Name______________________________ Date________

4.3 Long Division of Polynomials, Synthetic division, Remainder Theorem

Long Division of Polynomials

When you divide a polynomial *f(x)* by a nonzero polynomial *d(x)*, you get a quotient polynomial *q(x)* and a remainder polynomial *r(x)*.

$$\frac{f(x)}{d(x)} = q(x) + \frac{r(x)}{d(x)}$$

The degree of the remainder must be less than the degree of the divisor. When the remainder is 0, the divisor *divides evenly* into the dividend. Also, the degree of the divisor is less than or equal to the degree of the dividend f(x). One way to divide polynomials is called **polynomial long division**.

Example 1:

Divide 2x^4 + 3x^3 + 5x – 1 by x^2 + 3x + 2

Solution

Write polynomial division in the same format you use when dividing numbers. Include a “0” as the coefficient of x^2 in the dividend. At each stage, divide the term with the highest power in what is left of the dividend by the first term of the divisor. This gives the next term of the quotient.

$$\begin{array}{r l}
 & \underline{\quad\quad\quad\quad\quad 2x^2 - 3x + 5} \leftarrow \text{quotient} \\
x^2 + 3x + 2 \,\big) & 2\cancel{x}^4 + 3x^3 + 0x^2 + 5x - 1 \\
 & \underline{-2\cancel{x}^4 - 6x^3 - 4x^2} \downarrow \\
 & -3\cancel{x}^3 - 4x^2 + 5x \\
 & \underline{+3x^3 + 9x^2 + 6x} \downarrow \\
 & 5\cancel{x}^2 + 11x - 1 \\
 & \underline{-5\cancel{x}^2 - 15x - 10} \\
 & -4x - 11 \leftarrow \text{remainder}
\end{array}$$

Multiply Divisor by $\frac{2x^4}{x^2}$ $= 2x^2$

Subtract. Bring down next term

Multiply Divisor by $\frac{-3x^3}{x^2} = -3x$

Subtract. Bring down next term

multiply divisor by $\frac{5x^2}{x^2} =$

Answer: $2x^2 - 3x + 5 + \frac{-4x - 11}{x^2 + 3x + 2}$

Check

You can check the result of a division problem by multiplying the quotient by the divisor and adding the remainder. The result should be the dividend.

$$(2x^2 - 3x + 5)(x^2 + 3x + 2) + (-4x - 11)$$

$$= (2x^2)(x^2 + 3x + 2) - (3x)(x^2 + 3x + 2) + (5)(x^2 + 3x + 2) - 4x - 11$$

$$= 2x^4 + 6x^3 + 4x^2 - 3x^3 - 9x^2 - 6x + 5x^2 + 15x + 10 - 4x - 11$$

$$= 2x^4 + 3x^3 + 5x - 1$$ ✓

Divide using polynomial long division for the following problems.

1) (5x^4 – 2x^3 – 7x^2 – 39) / (x^2 + 2x – 4)

2) (4x^4 + 5x – 4) / (x^2 – 3x – 2)

Synthetic Division

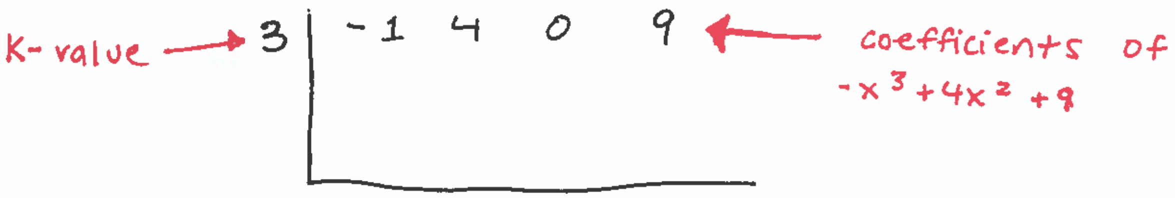
There is a shortcut for dividing polynomials by binomials of the form *x* – *k*. This shortcut is called **synthetic division**.

Example 2:

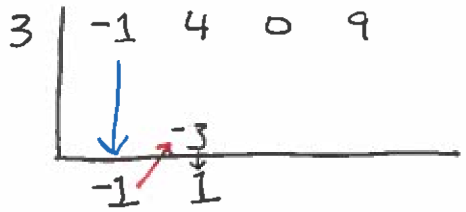
Divide –x^3 + 4x^2 + 9 by x – 3.

Solution

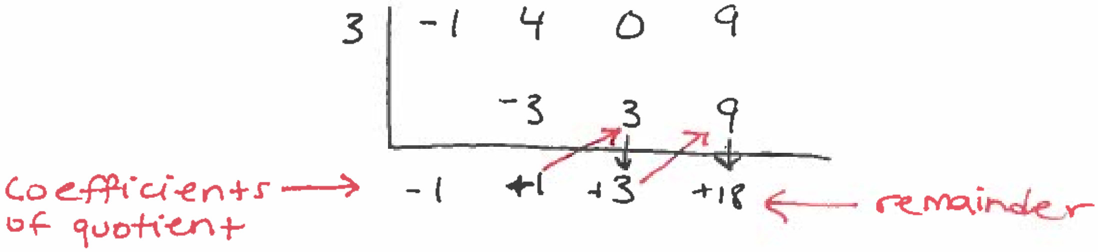
Step 1 Write the coefficients of the dividend in order of descending exponents. Include a "0" for the missing *x*-term. Because the divisor is *x* – 3, use *k* = 3. Write the *k*-value to the left of the vertical bar.



Step 2 Bring down the leading coefficient. Multiply the leading coefficient by the k-value. Write the product under the second coefficient. Add.



Step 3 Multiply the previous sum by the k-value. Write the product under the third coefficient. Add. Repeat this process for the remaining coefficient. The first three numbers in the bottom row are the coefficients of the quotient, and the last number is the remainder.



$$-x^2 + x + 3 + \frac{18}{x-3}$$

The Remainder Theorem

The remainder in the synthetic division process has an important interpretation. When you divide a polynomial *f(x)* by *d(x)* = *x* – *k*, the result is

$$\frac{f(x)}{d(x)} = q(x) + \frac{r(x)}{d(x)}$$

Polynomial division

$$\frac{f(x)}{x-k} = q(x) + \frac{r(x)}{x-k}$$

Substitute x-k for d(x)

$$f(x) = (x-k)q(x) + r(x)$$

Multiply both sides by x-k

Because either *r(x)* = 0 or the degree of *r(x)* is less than the degree of *x* – *k*, you know that *r(x)* is constant function. So, let *r(x)* = *r*, where *r* is a real number, and evaluate *f(x)* when *x* = *k*.

$$f(k) = (k-k)q(k) + r$$

Substitute k for x and r and r(x

$$f(k) = r$$

Simplify

This result is stated in the *Remainder Theorem*.

Example 3:

Use synthetic division to evaluate f(x) = 5x^3 – x^2 + 13x + 29 when x = -4

Solution

$$\begin{array}{r|rrrr} -4 & 5 & -1 & 13 & 29 \\ & \downarrow & -20 & 84 & -388 \\ \hline & 5 & -21 & 97 & -359 \end{array}$$

The remainder is -359. So, you can conclude from the Remainder Theorem that f(-4) = -359.

Check

Check this by substituting x = -4 in the original function.

$$f(-4) = 5(-4)^3 - (-4)^2 + 13(-4) + 29$$

$$= -320 - 16 - 52 + 29$$

$$= -359 \quad ✓$$

Use synthetic division to evaluate the function for the indicated value of x.

1) $f(x) = 3x^2 + 2x - 20; x = 3$

2) $f(x) = -x^4 - x^3 - 2; x = 5$

ANSWER KEY

Check

You can check the result of a division problem by multiplying the quotient by the divisor and adding the remainder. The result should be the dividend.

Divide using polynomial long division for the following problems.

1) (5x^4 – 2x^3 – 7x^2 – 39) / (x^2 + 2x – 4)

$$5x^2 - 12x + 37$$

$$x^2 + 2x - 4 \,\big)\, \cancel{5x^4} - 2x^3 - 7x^2 + 0x - 39$$

$$-\cancel{5x^4} - 10x^3 + 20x^2$$

$$-\cancel{12x^3} + 13x^2 + 0x$$

$$+\cancel{12x^3} + 24x^2 - 48x$$

$$\cancel{37x^2} - 48x - 39$$

$$-\cancel{37x^2} - 74x + 148$$

$$-122x + 109$$

$$\boxed{5x^2 - 12x + 37 + \frac{-122x + 109}{x^2 + 2x - 4}}$$

2) (4x^4 + 5x – 4) / (x^2 – 3x – 2)

$$4x^2 + 12x + 44$$

$$x^2 - 3x - 2 \,\big)\, \cancel{4x^4} + 0x^3 + 0x^2 + 5x - 4$$

$$-4x^4 + 12x^3 + 8x^2$$

$$\cancel{12x^3} + 8x^2 + 5x$$

$$-\cancel{12x^3} + 36x^2 + 24x$$

$$44x^2 + 29x - 4$$

$$-44x^2 + 132x + 88$$

$$161x + \cancel{8}84$$

$$\boxed{4x^2 + 12x + 44 + \frac{161x + 84}{x^2 - 3x - 2}}$$

ANSWER KEY

Divide using synthetic division for the problems below.

1) (4x^2 – 13x – 5) / (x – 2)

$k = 2$

$$\begin{array}{r|rrr} 2 & 4 & -13 & -5 \\ & \downarrow & 8 & -10 \\ \hline & 4 & -5 & -15 \end{array}$$

$$\boxed{4x - 5 - \frac{15}{x-2}}$$

2) (x^4 – 5x^3 – 8x^2 + 13x – 12) / (x – 6)

$k = 6$

$$\begin{array}{r|rrrrr} 6 & 1 & -5 & -8 & 13 & -12 \\ & \downarrow & 6 & 6 & -12 & 6 \\ \hline & 1 & 1 & -2 & 1 & -6 \end{array}$$

$$\boxed{x^3 + x^2 - 2x + 1 - \frac{6}{x-6}}$$

ANSWER KEY

Use synthetic division to evaluate the function for the indicated value of x.

1) f(x) = 3x^2 + 2x – 20; x = 3

$$\begin{array}{r|rrr} 3 & 3 & 2 & -20 \\ & \downarrow & 9 & 33 \\ \hline & 3 & 11 & 13 \end{array}$$

$$\boxed{f(3) = 13}$$

2) f(x) = -x^4 – x^3 – 2; x = 5

$$\begin{array}{r|rrrrr} 5 & -1 & -1 & 0 & 0 & -2 \\ & \downarrow & -5 & -30 & -150 & -750 \\ \hline & -1 & -6 & -30 & -150 & -752 \end{array}$$

$$\boxed{f(5) = -752}$$