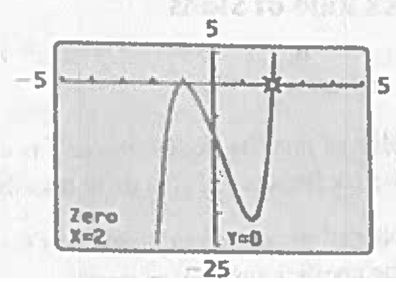
Since the polynomial degree is 4, there are 4 roots (-2, -2, -2, and 0)

<u>Make sure to count repeat solutions</u>

Equation	Degree	Solution(s)	Number of solutions
$2x - 1 = 0$	1	$\frac{1}{2}$	1
$x^2 - 2 = 0$	2	$\pm\sqrt{2}$	2
$x^3 - 8 = 0$	3	$2, -1 \pm i\sqrt{3}$	3
$x^3 + x^2 - x - 1 = 0$	3	$-1, -1, 1$	3

The graph of f and the real zeros are shown. Notice that only the *real* zeros appear as x-intercepts. Also, the graph of f touches the x-axis at the <u>repeated zero $x = -1$</u> and crosses the x-axis at $x = 2$. Real zeroes are x- intercepts





Core Concept

The Complex Conjugates Theorem

If f is a polynomial function with real coefficients, and $a + bi$ is an imaginary zero of f, then $a - bi$ is also a zero of f.

Ex: $f(x) = x + 4$

$f(x) = (x-2i)(x+2i)$

Solutions: $(2i, -2i)$

2i and -2i are complex conjugate. Every imaginary root has a complex conjugate. Radicals too

Practice:

1) Identify the number of solutions or zeroes

A) $X^4 + 2x^3 - 4x^2 + x = 0$ 4

B) $9t^6 - 14t^3 + 4t - 1 = 0$ 6

A) **Use Descarte's rule of Signs to find the total number of possible zeroes in the equation**

$-X^5 - 2x^4 - x^3 + 2x^2 + 7x - 1$

$f(x) = -x^5 - 2x^4 - x^3 + 2x^2 + 7x - 1$

$f(-x) = x^5 - 2x^4 + x^3 + 2x^2 - 7x + 1$

Positive Real Zeroes	Negative Real Zeroes	Imaginary Zeroes	Total Zeroes
1	4	0	5
1	2	2	5
1	0	4	5

2) Write a polynomial function f of least degree that has rational coefficients, and the given zeroes

-3i and 2-i

$(x-3i)(x+3i)(x-2+i)(x+2-i)$

$(x^2-9i^2)((x-2)-i^2)$

$(x^2+9)(x^2-4x+4+1)$

$x^4 - 4x^3 + 5x^2 + 9x^2 - 36x + 45$

$x^4 - 4x^3 + 14x^2 - 36x + 45$

3) Given f(X) $2x^4 + x^3 + 5x^2 + 4x - 12$

A) Use Descarte's Rule of Signs to find the possible Zeroes

Positive Real Zeroes	Negative Real Zeroes	Imaginary Zeroes	Total Zeroes
1	3	0	4
1	1	2	4

$f(x) = 2x^4 + x^3 + 5x^2 + 4x - 12$

$f(-x) = 2x^4 - x^3 + 5x^2 - 4x - 12$