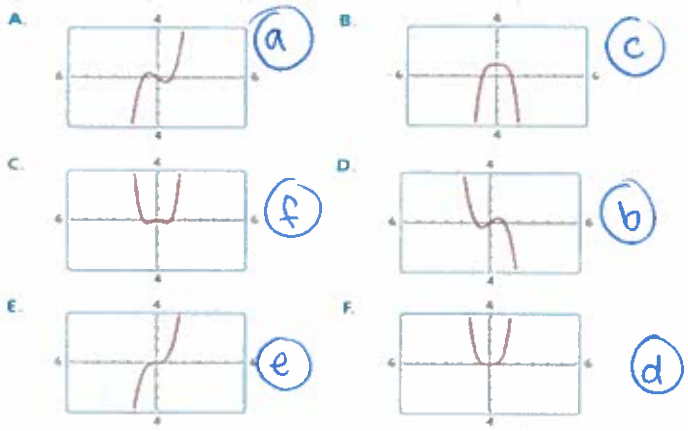
Key

Section 4.1: Graphing Polynomial Functions

SWBAT: identify polynomial functions & graph polynomial functions using tables & end behavior

- The end behavior of a function is the behavior of the graph as x approaches positive infinity or negative infinity.
- The degree of a function is the exponent of the first term; it helps determine the end behavior.
- The leading coefficient is the number prior to the leading term, for example in the term $2x^2$ the leading coefficient is 2.

Exercise 1: Match each polynomial function with its graph



a. $f(x) = x^3 - x$
b. $f(x) = -x^3 + x$
c. $f(x) = -x^4 + 1$
d. $f(x) = x^4$
e. $f(x) = x^3$
f. $f(x) = x^4 - x^2$

Exercise 2: Decide whether or not each function is a polynomial function. If so, write the degree, type, and leading coefficient.

a. $f(x) = -2x^3 + 5x + 8$

Yes — degree = 3 (cubic), leading coefficient = −2

b. $g(x) = -0.8x^3 + \sqrt{2x^4} - 12$

Yes — degree = 4 (quartic), leading coefficient = $\sqrt{2}$

c. $h(x) = -x^2 + 7x^{-1} + 4x$

No — exponent that's not positive/whole

d. $k(x) = x^2 + 3^x$

No — exponent that's not a whole number

Exercise 3: Describe the end behavior of the following functions

a. $f(x) = -x^3 + 3x^2 + 9$

$f(x) \to -\infty, x \to \infty$

$f(x) \to \infty, x \to -\infty$

b. $f(x) = 3x^5 - x^4 - 6x + 10$

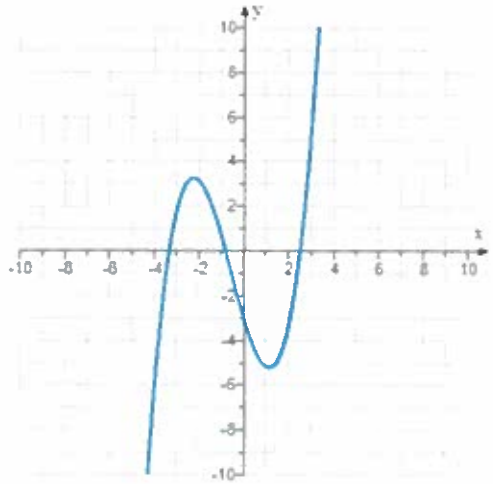
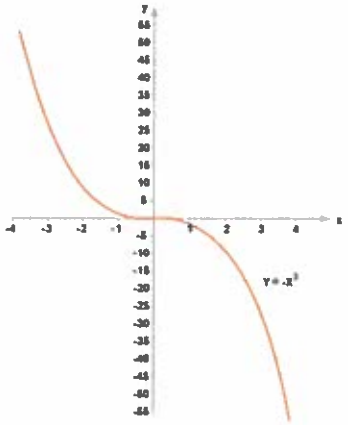
$f(x) \to -\infty, x \to -\infty$

$f(x) \to \infty, x \to \infty$

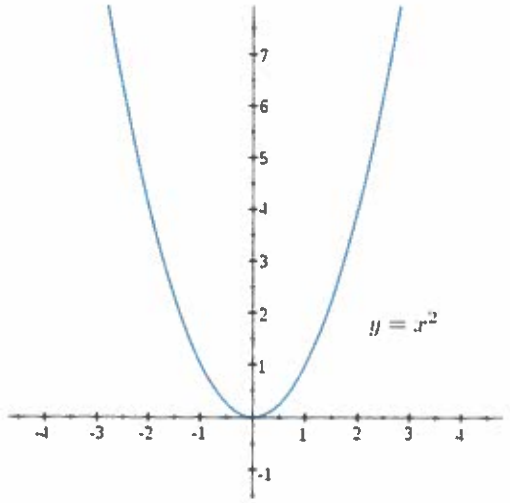
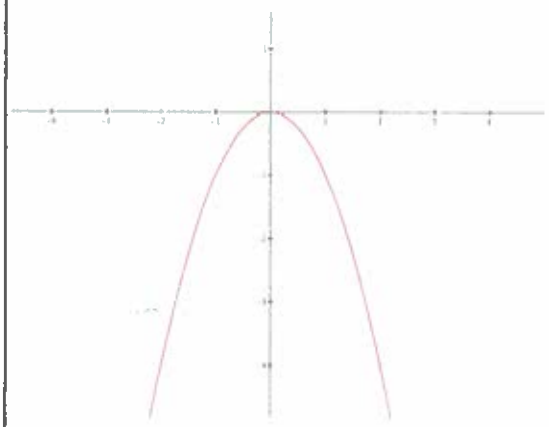
c. $f(x) = 0.25x^3 - x^2 + 10$

$f(x) \to -\infty, x \to -\infty$

$f(x) \to \infty, x \to \infty$

Degree: Odd Leading coefficient: Positive	Degree: Odd Leading Coefficient: Negative
	

*odd degree always has opposite end behaviors as $f(x) \to \infty$ and $f(x) \to -\infty$ but when the degree is EVEN the end behavior is the same in both directions (think parabola)

Degree: Even Leading Coefficient: Positive	Degree: Even Leading Coefficient: Negative
	

Exercise 4: Evaluate the function for the given value of x.

a. $f(x) = -x^3 + 3x^2 + 9, x = 4$

$-(4)^3 + 3(4)^2 + 9$

$-64 + 48 + 9$

$-64 + 57$

$f(4) = -7$

b. $f(x) = 3x^5 - x^4 - 6x + 10, x = -2$

$3(-2)^5 - (-2)^4 - 6(-2) + 10$

$-96 - 16 + 12 + 10$

$f(-2) = -90$

c. $f(x) = 0.25x^3 - x^2 - 1, x = 4$

$f(4) = 0.25(4)^3 - (4)^2 - 1$

$16 - 16 - 1$

$0 - 1$

$f(4) = -1$

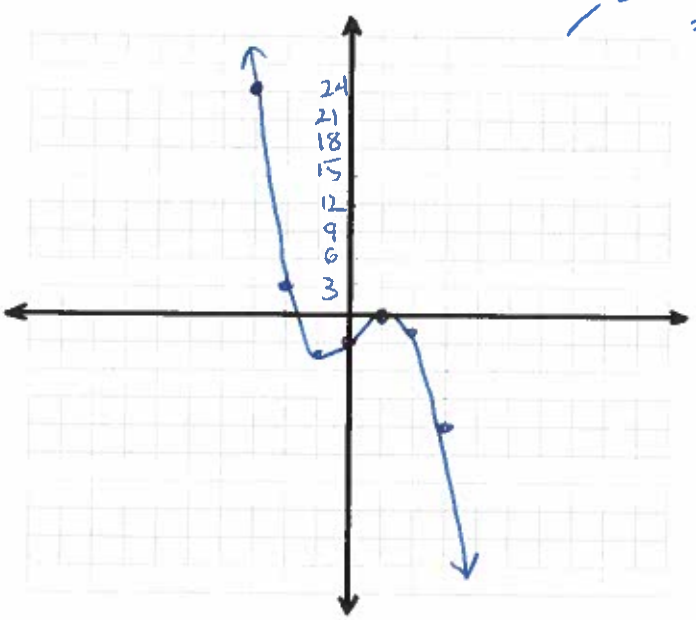
Exercise 4: Graph the function $f(x) = -x^3 + x^2 + 3x - 3$

x	-3	-2	-1	0	1	2	3
$f(x)$	24	3	-4	-3	0	-1	-12

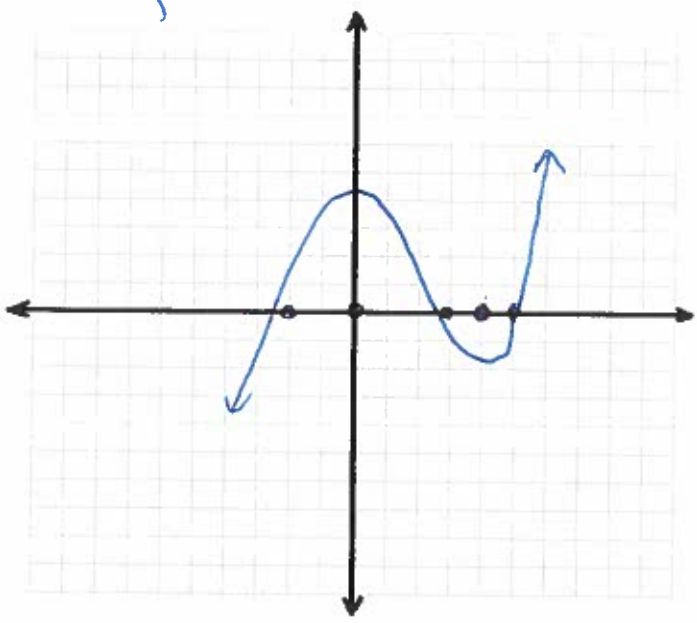
Exercise 5: Sketch the graph of the polynomial function that

- f is increasing when $x < 0$ and $x > 4$
- f is decreasing when $0 < x < 4$
- $f(x) > 0$ when $-2 < x < 3$ and $x > 5$
- $f(x) < 0$ when $x < -2$ and $3 < x < 5$

Exercise #4



Exercise #5



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Exercise 6: The estimated number V (in thousands) of electric vehicles used in the United States can be modeled by the function:

~~$V(t) = 0.151280t^3 + 23.7565t - 2.041$~~

$V(t) = 0.151280t^3 + 3.28234t^2 + 23.7565t - 2.041$

a. Graph the function for the interval $1 \leq t \leq 10$, and describe the behavior of the function over this interval.

2001 - 2004 → increasing

2005 → growth slowed • leveled off

2009 - 2010 → increasing

b. What was the average rate of change from 2001 to 2010?

$$\frac{58.57 - 18.5844}{9} \approx 4.443$$

Mixed Review:

1. Write the following function in standard form, give its degree, type, and leading coefficient: $f(x) = -3x + 5x^3 - 6x^2 + 2$

$f(x) = 5x^3 - 6x^2 - 3x + 2$

degree = 3 (cubic)

leading coefficient = 5

2. Evaluate the function for the given value of x: $g(x) = -x^3 + 3x^2 + 5x + 1, x = -12$

$-(-12)^3 + 3(-12)^2 + 5(-12) + 1$

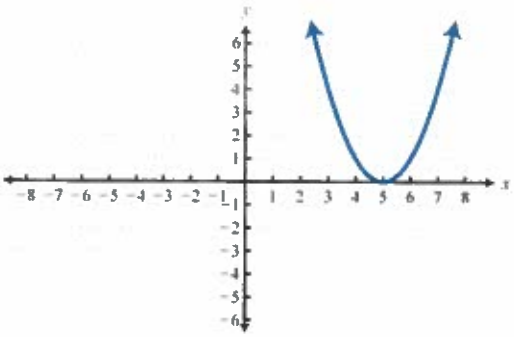
$1728 + 432 - 60 + 1$

$f(12) = 2101$

3. Describe the end behavior of the function: $h(x) = -5x^4 + 7x^3 - 6x^2 - 2x + 2$.

$f(x) \to -\infty, x \to -\infty$

$f(x) \to \infty, x \to -\infty$

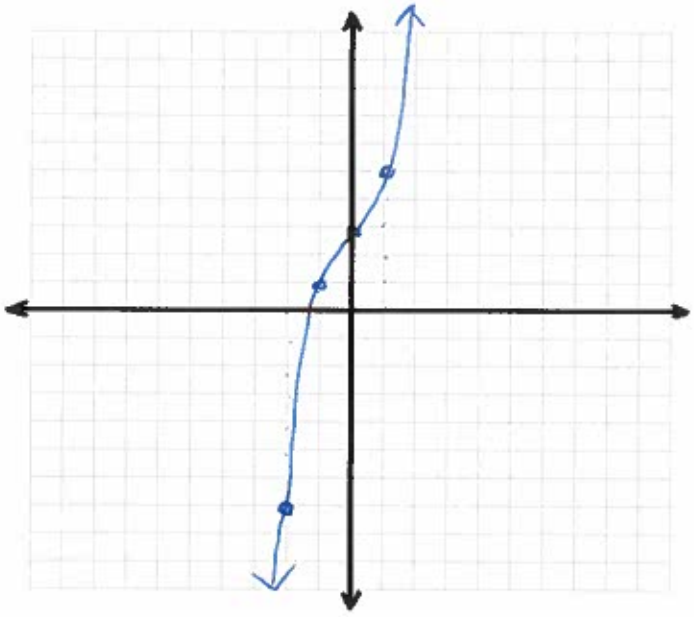


f is increasing $x > 5$

f is decreasing $x < 5$

$f \geq 0$ ALWAYS

5. Graph the following function: $p(x) = x^3 + x + 3$



-2	-1	0	1	2
-7	1	3	5	13

6. Graph the function $d(t) = -0.141t^3 + 9.64t^2 - 232.5t + 2421$ over the interval $0 \leq t \leq 27$ in your calculator. Describe the behavior of the graph for this interval.

Decreasing

* Use Table in calc