Edward Watson
Period 2

Long Division, Synthetic Division, and The Remainder Theorem

4.3

"I don't like long division; I always feel bad for the remainders."

Long Division of Polynomials

Divide $2x^4 + 3x + 5x - 1$ by $x^2 + 3x + 2$

The quotient is composed of the terms that you multiply by.

$2x^2 - 3x + 5$ ← **quotient**

$x^2 + 3x + 2 \,\overline{)\, 2x^4 + 3x^3 + 0x^2 + 5x - 1}$

$\underline{2x^4 + 6x^3 + 4x^2}$ ----------------- Multiply the divisor by $2x^2$ ($2x^4/x^2$).

$- 3x^3 - 4x^2 + 5x$ ---------- Subtract the polynomials. Bring down the next term.

$\underline{- 3x^3 - 9x^2 - 6x}$ --------- Multiply the divisor by $-3x$ ($-3x^3/x^2$).

$5x^2 + 11x - 1$ ----- Subtract the polynomials. Bring down the next term.

$\underline{5x^2 + 15x + 10}$ ----- Multiply the divisor by 5 ($5x^2/x^2$).

$- 4x - 11$ ← **remainder**

Let's try some examples…

1) Divide $x^3 - 4x^2 + 2x + 5$ by $x - 2$

2) Divide $12x^3 - 11x^2 + 9x + 18$ by $4x + 3$

3) $\dfrac{3x^3 - 5x^2 + 10x - 3}{3x + 1}$

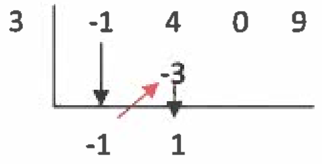
4) $\dfrac{64x^6 - 100x^4 - 144x^2 + 225}{4x + 5}$

Synthetic Division of Polynomials

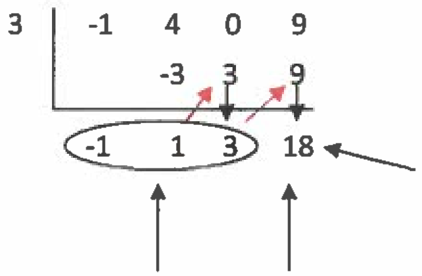
Divide $-x^3 + 4x^2 + 9$ by $x - 3$

$$\begin{array}{r|rrrr} 3 & -1 & 4 & 0 & 9 \\ & & & & \\ \hline \end{array}$$

Write the coefficients of the dividend in order of decreasing exponents. If an exponent is missing, write a "0" as the coefficient. With x – 3 as the divisor, k = 3 should be used and is placed to the left of the bar.



Bring down the leading coefficient. **Multiply** the leading coefficient by the k-value. Write the product under the second coefficient. **Add.**



Multiply the previous sum by the k-value. Write the value under the next coefficient, **add**, and then repeat for all remaining coefficients.

Remainder

$-x^2 + x + 3 + \frac{18}{x-3}$

The first three numbers in the bottom row are now used as the coefficients of the quotient. The last number is the remainder and is set over the original divisor.

Example Problems:

1) Divide $2x^3 + 5x^2 + 9$ by $x + 3$

2) Divide $5x^3 - 6x^2 + 3x + 14$ by $x + 1$

3) $\frac{x^4 + 2x^3 + x^2 + 5x + 6}{x + 2}$

4) $\frac{x^5 - 3x^3 - 4x - 1}{x - 1}$

Remainder Theorem

The Remainder Theorem is part of the synthetic division process. It allows you to evaluate a polynomial function.

Use synthetic division to evaluate $f(x) = 5x^3 - x^2 + 13x + 29$ when $x = -4$.

$$\begin{array}{r|rrrr} -4 & 5 & -1 & 13 & 29 \\ & & -20 & 84 & -388 \\ \hline & 5 & -21 & 97 & \boxed{-359} \end{array}$$

The remainder in this example is -359. Based on this, we can conclude that $f(-4) = -359$.

How about some practice…

1) Evaluate $f(x) = 4x^3 + x^2 - 2x + 3$ when $x = 1$

2) Use the Remainder Theorem to determine $f(-2)$ given that $f(x) = -3x^5 + x^3 - 2x^2 + x - 3$

Answer Key

Long Division

1) $$\begin{array}{r} x^2-2x-2 \\ x-2\overline{)x^3-4x^2+2x+5} \\ \underline{x^3-2x^2} \\ -2x^2+2x \\ \underline{-2x^2+4x} \\ -2x+5 \\ \underline{-2x+4} \\ 1 \end{array}$$

$$\boxed{x^2-2x-2+\frac{1}{x-2}}$$

2) $$\begin{array}{r} 3x^2-5x+6 \\ 4x+3\overline{)12x^3-11x^2+9x+18} \\ \underline{12x^3+9x^2} \\ -20x^2+9x \\ \underline{-20x^2-15x} \\ 24x+18 \\ \underline{24x+18} \\ 0 \end{array}$$

$$\boxed{3x^2-5x+6}$$

3) $$\begin{array}{r} x^2-2x+4 \\ 3x+1\overline{)3x^3-5x^2+10x-3} \\ \underline{3x^3+1x^2} \\ -6x^2+10x \\ \underline{-6x^2-2x} \\ 12x-3 \\ \underline{12x+4} \\ -7 \end{array}$$

$$\boxed{x^2-2x+4-\frac{7}{3x+1}}$$

4) $4x+5\overline{)64x^6-100x^4-144x^2+225}$

$$\begin{array}{r} 16x^5-20x^4+0+0-36x+45 \\ 4x+5\overline{)64x^6+0x^5-100x^4+0x^3-144x^2+0x+225} \\ \underline{64x^6+80x^5} \\ -80x^5-100x^4 \\ \underline{-80x^5-100x^4} \\ 0+0x^3 \\ \underline{0+0} \\ 0-144x^2 \\ \underline{0+0} \\ -144x^2+0x \\ \underline{-144x^2-180x} \\ 180x+225 \\ \underline{180x+225} \\ 0 \end{array}$$

$$\boxed{16x^5-20x^4-36x+45}$$

Synthetic Division

1)
$$\begin{array}{r|rrrr} -3 & 2 & 5 & 0 & 9 \\ & & -6 & 3 & -9 \\ \hline & 2 & -1 & 3 & 0 \end{array}$$

$$\boxed{2x^2-1x+3}$$

2)
$$\begin{array}{r|rrrr} -1 & 5 & -6 & 3 & 14 \\ & & -5 & 11 & -14 \\ \hline & 5 & -11 & 14 & 0 \end{array}$$

$$\boxed{5x^2-11x+14}$$

3)
$$\begin{array}{r|rrrrr} -2 & 1 & 2 & 1 & 5 & 6 \\ & & -2 & 0 & -2 & -6 \\ \hline & 1 & 0 & 1 & 3 & 0 \end{array}$$

$$\boxed{x^3+x+3}$$

4)
$$\begin{array}{r|rrrrrr} 1 & 1 & 0 & -3 & 0 & -4 & -1 \\ & & 1 & 1 & -2 & -2 & -6 \\ \hline & 1 & 1 & -2 & -2 & -6 & -7 \end{array}$$

$$\boxed{x^4+x^3-2x^2-2x-6-\frac{7}{x-1}}$$

Remainder Theorem

1)
$$\begin{array}{r|rrrr} 1 & 4 & 1 & -2 & 3 \\ & & 4 & 5 & 3 \\ \hline & 4 & 5 & 3 & 6 \end{array}$$

$$\boxed{f(1)=6}$$

2)
$$\begin{array}{r|rrrrrr} -2 & -3 & 0 & 1 & -2 & 1 & -3 \\ & & 6 & -12 & 22 & -40 & 78 \\ \hline & -3 & 6 & -11 & 20 & -39 & 75 \end{array}$$

$$\boxed{f(-2)=75}$$