Name ________________________ Date______________

**Final Review**

**Section 4.3: Long Division, Synthetic Division, Remainder Theorem**

**Overview:** Division can be used to determine whether a polynomial is a factor of another polynomial. Long division shows the step by step process as to how the smaller polynomial goes into the larger one, while synthetic division is a simplified version. When you divide polynomials and are left with a remainder, you add that to the end of your answer with the remainder as a numerator and the divisor as a denominator.

$$\text{Divisor}\,\overline{\big)\,\text{Dividend}}^{\;\text{Quotient}}$$

**Remainder Theorem:** If a polynomial $f(x)$ is divided by $x-k$, then the remainder is $r=f(k)$

The remainder theorem tells you that synthetic division can be used to evaluate a polynomial function. So, to evaluate $f(x)$ when $x=k$, divide $f(x)$ by $x-k$. The remainder will be $f(k)$.

**Sample Problem:** Divide $x^3+3x^2-4x-12$ by $x^2+x-6$ using long division.

$$x^2+x-6\overline{\big)\,x^3+3x^2-4x-12}$$

Step 1: Write the divisor and dividend in correct location of the division bracket. Make sure that the polynomials are written in decreasing order. If any terms are missing, fill in spot with zero.

$$\begin{array}{r} x \\ x^2+x-6\overline{\big)\,x^3+3x^2-4x-12} \\ \underline{x^3+x^2-6x\phantom{-12}} \\ 2x^2+2x-12 \end{array}$$

Step 2: Divide the term with the highest power inside the division symbol ($x^3$) by the term with the highest power outside the division symbol ($x^2$). Distribute the answer (x) to the remainder of the outside polynomial. Subtract.

$$\begin{array}{r} x+2 \\ x^2+x-6\overline{\big)\,x^3+3x^2-4x-12} \\ \underline{x^3+x^2-6x\phantom{-12}} \\ 2x^2+2x-12 \\ \underline{2x^2+2x-12} \\ 0 \end{array}$$

Step 3: Repeat step 2 using 2x2+2x-12 as your dividend.

$x+2$

Step 4: Write your final answer. In this case, there is no remainder.

**Practice Problems:** Divide the following using long division.

1) $2x^3+4x^2-30x \div x-3$

2) $x^3-4x^2+2x+5 \div x-2$

3) $12x^3+11x^2+9x+18 \div 4x+3$

4) $4x^3-2x^2-3 \div 2x^2-1$

5) $2x^4+13x^3+15x^2 \div 2x+3$

6) $-4x^3-21x^2+46x-21$

7) $64x^6-100x^4-144x^2+225 \div 4x+5$

8) $2x^3+4x^2-30x \div x-3$

**Sample Problem:** Divide $3x^3-2x^2+3x-4$ by $x-3$ using synthetic division.

$$\begin{array}{r|rrrr} 3 & 3 & -2 & 3 & -4 \\ & & & & \\ \hline & 3 & & & \end{array}$$

<u>Step 1:</u> write down all the coefficients, and put the zero from $x-3=0$ (so $x=3$) at the left. Carry down the leading coefficient. If any terms are missing, fill in spot with zero.

$$\begin{array}{r|rrrr} 3 & 3 & -2 & 3 & -4 \\ & & 9 & & \\ \hline & 3 & 7 & & \end{array}$$

$$\begin{array}{r|rrrr} 3 & 3 & -2 & 3 & -4 \\ & & 9 & 21 & \\ \hline & 3 & 7 & 24 & \end{array}$$

<u>Step 2:</u> Multiply the coefficient which you carried down (3) by the potential zero (3). Carry up to the next column, add down. Repeat with all coefficients.

$$\begin{array}{r|rrrr} 3 & 3 & -2 & 3 & -4 \\ & & 9 & 21 & 72 \\ \hline & 3 & 7 & 24 & 68 \end{array}$$

$$3x^2+7x+24+\frac{68}{x-3}$$

<u>Step 3:</u> Your answer (in the bottom row) will be of degree one less than what you'd started with, because you have divided out a linear factor. If you have a number left over after the ones place (24 is our ones place, 68 came after it), this number is your remainder. When writing the final answer, you write the remainder as a fraction: the remainder as a numerator, and the dividend as the denominator.

**Practice Problems:** Divide the following using synthetic division.

1) $x^3+5 \div x+2$

2) $1 - x + x^2 + x^3 \div x-2$

3) $2x^3 + 3x^2 - 2x - 1 \div x+1$

4) $2x^3-5x-7 \div x-2$

5) $x^3-4x^2+2x+5 \div x-2$

6) $2x^3+4x^2-30x \div x-3$