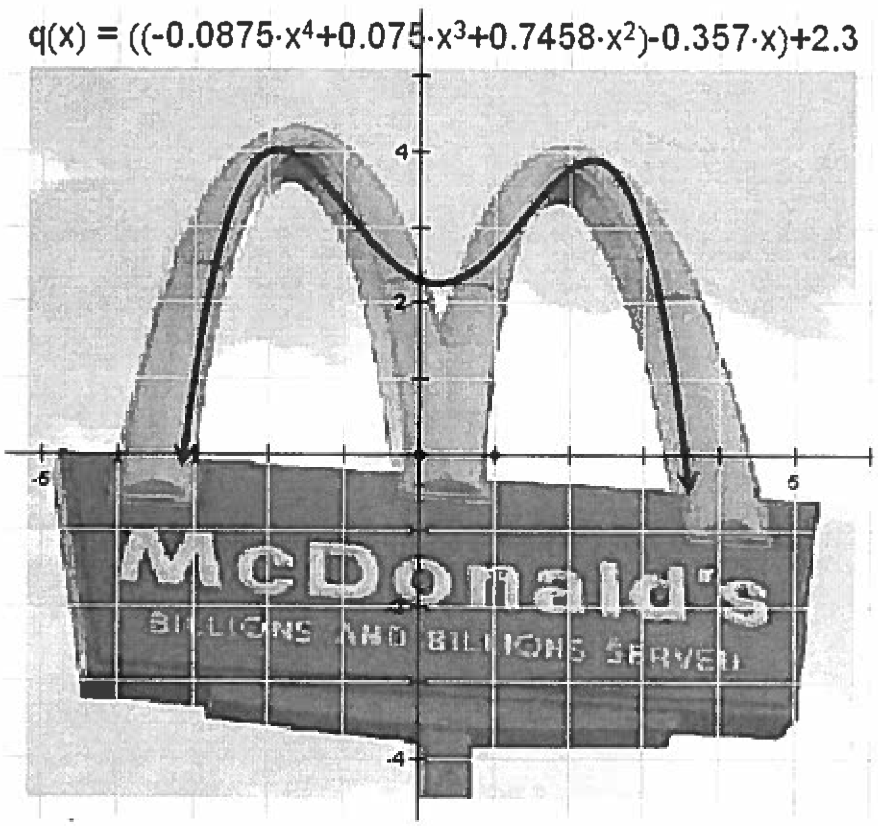
Alexandros Kerwick

Graphing Polynomial Functions

Lesson 4.1



A polynomial function is of the form

$f(x)= a_n x^n + a_{n-1} x^{n-1} + \ldots + a_1 x + a_0$

a_n is not equal to 0

all exponents are whole numbers

coefficients are real numbers

In this function a_n is the leading coefficient

n is the degree

a_0 is the constant term

A polynomial function is in standard for when written in descending order of exponents left to right

Common types of polynomial functions:

Common Polynomial Functions			
Degree	**Type**	**Standard Form**	**Example**
0	Constant	$f(x) = a_0$	$f(x) = -14$
1	Linear	$f(x) = a_1x + a_0$	$f(x) = 5x - 7$
2	Quadratic	$f(x) = a_2x^2 + a_1x + a_0$	$f(x) = 2x^2 + x - 9$
3	Cubic	$f(x) = a_3x^3 + a_2x^2 + a_1x + a_0$	$f(x) = x^3 - x^2 + 3x$
4	Quartic	$f(x) = a_4x^4 + a_3x^3 + a_2x^2 + a_1x + a_0$	$f(x) = x^4 + 2x - 1$

Example 1:

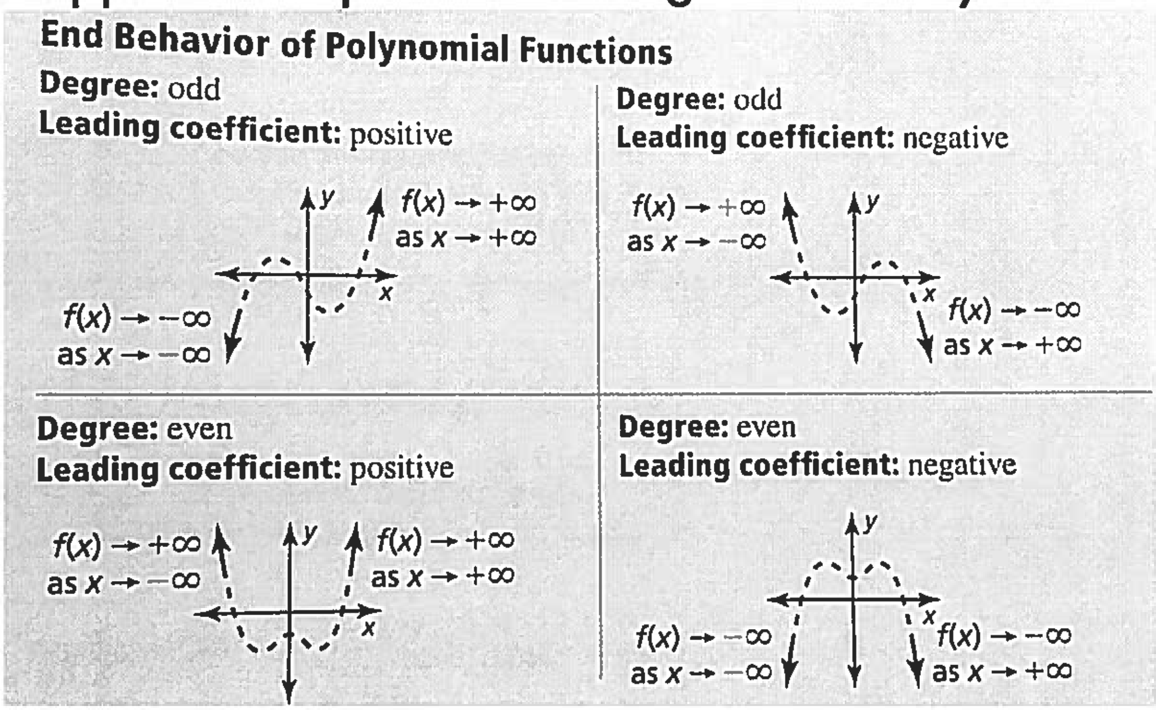
Decide if the function is a polynomial. If so, write in standard form and state degree, type and leading coefficient.

a. $f(x)= -2x^3 + 5x + 8$
b. $g(x)= -0.8x^3 + (\sqrt{2})x^4 - 12$
c. $h(x)= -x^2 + 7x^{-1} + 4x$
d. $k(x)= x^2 + 3^x$

Solutions:

a. This is a polynomial function. It is already in standard form. It has a degree of 3(cubic) and a leading coefficient of -2
b. This is a polynomial function written as $g(x)= (\sqrt{2})x^4 - 0.8x^3 - 12$ in standard form. It has a degree of 4 (quartic) and a leading coefficient of $\sqrt{2}$.
c. This is not a polynomial function because the term $7x^{-1}$ has an exponent that is not a whole number
d. This is not a polynomial function because 3^x does not have a variable base and it has a variable exponent

The end behavior is the behavior of the graph as x approaches positive or negative infinity



Example 2

Describe the end behavior of the graph of $f(x)= -0.5x^4 + 2.5x^2 +x -1$

Solution:

The degree is 4 and the leading coefficient is -0.5. Because the degree is even and the leading coefficient negative, f(x) »-∞ as x» -∞ and f(x) »-∞ as x» ∞. This can be checked on a graphing calculator.

Time for some problems!

State the degree, type and leading coefficient and write in standard form

1. $f(x) = -3x + 5x^3 - 6x^2 + 2$

2. $h(x) = (5/3)x^2 - (\sqrt{7})x^4 + 8x^3 - 1/2 + x$

3. $f(x) = 5x^3x + (5/2)x^3 - 9x^4 + (\sqrt{2})x^2 + 4x - 1 - x^{-5}x^5 - 4$

Graph the polynomial functions

1. $p(x) = 3 - x^4$
2. $f(x) = 4x - 9 - x^3$
3. $h(x) = x^4 - 2x^3 + 3x$
4. $g(x) = x^5 - 3x^4 + 2x - 4$

Describe the end behavior of each graph

1. $h(x) = -5x^4 + 7x^3 - 6x^2 + 9x + 2$
2. $f(x) = -2x^4 + 12x^8 + 17 + 15x^2$

1. $f(x) = 5x^3 - 6x^2 - 3x + 2$

degree: 3 (cubic)

leading coefficient: 5

2. $h(x) = -\sqrt{7}\,x^4 + 8x^3 + \frac{5}{3}x^2 + x - \frac{1}{2}$

degree: 4 (quartic)

leading coefficient: $-\sqrt{7}$

3. $f(x) = -4x^4 + \frac{5}{2}x^3 + \sqrt{2}\,x^2 + 4x - 6$

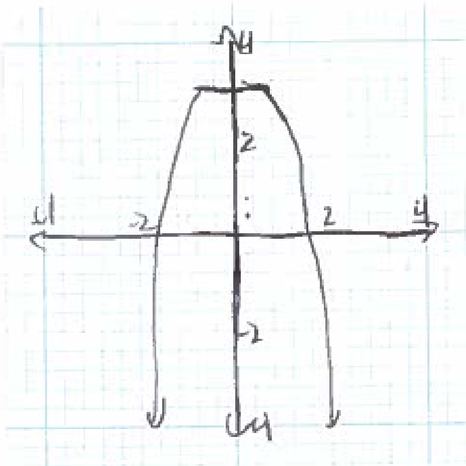
degree: 4 (quartic)

leading coefficient: -4

1. $h(x) \to -\infty$ as $x \to -\infty$ and $h(x) \to -\infty$ as $x \to \infty$

2. $f(x) \to \infty$ as $x \to -\infty$ and $f(x) \to \infty$ as $x \to \infty$

1.



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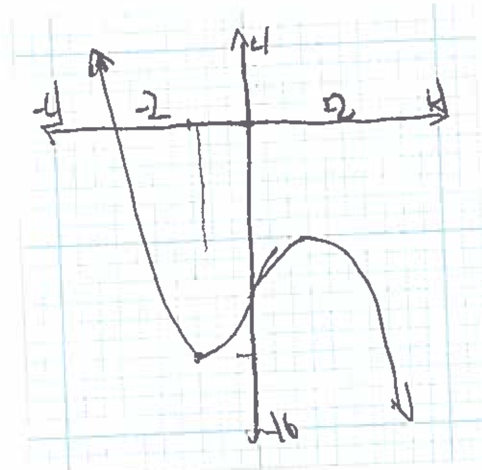

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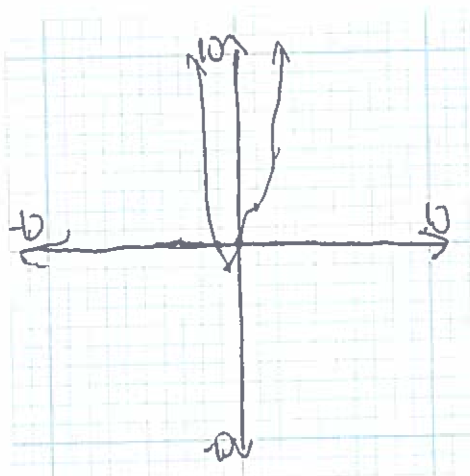

4.