

Solving Three-Variable Systems

STEP 1) Rewrite the system in three variables as a system with two variables either by substitution or elimination

STEP 2) Solve the new system for both its variables

STEP 3) Substitute the values found in step 2 into one of the original equations and solve for the remaining variable

$0=1$ means there is no solution

$0=0$ means there is infinitely many solutions

Example: *Solve the system*

$$4x + 2y + 3z = 12$$

$$2x - 3y + 5z = -7$$

$$6x - y + 4z = -3$$

Step 1: Rewrite the system

$$4x + 2y + 3z = 12$$

$$\underline{12x - 2y + 8z = -6}$$

$$16x \quad + 11z = 6$$

$$2x - 3y + 5z = -7$$

$$\underline{-18x + 3y - 12z = 9}$$

$$-16x \quad - 7z = 2$$

Step 2: Solve the new system

$$16x + 11z = 6$$

$$\underline{-16x - 7z = 2}$$

$$4z = 8$$

$$z = 2$$

$$x = -1$$

Step 3: Substitute $x = -1$ and $z = 2$ into an original equation and solve for y

$$6x - y + 4z = -3$$

$$6(-1) - y + 4(2) = -3$$

$$y = 5$$

Solution: $x = -1$, $y = 5$, and $z = 2$

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1) *Solve the system:*

$$x - y + z = -3$$

$$x - y - z = -3$$

$$5x - 5y + z = -15$$

2) *Solve the system:*

$$4r - 4s + 4t = -4$$

$$4r + s - 2t = 5$$

$$-3r - 3s - 4t = -16$$

3) *Solve the system*

$$x + y + z = 2$$

$$5x + 5y + 5z = 3$$

$$4x + y - 3z = -6$$

Solving Real- Life Problems

Example:

An amphitheater charges \$75 for each seat in Section A, \$55 for each seat in Section B, and \$30 for each lawn seat. There are three times as many seats in Section B as in Section A. The revenue

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from selling all 23,000 seats is \$870,000. How many seats are in each section of the amphitheater?

Step 1: Write a verbal model

of seats in B, $y = 3x$ # of seats in A, x

of seats in A, x + # of seats in B, y + # of lawn seats, z = Total # of seats

$75 \cdot$ # of seats in A, x + $55 \cdot$ # of seats in B, y + $30 \cdot$ # of lawn seats, z = total revenue

Step 2: Write a system

$$\begin{aligned}y &= 3x \\x + y + z &= 23,000 \\75x + 55y + 30z &= 87,000\end{aligned}$$

Step 3: Rewrite as a system in two variables

$$\begin{aligned}x + \quad + z &= 23,000 \\x + \quad + z &= 23,000 \\4x + z &= 23,000 \\75x + 55 \quad + 30z &= 870,000 \\75x + 55(\quad) + 30z &= 870,000 \\240x + 30z &= 870,000\end{aligned}$$

Step 4: Solve

$$\begin{aligned}-120x - 30z &= -690,000 \\ \underline{240x + 30z} &= \underline{870,000} \\ 120x &= 180,000 \\ x &= 1500 \\ y &= 4500 \\ z &= 17,000\end{aligned}$$

Solution

1) An ultimate Frisbee team has to order jerseys, shorts, and hats. They have a budget of \$1350 to spend on \$50 jerseys, \$20 shorts, and \$15 hats. They want to buy 40 items in preparation for the oncoming season and must order as many jerseys as shorts and hats combined

- o How many of each item should they order? Write a system of equations to help you solve this problem.

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2) Mike is doing a math quiz and he has 2 hours. It takes him 5 minutes to complete a multiple-choice question, 5 minutes to complete a calculation question and 10 minutes to complete a word problem. He must complete 20 questions and as many as multiple-choice questions as calculation questions and word problems combined.

- How many of each kind of question should he complete? Write a system of equations to help you solve this problem.

3) Tracy wants to buy some pies for her sisters and she has a budget of \$82 to spend on \$5 apple pies, \$4.5 banana pies, and \$5.5 chocolate pies. She wants to buy 16 pies for her sisters and must buy as many chocolate pies as apple pies and banana pies combined.

- How many of each item should she buy? Write a system of equations to help you solve this problem.