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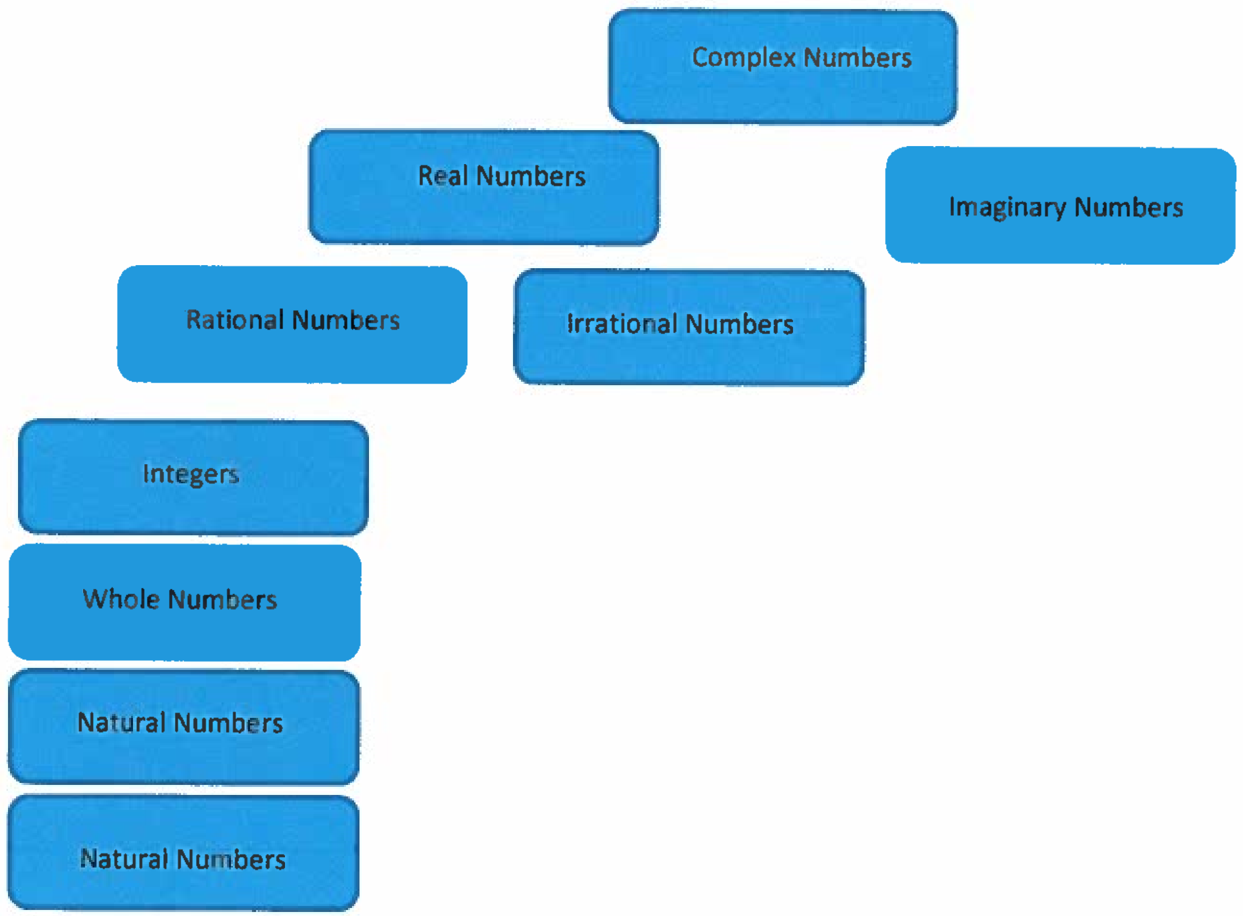
Complex Numbers

Definition: A number that can be expressed I the form of $a+bi$ where a and b are real number and i is the imaginary unit



The imaginary unit can also be defined as $\sqrt{-1}$

Example 1

Find the square root of each number

a. $\sqrt{-25}$
b. $\sqrt{-72}$
c. $-5\sqrt{-9}$

Solution

a. $\sqrt{-25} = \sqrt{25} * \sqrt{-1} = 5i$
b. $\sqrt{-72} = \sqrt{72} * \sqrt{-1} = \sqrt{36} * \sqrt{2} * i = 6\sqrt{2}\, i = 6i\sqrt{2}$
c. $-5\sqrt{-9} = -5\sqrt{9} * \sqrt{-1}$=-5*3*i=-15i

Example 2

Find the values of x and y that satisfy the equation 2x-7i=10=yi

Solution

Set the real parts equal to each other and the imaginary parts equal to each other

2x=10	7i=yi
X=5	-7=y

So, x=5 and y=-7

Example 3

Add or Subtract. Write the answer in standard for,

a. (8-i)+(5+4i)
b. (7-6i)-(3-6i)
c. 13-(2+7i)+5i

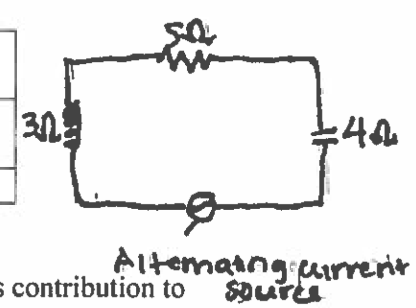
Solution

a. (8-i)=(5+4i)=(8+5)+(-1+4)i	Definition of complex addition
=13+3i	Write in standard form
b. (7-6i)-(3-6i)=(7-3)+(-6+6)I	Definition of complex subtraction
=4+0i	Simplify
=4	Write in standard form
c. 13-(2-7i)+5i=[913-20-7i]+5i	Definition of complex subtraction
= (11-7i) +5i	Simplify
=11+ (-7+5) I	Definition of complex addition
=11-2i	Write in standard form

Example 4

Electrical circuit components, such as resistors, inductors, and capacitors, all oppose the flow of current. This opposition is called resistance for resistors and reactants for inductors and capacitors. Each of these quantities is measured in ohms. The symbol used for ohms is Ω, the uppercase Greek letter omega.

Component and symbol	Resistor	Inductor	Capacitor
Resistance or reactance (in ohms)	R	L	C
Impedance (in ohms)	R	Li	-Ci



The table shows the relationship between a component's resistance or reactance and its contribution to impedance. A series circuit is also shown with the resistance or reactance of each component labeled. The impedance for a series circuit is the sum of the impedances for the individual components. Find the impedance of the circuit.

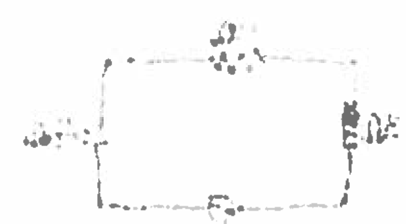
Solution

The resistor has a resistance of 5 ohms, so its impedance is 5 ohms. The inductor has a reactance of 3 ohms, so its impedance is 3i ohms. The capacitor has a reactance of 4 ohms, so its impedance is -4i ohms.

Impedance of circuit=5+3i+ (-4i) =5-i

The impedance of the circuit is (5-i) ohms

Example 5



Multiply. Write the answer in standard form.

a. $4i(-6+i)$
b. $(9-2i)(-4+7)$

Solution

a. $4i(-6+i)=-24i+4i^2$	Distributive Property
$=-24i+4(-1)$	Use $i^2=-1$
$=-4-24i$	Write in standard form
b. $(9-2i)(-4=7i)=-36+63i+8i-14i^2$	Multiply using FOIL
$=-36+71i-14(-1)$	Simplify and use $i^2=-1$
$=-36+71i+14$	Simplify
$=-22+71i$	Write in standard form

Example 6

Solve (a)$x^2+4=0$ and (b) $2x^2-11= -47$

Solution

a. $x^2+4=0$
$x^2=-4$
$x=\pm\sqrt{-4}$
$x=\pm 2i$
The solutions are 2i and -2i

b. $2x^2-11= -47$
$2x^2=-36$
$x^2=-18$
$x=\pm\sqrt{-18}$
$x=\pm i\sqrt{18}$
$x=\pm 3i\sqrt{2}$
The solutions are $+3i\sqrt{2}$ and $-3i\sqrt{2}$

Example 7

Find the zeros of $f(x)$ $=4x^2+20$

Solution

$4x^2+20=0$

$4x^2=-20$	Set f(x) equal to 1
$x^2=-5$	Subtract 20 from each side
$x=\pm\sqrt{-5}$	Divide each side by 4
$x=\pm i\sqrt{5}$	Write in terms of i

Name:______________________ Date:________

Complex Number Practice

Find the square root of the number:

1. $\sqrt{-36}$
2. $\sqrt{-64}$
3. $\sqrt{-18}$
4. $-3\sqrt{-49}$
5. $6\sqrt{-63}$

Find the values of x and y that satisfy the equation

1. $4x+2i=8+yi$
2. $9x-18i=-36+6yi$
3. $2x-yi=14+12i$
4. $54-(1/7)yi=9x-4i$
5. $15-3yi=\frac{1}{2}x+2i$

Add, subtract or multiply and write the answer in standard form

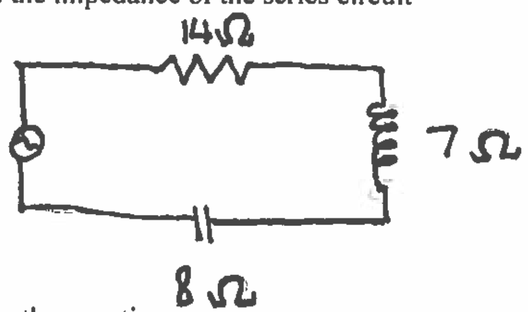
1. $(6-i)+(7+3i)$
2. $(12-3i)+(7+3i)$
3. $-3+(8+2i)+7i$
4. $3i(-5+i)$
5. $(8+3i)^2$

Write each expression as a complex number in standard form

1. $\sqrt{-9}+\sqrt{-4}-\sqrt{16}$
2. $\sqrt{-16}+\sqrt{8}+\sqrt{-36}$

Find the additive inverse of z=-2+8i

Find the impedance of the series circuit



Solve the equation

1. $x^2+9=0$
2. $2x^2+6=-34$
3. $x^2+49=0$

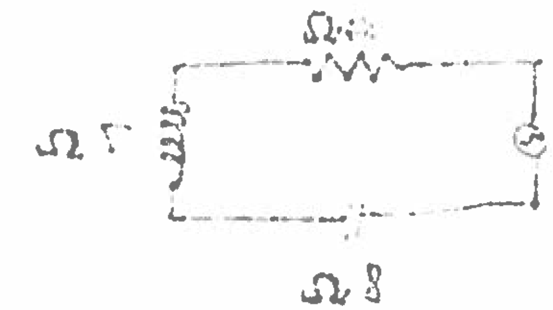
Find the zeros of the function

1. $f(x)=3x^2+6$
2. $k(x)=-5x^2-125$
3. $f(x)=(-1/5)x^2-10$

True or false?

a. The sum of two imaginary numbers is an imaginary number
b. The real product of two pure imaginary numbers is a real number
c. A pure imaginary number is an imaginary number
d. A complex number is a real number

Create a circuit that has an impedance of 14-3i


8 Ω

Answer Key

Please excuse the weird numbering

1. 6i
2. 8i
3. $3i\sqrt{2}$
4. -21i
5. $18i\sqrt{7}$

6. x=2 and y=2

7. x=-4 and y=-3

8. x=7 and y=-12

9. x=6 and y=28

10. x=30 and y=-2/3

11. 13+2i

12. 19

13. 5+9i

14. -3-15i

15. 55+48i

16.

    a. -4+5i
    b. $2\sqrt{2} + 10i$

1. Z =2-8i
2. (14-i) ohms

3. $X = \pm 3i$
4. $X = \pm 2i\sqrt{5}$
5. $X = \pm 7i$

6. $X = \pm i\sqrt{2}$

7. $X = \pm 5i$

8. $X = \pm 5i\sqrt{2}$

9.

   a. False
   b. True
   c. True
   d. false