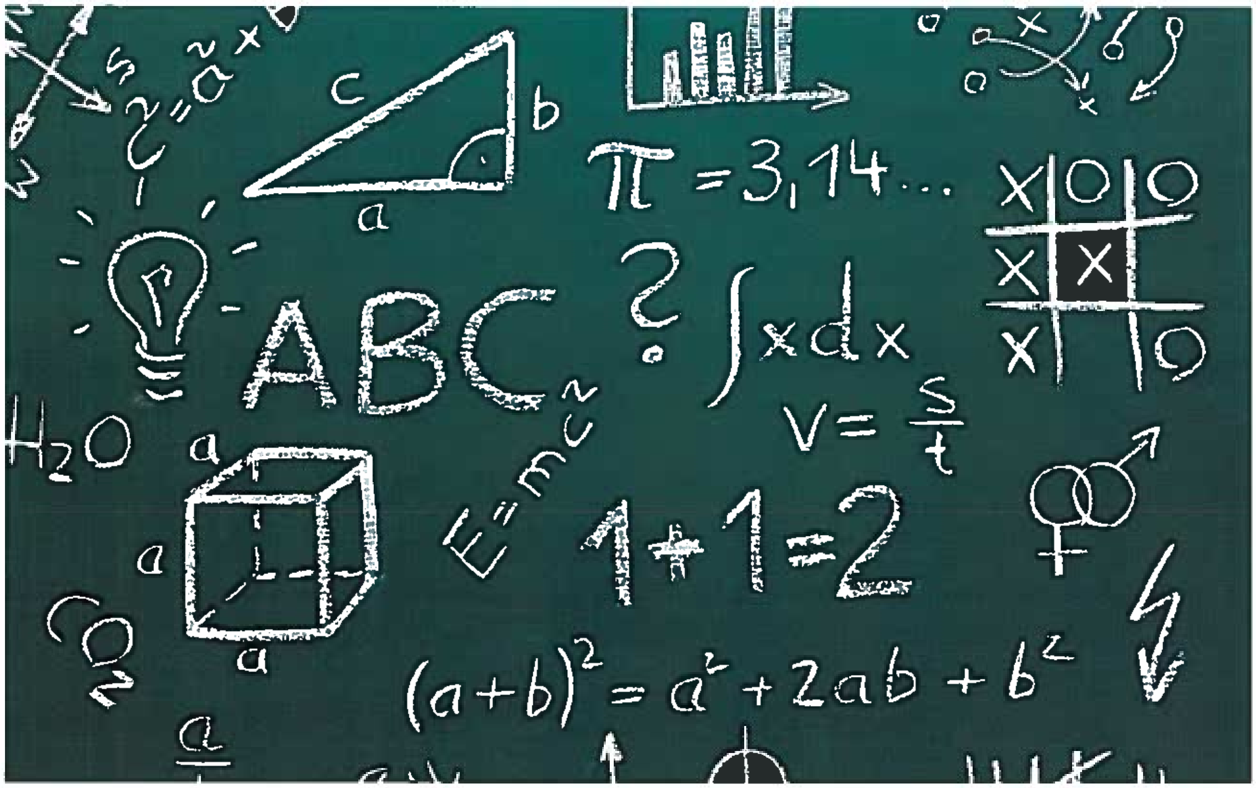

Lesson 4.5 Solving Polynomial Equations

Review

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Name: ______________________________

Teacher: ____________________________

Pd: ___________

Lesson 4.6: Fundamental Theorem of Algebra

By: Lily

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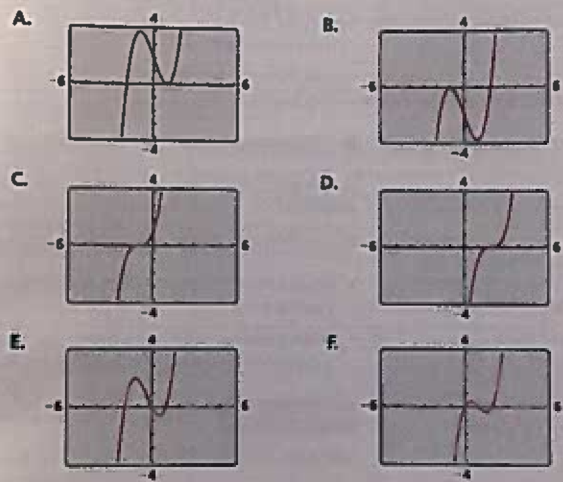
Lesson 4.5 Review- Solving Polynomial Equations

a. $x^3 - 6x^2 + 12x - 8 = 0$
b. $x^3 + 3x^2 + 3x + 1 = 0$
c. $x^3 - 3x + 2 = 0$
d. $x^3 + x^2 - 2x = 0$
e. $x^3 - 3x - 2 = 0$
f. $x^3 - 3x^2 + 2x = 0$



Your Task:

1) Match each cubic polynomial equation with the graph of its related polynomial function

2) Solve each equation

a. ______ x = ______________
b. ______ x = ______________
c. ______ x = ______________
d. ______ x = ______________
e. ______ x = ______________
f. ______ x = ______________

EXAMPLE 1 **Solving a Polynomial Equation by Factoring**

Solve $2x^3 - 12x^2 + 18x = 0$.

SOLUTION

$2x^3 - 12x^2 + 18x = 0$	Write the equation.
$2x(x^2 - 6x + 9) = 0$	Factor common monomial.
$2x(x - 3)^2 = 0$	Perfect Square Trinomial Pattern
$2x = 0$ or $(x - 3)^2 = 0$	Zero-Product Property
$x = 0$ or $x = 3$	Solve for x.

▶ The solutions, or roots, are $x = 0$ and $x = 3$.

REPEATED SOLUTION:

- When a factor appears more than once
- such as (x – 3)
- x = 0 and x = 3
- NOTE: the graph touches the x-axis at x = 3 but does not cross the x-axis

Rules for a Repeated Solution

- When (x – k) is raised to an *even* power, the graph touches the x-axis at x = k but does NOT cross it
- When (x – k) is raised to an *odd* power, the graph crosses the x-axis at x = k

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Solve the equation.

1. $4x^4 - 40x^2 + 36 = 0$
2. $2x^5 + 24x = 14x^3$

Find the zeros of the function. Then sketch a graph of the function.

3. $f(x) = 3x^4 - 6x^2 + 3$
4. $f(x) = x^3 + x^2 - 6x$

Your Task:

1) Do problems 1 and 3

Lesson 4.5 Review- Solving Polynomial Equations

Core Concept

The Rational Root Theorem

If $f(x) = a_nx^n + \cdots + a_1x + a_0$ has *integer* coefficients, then every rational solution of $f(x) = 0$ has the following form:

$$\frac{p}{q} = \frac{\text{factor of constant term } a_0}{\text{factor of leading coefficient } a_n}$$

STUDY TIP

Notice that you can use the Rational Root Theorem to list possible zeros of polynomial functions.

The Rational Root Theorem can be a starting point for finding solutions of polynomial equations. However, the theorem lists only *possible* solutions. In order to find the *actual* solutions, you must test values from the list of possible solutions.

Rational Root Theorem EXAMPLE

$64x^3 + 152x^2 - 62x - 105$

Leading coefficient: ___ Factors: ______________________________

Constant term: ___ Factors: ______________________________

Possible Zeros (list at least 10):

Once you have all the possible zeros, use synthetic division to find a factor

Your Task:

1) Do problem 6

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5. Find all real solutions of $x^3 - 5x^2 - 2x + 24 = 0$.
6. Find all real zeros of $f(x) = 3x^4 - 2x^3 - 37x^2 + 24x + 12$.

Core Concept

The Irrational Conjugates Theorem

Let f be a polynomial function with rational coefficients, and let a and b be rational numbers such that $\sqrt{b}$ is irrational. If $a + \sqrt{b}$ is a zero of f, then $a - \sqrt{b}$ is also a zero of f.

Lesson 4.5 Review- Solving Polynomial Equations

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EXAMPLE 5 **Using Zeros to Write a Polynomial Function**

Write a polynomial function f of least degree that has rational coefficients, a leading coefficient of 1, and the zeros 3 and $2 + \sqrt{5}$.

SOLUTION

Because the coefficients are rational and $2 + \sqrt{5}$ is a zero, $2 - \sqrt{5}$ must also be a zero by the Irrational Conjugates Theorem. Use the three zeros and the Factor Theorem to write $f(x)$ as a product of three factors.

$f(x) = (x-3)[x-(2+\sqrt{5})][x-(2-\sqrt{5})]$	Write $f(x)$ in factored form.
$= (x-3)[(x-2)-\sqrt{5}][(x-2)+\sqrt{5}]$	Regroup terms.
$= (x-3)[(x-2)^2-5]$	Multiply.
$= (x-3)[(x^2-4x+4)-5]$	Expand binomial.
$= (x-3)(x^2-4x-1)$	Simplify.
$= x^3-4x^2-x-3x^2+12x+3$	Multiply.
$= x^3-7x^2+11x+3$	Combine like terms.

Check

You can check this result by evaluating f at each of its three zeros.

$f(3) = 3^3 - 7(3)^2 + 11(3) + 3 = 27 - 63 + 33 + 3 = 0$ ✓

$f(2+\sqrt{5}) = (2+\sqrt{5})^3 - 7(2+\sqrt{5})^2 + 11(2+\sqrt{5}) + 3$

$= 38 + 17\sqrt{5} - 63 - 28\sqrt{5} + 22 + 11\sqrt{5} + 3$

$= 0$ ✓

Because $f(2+\sqrt{5}) = 0$, by the Irrational Conjugates Theorem $f(2-\sqrt{5}) = 0$. ✓

- $[x-(a+\sqrt{b})][x-(a-\sqrt{b})]$
- $[(x-a)-\sqrt{b}][(x-a)+\sqrt{b}]$
- $[(x-a)^2-b]$
- $[(x^2+a^2x+a^2)-b]$
- Multiply by the remaining factor
- Combine like terms

Your Task:

Solve the equations.

1) $2x^4 - 4x^3 = -2x^2$
2) $a^3 - 4a^2 + 4a = 0$
3) $y^3 - 27 = 9y^2 - 27y$

Find the zeros of the function. Then sketch the function of the graph.

4) $h(x) = x^4 + x^3 - 6x^2$
5) $n(x) = -x^3 - 2x^2 + 15x$
6) $p(x) = x^3 - 5x^2 - 4x + 20$

Write a polynomial function f of least degree with a leading coefficient of one and the given zeros.

11) $-2, 1+\sqrt{7}$

12) $4, 6-\sqrt{7}$

13) $-6, 0, 3-\sqrt{5}$

DN) a $\underline{D}$ d $\underline{E}$

(2,0) (-2,0)(0,0)(1,0)

b $\underline{C}$ e $\underline{B}$

(-1,0) (-1,0)(0,0)

c $\underline{A}$ f $\underline{F}$

(1,0)(-2,0) (0,0)(1,0)(2,0)

1) $4x^4 - 40x^2 + 36 = 0$

$4(x^4 - 10x^2 + 9)$

$4(x^2-1)(x^2-9)$

$4(x+1)(x-1)(x+3)(x-3)$

$\boxed{x = \pm 1, \pm 3}$

3) $f(x) = 3x^4 - 6x^2 + 3$

$3x^4 - 6x^2 + 3$

$3(x^4 - 2x^2 + 1)$

$3(x^2-1)(x^2-1)$

$3(x+1)^2(x-1)^2$

$\boxed{x = \pm 1}$

2RT) $64x^3 + 152x^2 - 62x - 105$

Leading coefficient: $\underline{64}$ Factors: $\pm 1, \pm 2, \pm 4, \pm 8, \pm 16, \pm 32, \pm 64$

Constant term: $\underline{105}$ Factors: $\pm 1, \pm 3, \pm 5, \pm 7, \pm 15, \pm 21, \pm 35, \pm 105$

Possible Zeros:

$\pm \frac{3}{2}, \pm \frac{5}{2}, \pm \frac{7}{2}, \pm \frac{15}{2}, \pm \frac{35}{2}, \pm \frac{105}{2}, \pm \frac{21}{2}, \pm \frac{35}{4}, \pm \frac{35}{16}, \pm 3, \pm 7, \pm 15, \pm 105$

(ANSWERS MAY VARY → SAMPLE ANSWERS)

6) $3x^4 - 2x^3 - 37x^2 + 24x + 12$

$\pm 12, \pm 6, \pm 4, \pm 3, \pm 2, \pm 1, \pm \frac{4}{3}, \pm \frac{2}{3}, \pm \frac{1}{3}$

1	3	-2	-37	24	12
	↓	3	1	-36	-12
	3	1	-36	-12	0

$3x^3 + x^2 - 36x - 12$ → $[x^2(3x+1) - 12(3x+1)](x-1)$

$(x^2-12)(3x+1)(x-1)$

$(x+\sqrt{12})(x-\sqrt{12})(3x+1)(x-1)$

$x = \pm\sqrt{12}, -\frac{1}{3}, 1$

1) $2x^4 - 4x^3 = -2x^2$

$2x^4 - 4x^3 + 2x^2 = 0$

$2x^2(x^2 - 2x + 1) = 0$

$2x^2(x-1)^2$

$\boxed{x=0}$ $\boxed{x=1}$

2) $a^3 - 4a^2 + 4a = 0$

$a(a^2 - 4a + 4) = 0$

$a(a-2)^2 = 0$

$\boxed{a=0}$ $\boxed{a=2}$

3) $y^3 - 27 = 9y^2 - 27y$

$(y-3)(y^2+3y+9) = 9y(y-3)$

$(y-3)(y^2+3y+9) - 9y(y-3) = 0$

$(y^2+3y+9-9y)(y-3) = 0$

$(y^2-6y+9)(y-3) = 0$

$(y-3)^2(y-3) = 0$

$(y-3)$

$\boxed{y=3}$

4) $h(x) = x^4 + x^3 - 6x^2$

$h(x) = x^2(x^2 + x - 6)$

$h(x) = x(x+5)(x-1)$

$\boxed{x=0}$ $\boxed{x=-5}$ $\boxed{x=1}$

5) $n(x) = -x^3 - 2x^2 + 15x$

$n(x) = -x(x^2 + 2x - 15)$

$n(x) = -x(x+5)(x-3)$

$\boxed{x=0}$ $\boxed{x=-5}$ $\boxed{x=3}$

6) $p(x) = x^3 - 5x^2 - 4x + 20$

$p(x) = x^2(x-5) - 4(x-5)$

$p(x) = (x^2-4)(x-5)$

$p(x) = (x+2)(x-2)(x-5)$

$\boxed{x=2}$ $\boxed{x=-2}$ $\boxed{x=5}$

(11) $\therefore -2,\ 1+\sqrt{7}$

$(x+2)\left[x-(1+\sqrt{7})\right]\left[x-(1-\sqrt{7})\right]$

$(x+2)\left[(x-1)^2-7\right]$

$(x+2)\left[x^2-2x+1-7\right]$

$(x+2)(x^2-2x-6)$

$x^3-2x^2-6x+2x^2-4x-12$

$\boxed{x^3-10x-12}$

(12) $4,\ 6-\sqrt{7}$

$(x-4)\left[x-(6-\sqrt{7})\right]\left[x-(6+\sqrt{7})\right]$

$(x-4)\left[(x-6)^2-7\right]$

$(x-4)\left[x^2-12x+36-7\right]$

$(x-4)(x^2-12x+29)$

$x^3-12x^2+29x-4x^2+48x-116$

$\boxed{x^3-16x^2+77x-116}$

(13) $-6,\ 0,\ 3-\sqrt{5}$

$x(x+6)\left[x-(3-\sqrt{5})\right]\left[x-(3+\sqrt{5})\right]$

$x(x+6)\left[(x-3)^2-5\right]$

$x(x+6)\left[x^2-6x+9-5\right]$

$x(x+6)(x^2-6x+4)$

$x(x^3-6x^2+4x+6x^2-36x+24)$

$x^4-\cancel{6x^3}+4x^2+\cancel{6x^3}-36x^2+24$

$\boxed{x^4-32x^2+24}$