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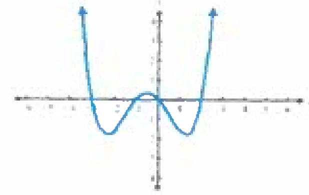
Lesson 4.5 Solving Polynomial Equations



What we will explore throughout this lesson:

- Finding the solutions of polynomial equations and zeros of polynomial functions
- How to use the Rational Root Theorem
- How to use the Irrational Conjugates Theorem

Finding Solutions and Zeros

The Zero Product Property that was used to solve factorable quadratic equations can be used to solve higher degree polynomial equations such as cubic and quartic equations.

Example Problem #1: *Solving a Polynomial Equation by Factoring*

Solve $X^3+6x^2+5x=0$

- $X^3+6x^2+5x=0$ Write the equation
- $X(x^2+6x+5)=0$ Factor common monomial
- $X(x+5)(x+1)=0$ Factor thetrinomial

- $X=0$, $X+5=0$, $X+1=0$ — Zero Product Property
- $X=0$, $X=-5$, $X=-1$ — Solve for X

Example Problem #2: Finding Zeros of a Polynomial Function

Find the zeros of f(x)= 3x^4-24x^2+48 Then sketch a graph of the function.

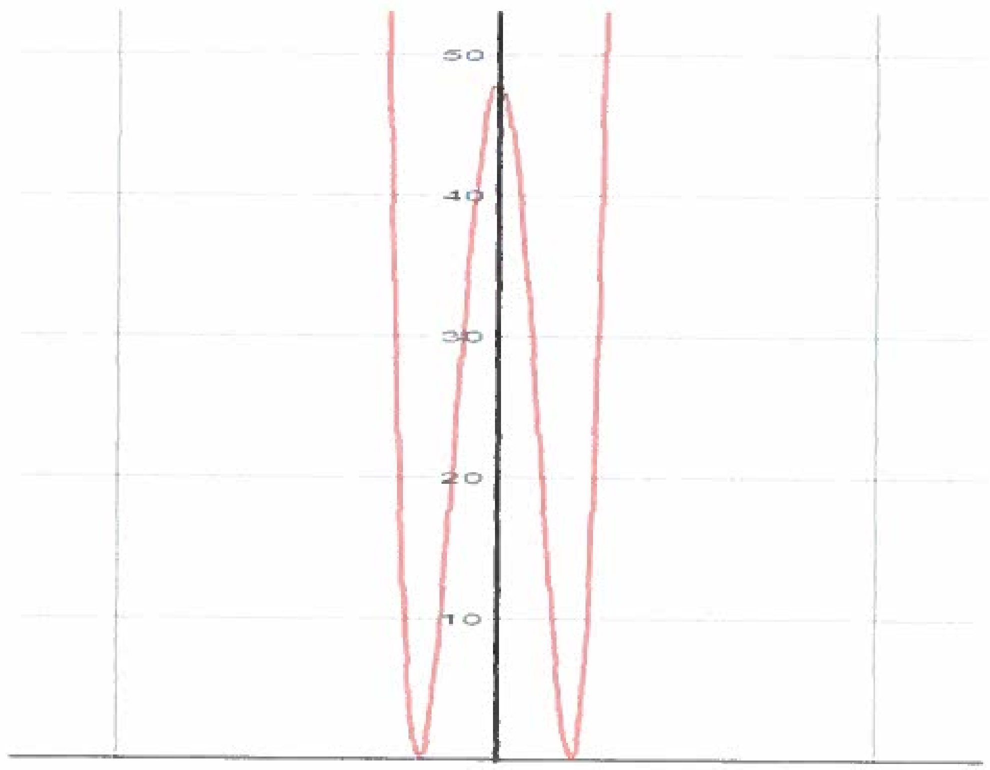
- 0= 3x^4-24x^2+48 — Set f(x) equal to 0
- 0= 3(x^4-8x^2+16) — Factor out 3
- 0= 3(x^2-4)(x^2-4) — Factor trinomial
- 0= 3(x+2)(x-2)(x+2)(x-2) — Difference of 2 perfect squares
- 0= 3[(x+2)^2][(x-2)^2] — Rewrite using exponents

Graph of *f* touches the x axis at zeros x=-2 and x=2

By analyzing the function the determined y intercept is +48

Positive coefficient means f(x) -> ∞ as x-> ∞ and f(x) -> ∞ as x-> -∞

Use these characteristics to sketch a graph of the function



The Rational Root Theorem

The Rational Root Theorem is used for finding solutions of polynomial equations. However, it only lists possible solutions. To find the actual solutions, you must test values from the list of possible solutions.

If the polynomial has integer coefficients, then every rational solution of f(x)=0 has the following form:

p/q= factor of constant term/factor or leading coefficient

Example #3: Using the Rational Root Theorem

Find all real solutions of x^3-5x^2-2x+24=0

Step 1: Use the rational root theorem by listing all rational solutions.

The leading coefficient of f(x) is 1 and the constant is 24. The possible solutions are:

X= ±1/1, ±2/1,±3/1,±4/1,±6/1,±8/1,±12/1,±24/1

Step 2: Test possible solutions using synthetic division until a solution is found.

Test x= 2

```
2 | 1  -5  -2  24
  |     2  -6 -16
  ----------------
    1  -3  -8   8
```

<-f(2)≠0, so x-2 isn't a factor of f(x)

Test x=-2

```
-2 | 1  -5  -2  24
   |    -2  14 -24
   ----------------
     1  -7  12   0
```

<-f(-2)=0, so x+2 is a factor of f(x)

Step 3: Factor completely using the result of the synthetic division.

$(x+2)(x^2-7x+12)=0$ Write as a product of factors

$(x+2)(x-4)(x-3)=0$ Factor the trinomial

Solutions: $x=-2$, $x=4$, $x=3$

The Irrational Conjugates Theorem

Let f be a polynomial function with rational coefficients, and let a and b be rational numbers such that $\sqrt{b}$ is irrational. If $a+\sqrt{b}$ is a zero of f, then $a-\sqrt{b}$ is also a zero of f.

Example #5: Using Zeros to Write a Polynomial Function

Write a polynomial function f of least degree that has rational coefficients, a leading coefficient of 1, and zeros 4 and $1-\sqrt{5}$.

$F(x)= (x-4)[x-(1-\sqrt{5})][x-(1+\sqrt{5})]$ Write f(x) in factored form

$(x-4)[(x-1)-\sqrt{5}][(x-1)+\sqrt{5}]$ Regroup terms

(x-4)[(x-1)^2-5]	Multiply
(x-4)[(x^2-2x+1)-5]	Expand Binomial
(x-4)(x^2-2x-4)	Simplify
X^3-2x^2-4x-4x^2+8x+16	Multiply
X^3-6x^2+4x+16	Combine like terms

Practice Problems

For Problems 1-3, solve the equation.

1. $9m^5 = 27m^3$

2. $z^3-z^2-12z = 0$

For Problems 3-4, find the zeros of the function. Then sketch a graph of the function.

3. $h(x) = x^4+x^3-6x^2$

4. $f(x) = x^4-18x^2+81$

For Problem 5, find all real solutions of $x^3-8x^2+11x+20 = 0$

For Problem 6 write a polynomial function f of least degree that has rational coefficients, a leading coefficient of 1, and zeros 3 and $2+\sqrt{5}$.

Answer Key

Practice Problems

For Problems 1-3, solve the equation.

1. 9m^5 = 27m^3

$9m^5 - 27m^3 = 0$

$9m^3(m^2 - 3) = 0$

$9m^3 = 0$ $\quad$ $m^2 - 3 = 0$

$m = 0$ or $m = \sqrt{3}$

2. z^3-z^2-12z = 0

$z(z^2 - z - 12) = 0$

$z(z-4)(z+3) = 0$

$z = 0$, $z = 4$, $z = -3$

For Problems 3-4, find the zeros of the function. Then sketch a graph of the function.

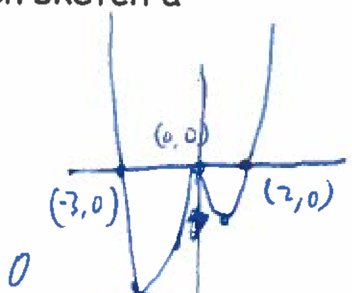
3. h(x) = x^4+x^3-6x^2

$x^4 + x^3 - 6x^2 = 0$

$x^2(x^2 + x - 6) = 0$

$x^2(x+3)(x-2) = 0$

zeros: $x = 0$, $x = -3$, $x = 2$



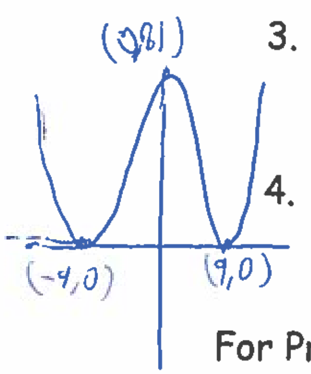
4. f(x) = x^4-18x^2+81

$x^4 - 18x^2 + 81$

$(x^2 - 9)(x^2 - 9x) = 0$

$x(x-9)(x-9) = 0$

zeros: $x = 0$, $x = \pm 9$



For Problem 5, find all real solutions of x^3-8x^2+11x+20 = 0

Find possible solutions:

$\pm\frac{1}{1}, \pm\frac{2}{1}, \pm\frac{4}{1}, \pm\frac{5}{1}, \pm\frac{10}{1}, \pm\frac{20}{1}$

$-1\,|\ 1 \quad -8 \quad 11 \quad 20$
$\quad\ \ -1 \quad 9 \quad -20$
$\quad\ \ 1 \quad -9 \quad 20 \quad 0$

Solution found b/c $f(x) = 0$

Test:

$1\,|\ 1 \quad -8 \quad 11 \quad 20$
$\quad\ \ 1 \quad -7 \quad 4$
$\quad\ \ 1 \quad -7 \quad 4 \quad 24$

Factor: $(x+1)(x^2 - 9x + 20) = 0$

$(x+1)(x-4)(x-5)$

zeros: $x = -1$, $x = 4$, $x = 5$

For Problem 6 write a polynomial function f of least degree that has rational coefficients, a leading coefficient of 1, and zeros 3 and 2+√5.

$f(x) = (x-3)[x - (2+\sqrt{5})][x - (2-\sqrt{5})]$

$(x-3)[(x-2) - \sqrt{5}][(x-2) + \sqrt{5}]$

$(x-3)[(x-2)^2 - 5]$

$(x-3)[(x^2 - 4x + 4) - 5]$

$(x-3)(x^2 - 4x - 1)$

$x^3 - 4x^2 - x - 3x^2 + 12x + 3 = x^3 - 7x^2 + 11x + 3$