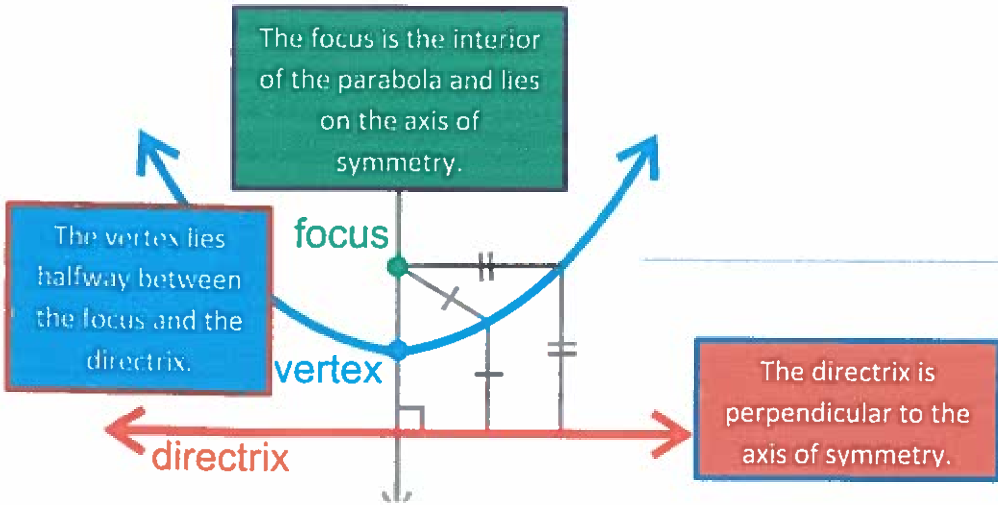
Name________________________________ Period______ Date__________

Focus of a Parabola



How to graph a parabola in the form y = a(x-h) ² + k?

Step 1: Find the vertex. Since the equation is in vertex form, the vertex will be at the point (h, k).

Step 2: Find the y-intercept. To find the y-intercept let x = 0 and solve for y.

Step 3: Find the x-intercept(s). To find the x-intercept let y = 0 and solve for x. You can solve for x by using the square root principle or the quadratic formula (if you simplify the problem into the correct form).

Step 4: Graph the parabola using the points found in steps 1 – 3.

Core Concept

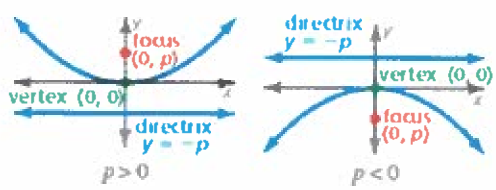
Standard Equations of a Parabola with Vertex at the Origin

Vertical axis of symmetry ($x = 0$)

Equation: $y = \frac{1}{4p}x^2$

Focus: $(0, p)$

Directrix: $y = -p$

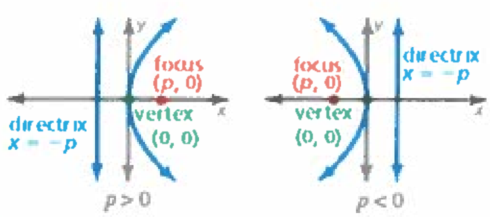


Horizontal axis of symmetry ($y = 0$)

Equation: $x = \frac{1}{4p}y^2$

Focus: $(p, 0)$

Directrix: $x = -p$



Core Concept

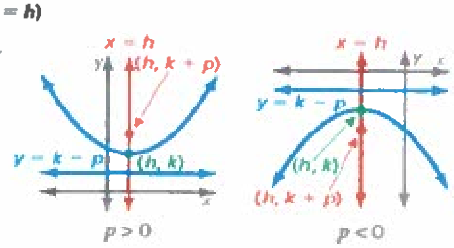
Standard Equations of a Parabola with Vertex at (h, k)

Vertical axis of symmetry ($x = h$)

Equation: $y = \frac{1}{4p}(x - h)^2 + k$

Focus: $(h, k + p)$

Directrix: $y = k - p$

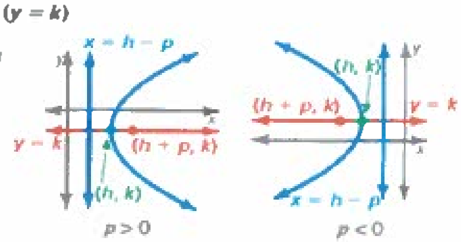


Horizontal axis of symmetry ($y = k$)

Equation: $x = \frac{1}{4p}(y - k)^2 + h$

Focus: $(h + p, k)$

Directrix: $x = h - p$



EXAMPLE 1 Graphing an Equation of a Parabola

Identify the focus, directrix, and axis of symmetry of $-4x = y^2$. Graph the equation.

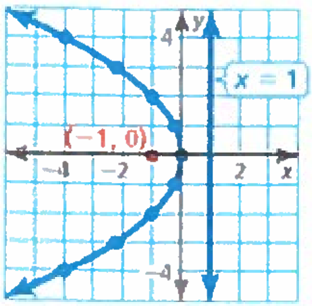
SOLUTION

Step 1 Rewrite the equation in standard form.

$$-4x = y^2 \quad \text{Write the original equation.}$$

$$x = -\frac{1}{4}y^2 \quad \text{Divide each side by } -4.$$

Step 2 Identify the focus, directrix, and axis of symmetry. The equation has the form $x = \frac{1}{4p}y^2$, where $p = -1$. The focus is $(p, 0)$, or $(-1, 0)$. The directrix is $x = -p$, or $x = 1$. Because y is squared, the axis of symmetry is the x-axis.

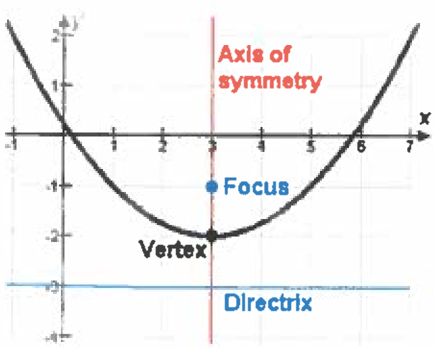


Step 3 Use a table of values to graph the equation. Notice that it is easier to substitute y-values and solve for x. Opposite y-values result in the same x-value.

y	0	±1	±2	±3	±4
x	0	−0.25	−1	−2.25	−4

EXAMPLE 2 Writing an Equation of a Parabola

Write an equation of the parabola shown.

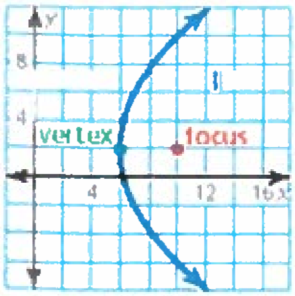


Because the vertex is at point (3, -2), and because the axis of symmetry is vertical, the equation will take the form $y = \frac{1}{4p}(x - h)^2 + k$. The directrix is y = p = 2, so p = 2. Substitute 2 for p to write an equation of the parabola.

$$Y = \frac{1}{4(2)}(x-3)^2 - 2 = \frac{1}{8}(x-3)^2 - 2$$

➤ So, an equation for the parabola is $Y = \frac{1}{8}(x-3)^2 - 2$

EXAMPLE 3 Writing an Equation of a Translated Parabola



Write an equation of the parabola shown.

SOLUTION

Because the vertex is not at the origin and the axis of symmetry is horizontal, the equation has the form $x = \frac{1}{4p}(y-k)^2 + h$. The vertex (h, k) is $(6, 2)$ and the focus $(h+p, k)$ is $(10, 2)$, so $h = 6$, $k = 2$, and $p = 4$. Substitute these values to write an equation of the parabola.

$$x = \frac{1}{4(4)}(y-2)^2 + 6 = \frac{1}{16}(y-2)^2 + 6$$

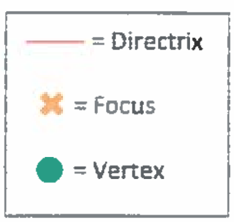
So, an equation of the parabola is $x = \frac{1}{16}(y-2)^2 + 6$.

EXAMPLE 4 Writing Equations in Vertex Form given the Focus and Directrix

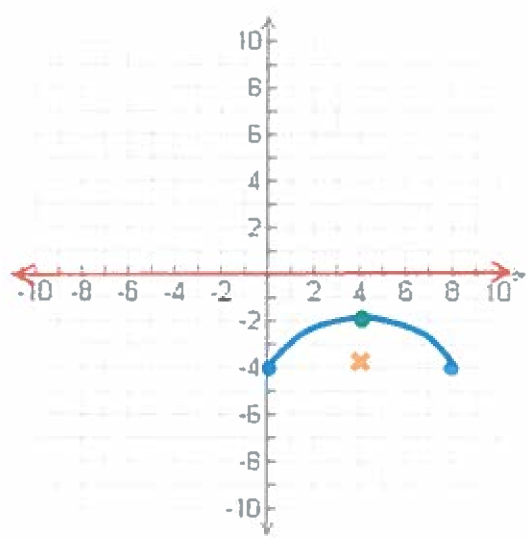
Focus: (4, -2) Directrix: x-axis

Vertex: (4, -2)

P = 2



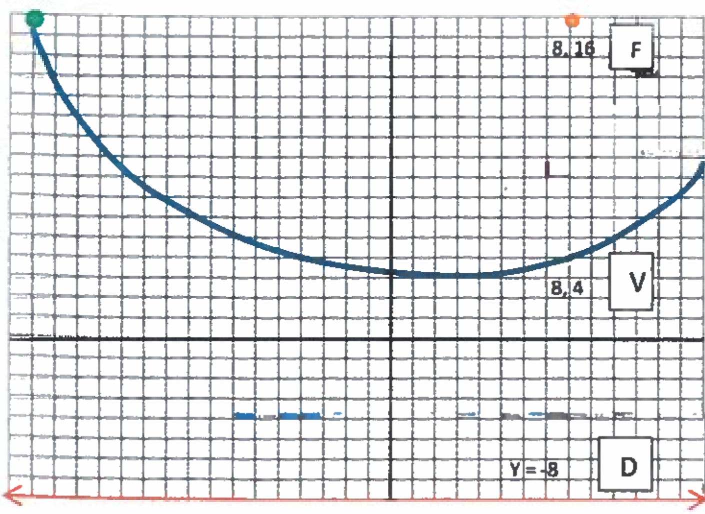
$$Y = -\frac{1}{8}(x-4)^2 - 2$$



EXAMPLE 5 Solving a Real Life Problem based on Parabolas

Write an equation of the parabola shown.

In order to successfully perform a trick, a flying trapeze artist must swing along a parabolic path that is equidistant from the floor and the pivot point where the trapeze rope is attached. The rope is attached to the ceiling 8 feet out of 16 feet above her starting point and the floor is 8 feet below her starting point. Use the focus of (8, 16) and directrix at $y = -8$ to determine the equation of the parabola.



Because the vertex is at (8, 4), the axis of symmetry is vertical, which means it takes the equation, $Y = \frac{1}{4p}(x-h)^2 - k$. The rope is where the focus is, at (8, 16). It's also 12 feet above the vertex. So $p =$ 12. Substitute 12 for p to write the equation.

So, we plug in 12 for p, 8 for h, since the rope is 8 feet out, then we substitute 4 for k since 4 is between the focus (8, 16) and the directrix, y = -8. You can also look at it by substituting (h, k) with the vertex.

$$Y = \frac{1}{4(12)}(x-8)^2 + 4$$

Next, you multiply 12 and 4 and the equation is complete.

➢ $Y = \frac{1}{48}(x-8)^2 + 4$

Practice Questions: Choose at least 3 questions from each section. Complete on a separate sheet of paper.

11. ANALYZING RELATIONSHIPS Which of the given characteristics describe parabolas that open down? Explain your reasoning.

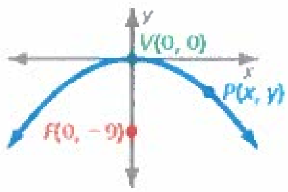
(A) focus: $(0, -6)$ directrix: $y = 6$

(B) focus: $(0, -2)$ directrix: $y = 2$

(C) focus: $(0, 6)$ directrix: $y = -6$

(D) focus: $(0, -1)$ directrix: $y = 1$

12. REASONING Which of the following are possible coordinates of the point P in the graph shown? Explain.

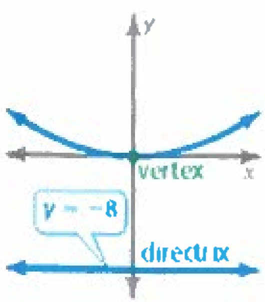


(A) $(-6, -1)$ (B) $\left(3, -\frac{1}{4}\right)$ (C) $\left(4, -\frac{4}{9}\right)$

(D) $\left(1, \frac{1}{36}\right)$ (E) $(6, -1)$ (F) $\left(2, -\frac{1}{18}\right)$

In Exercises 25–28, write an equation of the parabola shown. *(See Example 3.)*

25.



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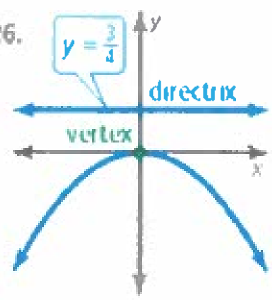

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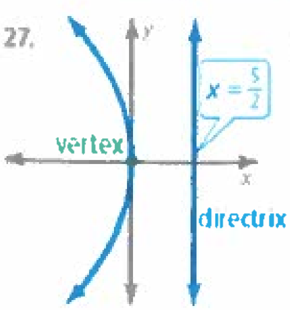

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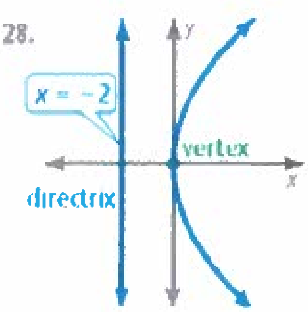
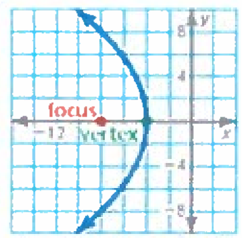


In Exercises 37–40, write an equation of the parabola shown. *(See Example 4.)*

37.



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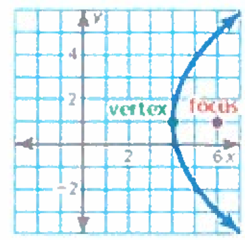

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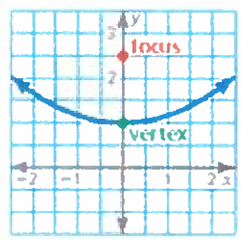

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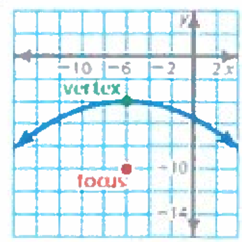


In Exercises 41–46, identify the vertex, focus, directrix, and axis of symmetry of the parabola. Describe the transformations of the graph of the standard equation with $p = 1$ and vertex $(0, 0)$.

41. $y = \frac{1}{8}(x - 3)^2 + 2$ **42.** $y = -\frac{1}{4}(x + 2)^2 + 1$

43. $x = \frac{1}{16}(y - 3)^2 + 1$ **44.** $y = (x + 3)^2 - 5$

45. $x = -3(y + 4)^2 + 2$ **46.** $x = 4(y + 5)^2 - 1$

In Exercises 13–20, identify the focus, directrix, and axis of symmetry of the parabola. Graph the equation. *(See Example 2.)*

13. $y = \frac{1}{8}x^2$ **14.** $y = -\frac{1}{12}x^2$

15. $x = -\frac{1}{20}y^2$ **16.** $x = \frac{1}{24}y^2$

17. $y^2 = 16x$ **18.** $-x^2 = 48y$

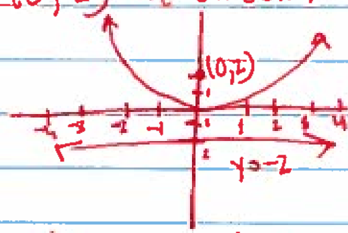
19. $6x^2 + 3y = 0$ **20.** $8x^2 - y = 0$

Focus of a Parabola – Answer Key

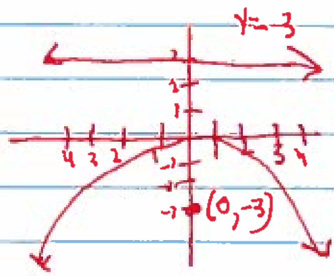
11. A, B, and D. Each has a value for p that is negative. Substituting in a negative value for p in $y=\frac{1}{4p}x^2$ results in a parabola that has been reflected across the x-axis.

12. B, C and E. Use the focus to create the equation $y=-\frac{1}{36}x^2$. Point B, C, and E are the fourth quadrant points that satisfy the equation.

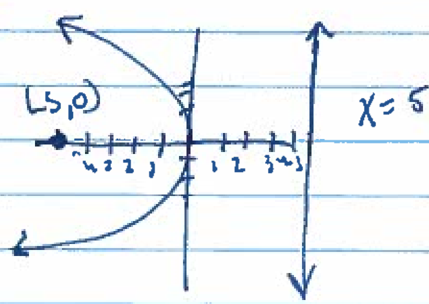
13. The focus is (0, 2) The directrix is y=-2 The axis of symmetry is the y-axis



14. The focus in (0,-3). The directrix is y=3. The axis of symmetry is the y-axis

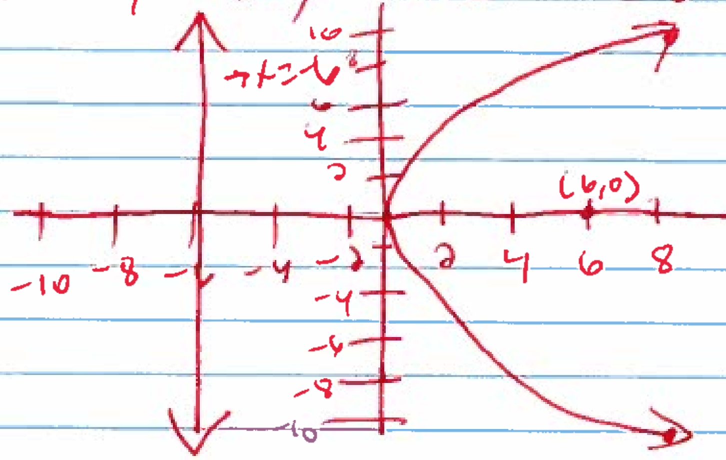


15. The focus is (-5,0) the directrix is x=5. The axis of symmetry is the x-axis

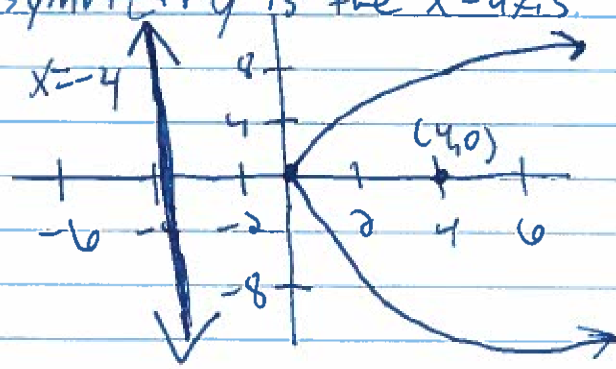


Focus of a Parabola - Answer Key

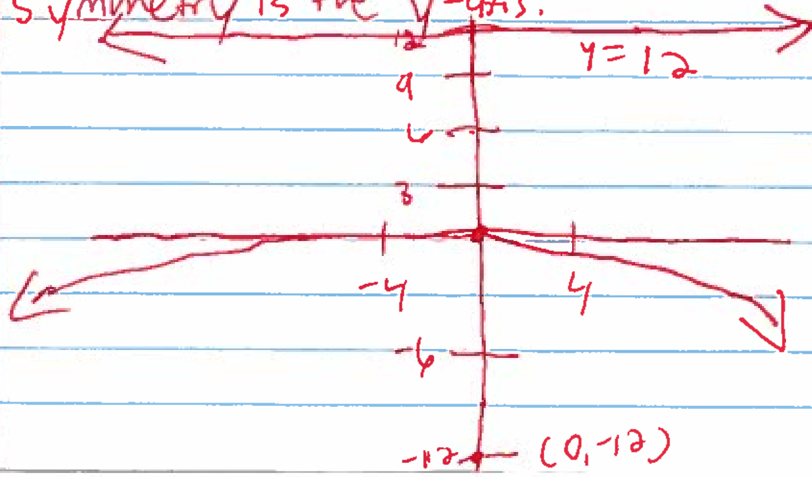
16. The focus is $(6,0)$. The directrix is $x=-6$. Because the y is squared, the axis of symmetry is the x-axis



17. The focus is $(4,0)$. The directrix is $x=-4$. Because y is squared, the axis of symmetry is the x-axis.

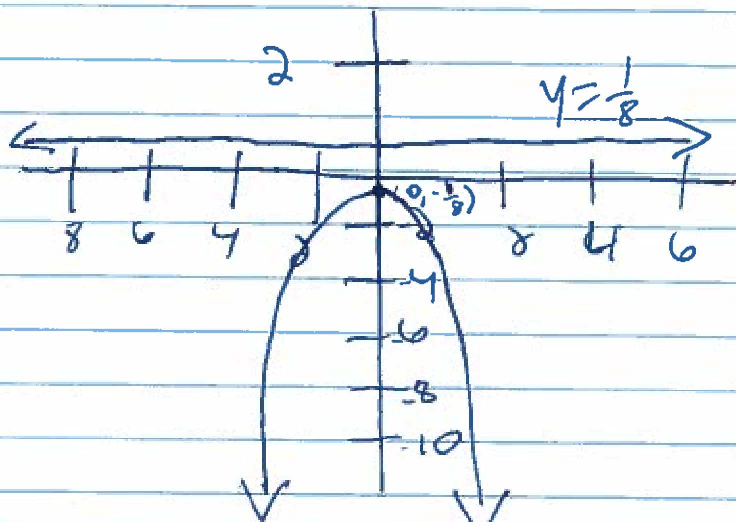


18. The focus is $(0,-12)$. The directrix is $y=12$. Because the x is squared, the axis of symmetry is the y-axis.

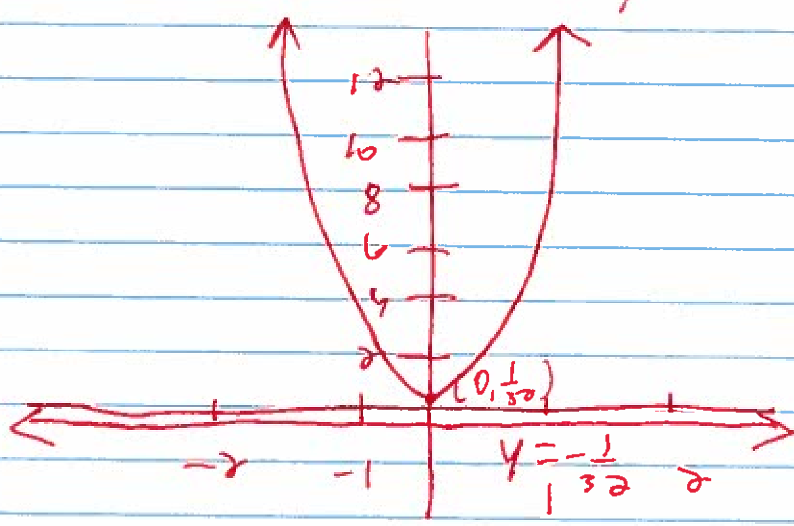


Focus of a Parabola - Answer Key

19. The focus is $(0, -\frac{1}{8})$. The directrix is $y = \frac{1}{8}$. Because x is squared, the axis of symmetry is the y-axis.



20. The focus is $(0, \frac{1}{32})$. The directrix is $y = -\frac{1}{32}$. Because x is squared, the axis of symmetry is the y-axis.



Focus of Parabola Answer Key

25. $y = \frac{1}{32}x^2$

37. $x = -\frac{1}{16}y^2 - 4$

26. $y = -\frac{1}{3}x^2$

38. $x = \frac{1}{8}(y-1)^2 + 4$

27. $x = -\frac{1}{10}y^2$

39. $y = \frac{1}{4}x^2 + 1$

28. $x = \frac{1}{8}y^2$

40. $y = \frac{1}{24}(x+6)^2 - 4$

41. The vertex is $(-2, 1)$. The focus is $(3, 4)$. The directrix is $y = 0$. The axis of symmetry is $x = 3$. The graph is a vertical shrink by a factor of $1/2$ followed by a translation 3 units right and 2 units up.

42. The vertex is $(-2, 1)$. The focus is $(-2, 0)$. The directrix is $y = 2$. The axis of symmetry is $x = -2$. The graph is a reflection in the x-axis and a translation 2 units left and 1 unit up.

43. The vertex is $(1, 3)$. The focus is $(5, 3)$. The directrix is $y = 2$. The axis of symmetry is $y = 3$. The graph is a horizontal shrink by a factor of $1/4$ followed by a translation 1 unit right and 3 units up.

44. The vertex is $(-3, -5)$. The focus is $(-3, -4.75)$. The directrix is $y = -5.25$. The axis of symmetry is $x = -3$. The graph is a vertical stretch by a factor of 4 followed by a translation 3 units left and 5 units down.

Focus of a Parabola - Answer Key

45.) The vertex is $(2,-4)$. The focus is $(\frac{23}{12},-4)$. The directrix is $x=\frac{25}{12}$. The axis of symmetry is $y=-4$. The graph is a horizontal stretch by a factor of 12 followed by a reflection in the y-axis and a translation 2 units right and 4 units down.

46.) The vertex is $(-1,-5)$. The focus is $(\frac{-15}{16},-5)$. The directrix is $x=\frac{-17}{16}$. The axis of symmetry is $y=-5$. The graph is a horizontal stretch by a factor of 16 followed by a translation 1 unit left and 5 units down.