Hannah Fuchs
A2CC Honors
Extra Credit

End Behavior and Graphing Polynomial Expressions

The goal: Learn the concept end behavior regarding polynomial functions and learn how to graph different types of polynomial expressions

The Basics: A polynomial function has the form

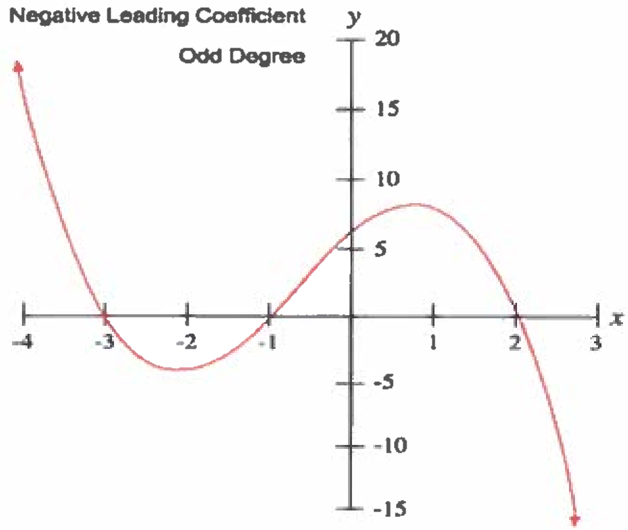
$f(x) = a_n x^n = a_{n-1} x^n - 1 + \ldots + a_1 x + a_0$, where

$a_n, a_{n-1}, \ldots, a_0$ are real numbers and n is a nonnegative integer. (defined by chegg.com) Also note that **N** represents the degree of the function.

End Behavior: What does that mean?

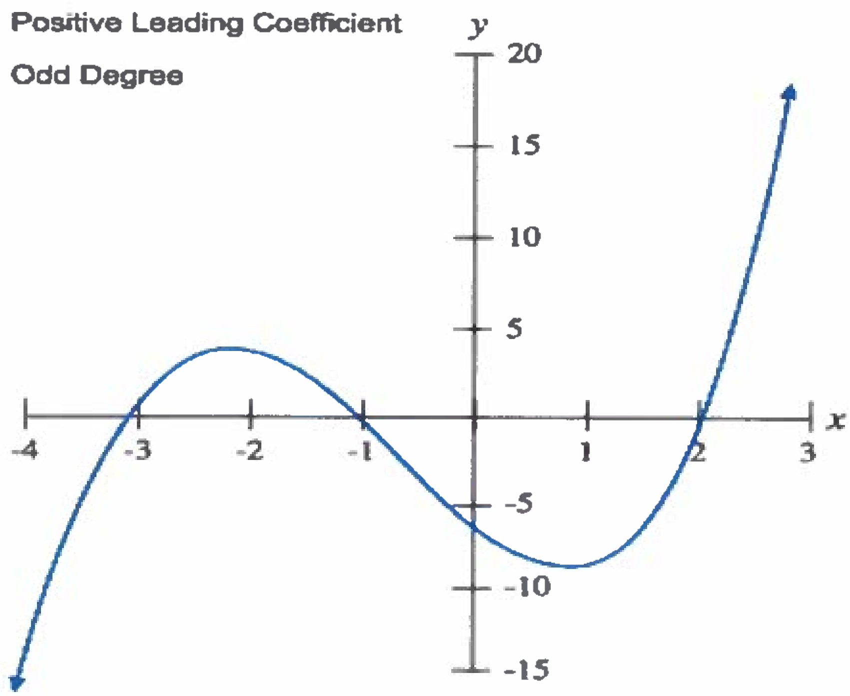
-The end behavior of a function is what happens to its graph as the x-values approach both positive and negative infinity.
-End behavior is determined by the function's degree and whether the leading coefficient is a positive or negative number.
NOTE: "f(x)" is another way of writing "y"
-"x→∞" is another way of saying "as x approaches positive infinity"

An example/Model problem:

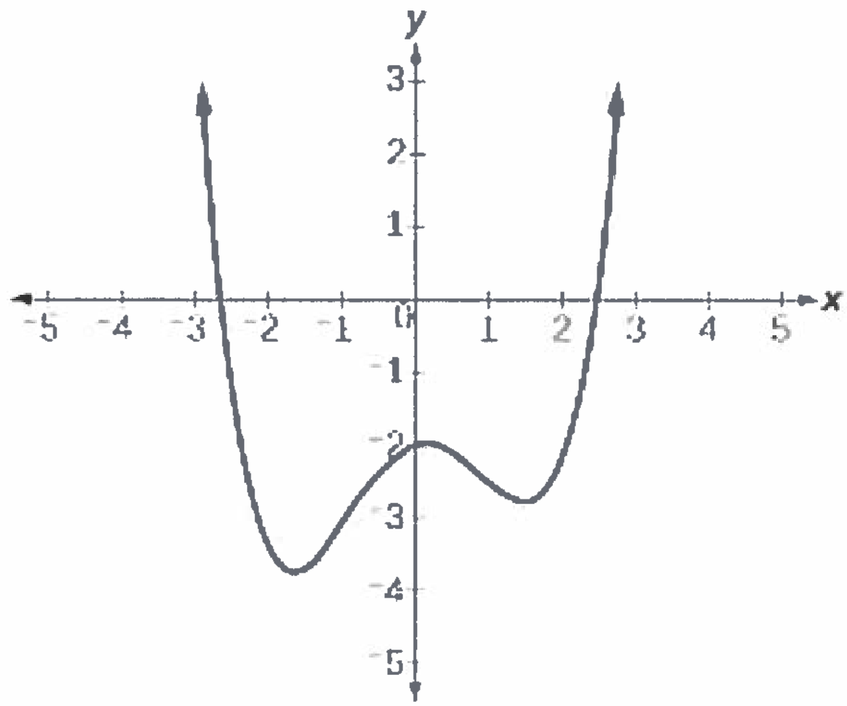


The function graphed above has an odd degree and a negative leading coefficient.
Therefore, we can say that:
as x approaches positive infinity, f(x) approaches negative infinity and
as x approaches negative infinity, f(x) approaches positive infinity.
This is the method we use to write and determine the end behavior of polynomial functions.

Try it yourself!



PROBLEM !: Degree: Odd
Leading Coefficient: Positive
End Behavior:
As $x \to \infty$, $f(x) \to ?$
As $x \to -\infty$, $f(x) \to ?$



PROBLEM 2: Degree: Even
Leading Coefficient: Positive
End Behavior:
As x→∞, f(x)→?
As x→-∞, f(x)→?

How to graph polynomial functions:

Step one: When given the equation to a polynomial function, make a table of values. (Your calculator can help you!)

Step Two: Plot these points to determine the general shape of the function's graph.

Step three: Connect the points with a smooth curve to complete the graph.

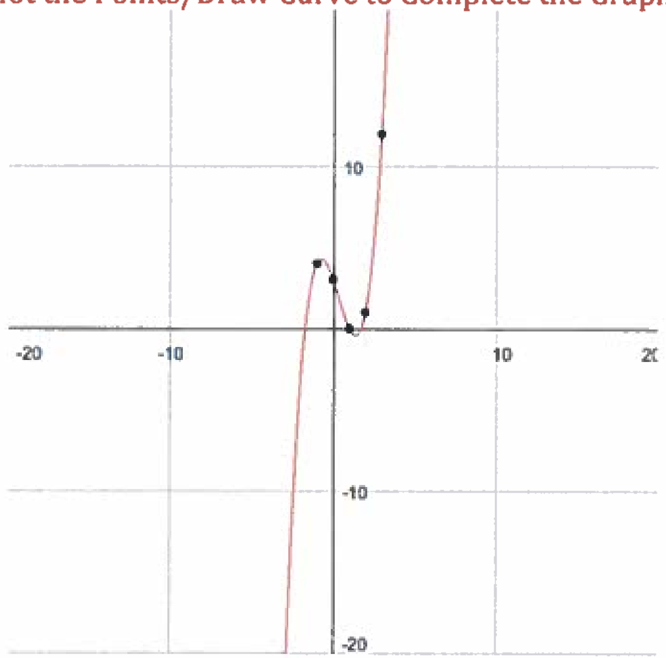
Step four: Check the end behavior of the graph and write it down like we learned above.

Example: Graph the function f(x)= $x^3 - x^2 - 3x + 3$

Table of Values:

x	y
-1	4
0	3
1	0
2	1
3	12

Plot the Points/Draw Curve to Complete the Graph:



End Behavior:
As $x \to \infty$, $f(x) \to \infty$
As $x \to -\infty$, $f(x) \to -\infty$

Sometimes, you may be given information regarding a polynomial function without its equation and be asked to sketch its graph and/or describe the degree and leading coefficient of the function. In many cases, *f* represents the function.

Let's look at an example and break it down:

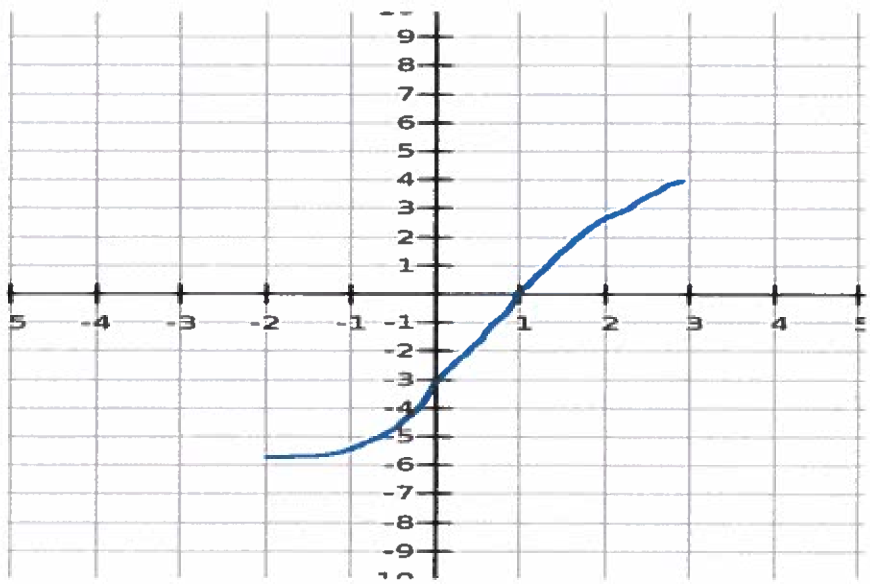
(Question 38 from Chapter 4.1 Exercises):

- "*f* is increasing when $-2 < x < 3$; *f* is decreasing when $x < -2$ and $x > 3$."
- "$f(x) > 0$ when $x < -4$ and $1 < x < 5$; $f(x) < 0$ when $-4 < x < 1$ and $x > 5$."

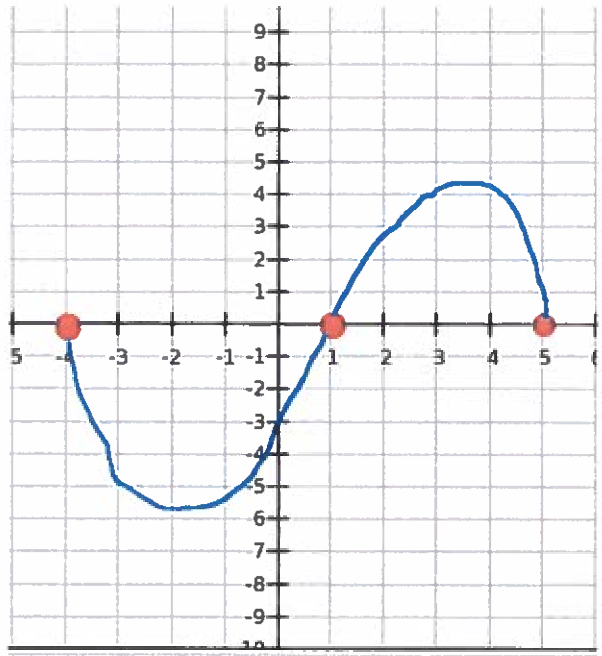
So where should we start?

- Well, we know that our function's y-values are increasing between the x-values of -2 and 3.
- We also know that our function's y-values are decreasing when the x-values are less than -2 and greater than 3.
- The second bullet point of the problem tells us that the graph is above the x-axis (greater than zero) for the given intervals, so when x is greater than -4 and when the x-value of the function is in between 1 and 5.
- It also tells us that the graph is below the x-axis (less than zero) between the x-values of -4 and 1 and when x is greater than 5.
- Now, let's start to sketch our graph.

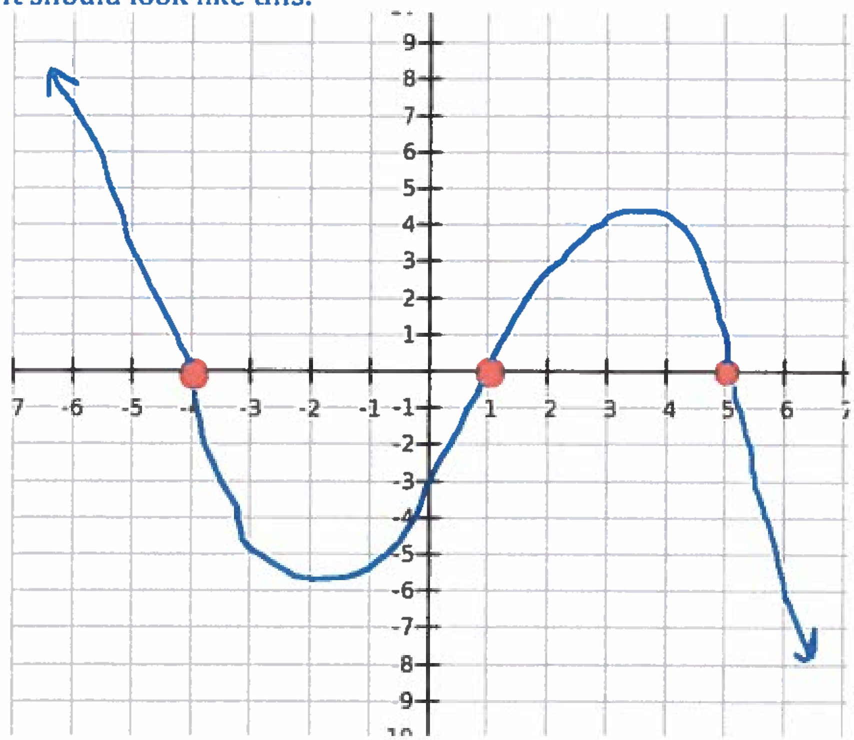
First, we draw a sketch indicating that *f* is increasing between x=-2 and x=3.



The next easiest step is to locate our zeroes. We determine the zeroes by seeing at which x-values f(x) is greater than zero and the x-values where f(x) are less than zero. In this case, we have zeros at -4, 1 and 5. After determining the zeroes, connect between them, following the instructions (seeing if you connect above or below the x-axis).



Finish up the graph by continuing onwards where the graph increases and decreases. It should look like this:



The problem goes on to ask us to use our graph to describe the degree and leading coefficient of the problem. Judging by our previous knowledge, it is clear that:

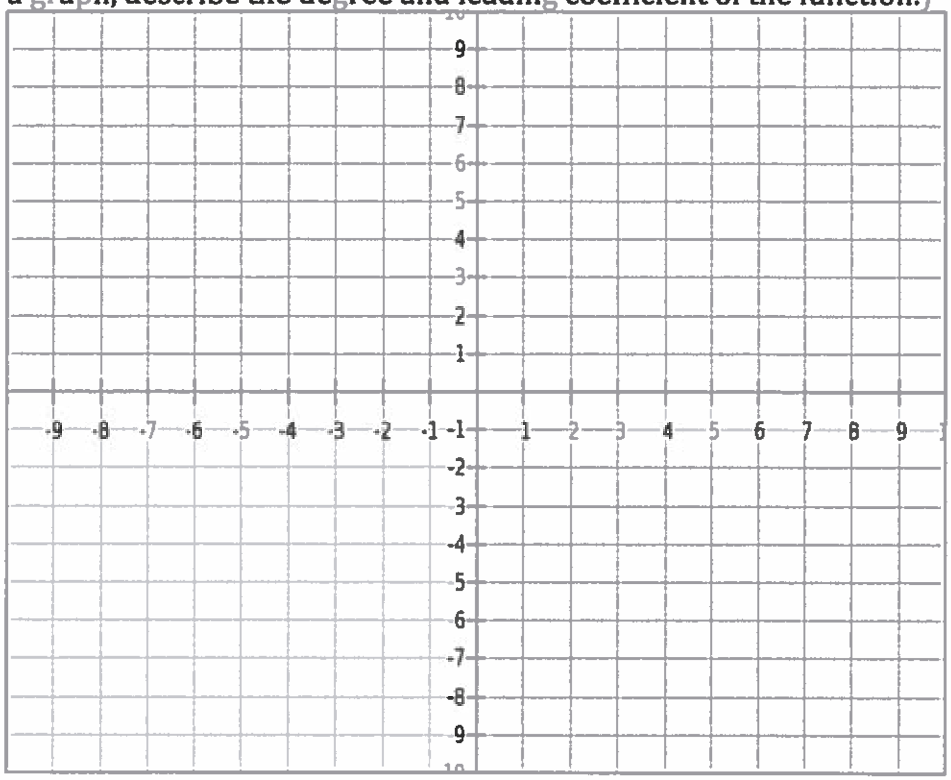
Degree: ODD (we also know this because there are 3 zeroes)
Leading Coefficient: NEGATIVE (determine this by looking at the end behavior of the function like we learned earlier!)

Need more practice?

Question 37:

- *f* is increasing when x > 0.5 ; *f* is decreasing when x < 0.5
- *f(x)* > 0 when x < -2 and x > 3 ; *f(x)* < 0 when -2 < x < 3

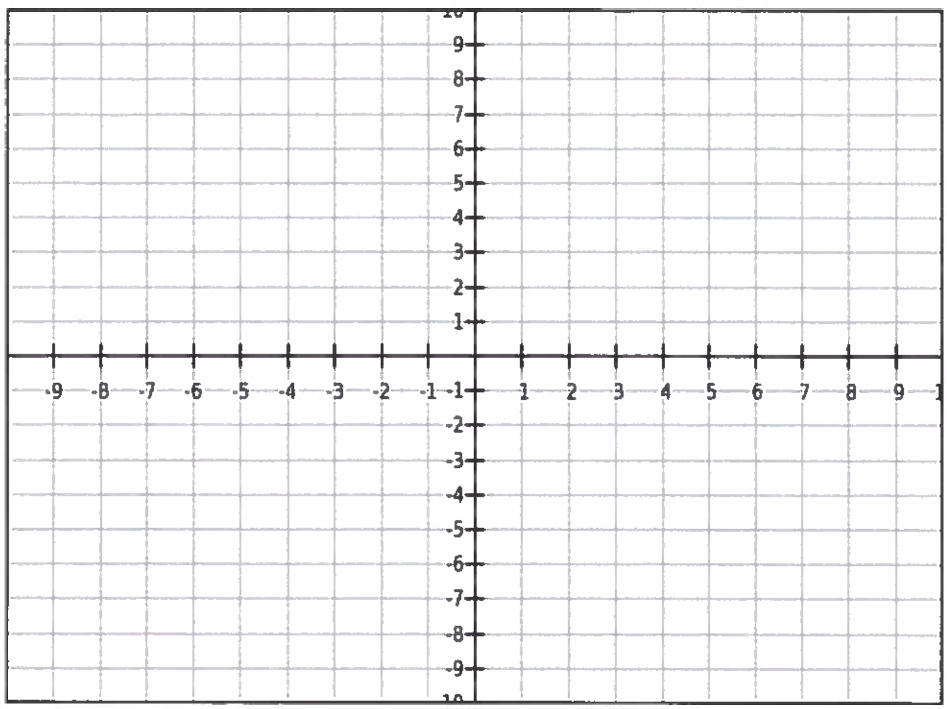
(Sketch a graph, describe the degree and leading coefficient of the function.)



Question 40:

- *f* is increasing when x < -1 and x > 1 ; *f* is decreasing when -1 < x < 1
- *f(x)* > 0 when -1.5 < x < 0 and x > 1.5 ; *f(x)* < 0 when x < -1.5 and 0 < x < 1.5

(Sketch a graph, describe the degree and leading coefficient of the function.)



What's the deal with Multiplicity?

The textbook defines multiplicity as a "repeated solution", meaning that when the zeroes of an equation are determined, some may repeat more than once. Say when you factor out an equation, you get something like:

$$2x(x-4)^2 = 0$$

We notice that the factor x-4 appears more than once. Therefore, our repeated solution is x = 4. When this function is graphed, it touches the x-axis at 4 without crossing it, almost looking as if it has been flattened.

Since the factor of x-4 has been raised to the power of 2, we can say that it has a multiplicity of 2, meaning this zero repeats 2 times.

In this case, since the factor is raised to an even power, it does not **cross the x-axis. However, if the factor is raised to an odd power, it** does **cross the x-axis.**

Try it yourself!

PROBLEM !: **Give the zeros of polynomial P and list their multiplicities :** $P(x) = (x + 3)^3(x - 5)(-x + 5)$.

PROBLEM 2: **Give the zeros of polynomial P and list their multiplicities :** $P(x) = (x - 2)(x^2 + 4)^2$.

ANSWER KEY:

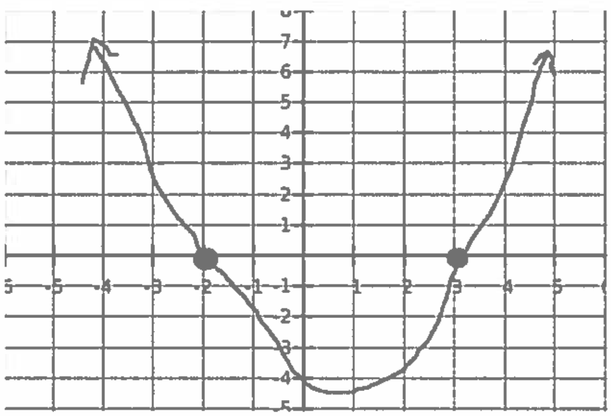
[redacted]

PROBLEM 1: As $x \to \infty$ $f(x) \to \infty$

PROBLEM 2: As $x \to -\infty$ $f(x) \to -\infty$

[redacted]

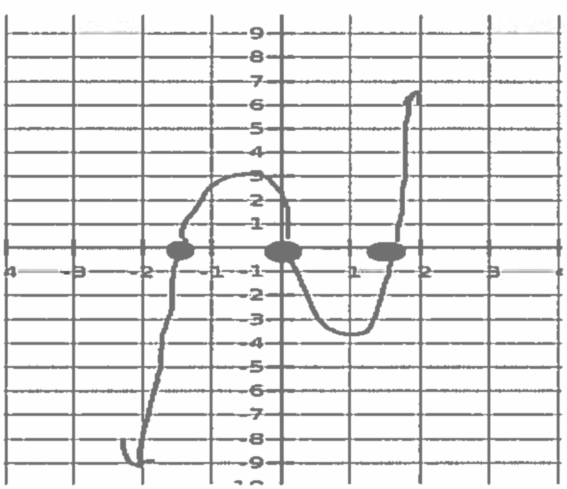
QUESTION 37:



DEGREE: EVEN

LEADING COEFFICIENT: POSITIVE

QUESTION 40:



DEGREE: ODD

LEADING COEFFICIENT: POSITIVE

[redacted]

Problem 1:

- x = -3 with a multiplicity of 3
- x = 5 with a multiplicity of 1
- x = -5 with a multiplicity of 1

Problem 2:

- x = 2 with a multiplicity of 1
- x = 2i with a multiplicity of 2
- x = -2i with a multiplicity of 2