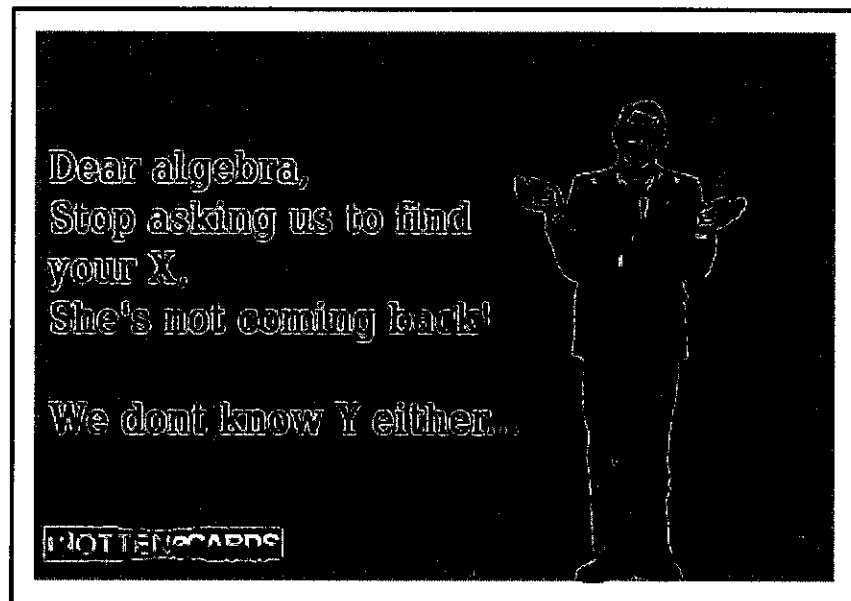


Chapter 3:

Solving Rational Equations



Day 1: Proportions

A-REI.3 Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters.

Warm-Up Evaluate each expression. Express the answer in simplest form.

$$1) \frac{3}{4} \times \frac{5}{6}$$

$$2) 4 \times \frac{5}{12}$$

$$3) \frac{3}{8} + \frac{1}{4}$$

A **rational equation** is an equation that contains two or more fractions. Rational equations differ from rational expressions in that rational equations contain an equals sign (=) and must be solved, rather than simplified.

There are two types of rational equations: proportions, and equations that include sums and differences.

The Proportion Structure
$\frac{\square}{\square} = \frac{\square}{\square}$
<i>To solve: Cross-multiply.</i>

Model Problem A

$$-\frac{x}{5} = 4$$

Rewrite
for Structure:

Model Problem B

$$\frac{2}{3}x = 18$$

Rewrite:

Remember:

Remember:

Model Problem C

$$-6 = \frac{y}{5} + 4$$

Rewrite:

Remember:

Exercise Solve and check:

1. $\frac{d}{8} = 6$

2. $\frac{-t}{2} = 12$

3. $\frac{2r}{3} = 16$

4. $-15 = -\frac{3n}{5}$

5. $\frac{2}{5}b + 6 = 10$

6. $\frac{5}{8}n + 6 = 7$

The first strategy to try when you see fractions is _____

Binomials in Proportions

<p><u>Model Problem A</u></p> $\frac{x + 1}{6} = \frac{4}{5}$	<p><u>Model Problem B</u></p> $\frac{4(-1 - c)}{8} = \frac{-2(c - 8)}{-9}$
<p><i>When multiplying a number by a binomial:</i></p>	<p><i>If the binomial has a coefficient already:</i></p>

<p><u>Model Problem C</u></p> $2x - 1 = \frac{5}{2}$	<p><u>Model Problem D</u></p> $15 = -\frac{1}{2}(-12x + 2)$
<p><i>Making a proportion:</i></p>	<p><i>Dealing with parentheses on the side:</i></p>

Homework

Chapter 3, Day 1



Study the examples below. They will help you solve the problems on the right.

1) $\frac{2}{3}x = \frac{14}{6}$

$$\frac{2x}{3} = \frac{14}{6}$$

Put the variable on top to put into structure.

~~$$\frac{2x}{3} = \frac{14}{6}$$~~

Cross-multiply to eliminate the fraction.

$$12x = 42$$

$$x = \frac{42}{12}$$

Isolate the variable to solve.

$$x = \frac{7}{2}$$

2) $\frac{4}{5}x = \frac{12}{6}$

3) $-\frac{4}{5}g - 3 = 9$

$$\begin{array}{r} +3 +3 \\ \hline -\frac{4}{5}g = 12 \end{array}$$

Isolate the variable term.

$$-\frac{4g}{5} = \frac{12}{1}$$

Put the variable on top to put into structure.

~~$$-\frac{4g}{5} = \frac{12}{1}$$~~

Cross multiply to eliminate the fraction.

$$-4g = 60$$

$$g = -15$$

Isolate the variable to solve.

4) $-\frac{3}{5}g - 2 = 7$

$$5) \frac{2x-18}{4} = \frac{3x+1}{2}$$

~~$$\frac{2x-18}{4} = \frac{3x+1}{2}$$~~

Cross-multiply to eliminate the fraction.

$$2(2x-18) = 4(3x+1)$$

Write as a product, coefficient in front.

$$4x - 36 = 12x + 4$$

Distribute.

$$-40 = 8x$$

Isolate the variable to solve.

$$\frac{-40}{8} = x$$

$$\boxed{-5 = x}$$

$$6) \frac{x+9}{5} = \frac{x-7}{10}$$

$$7) \frac{x}{3} = x + 4$$

$$\frac{x}{3} = \frac{x+4}{1}$$

Give the binomial a denominator of 1.

~~$$\frac{x}{3} = \frac{x+4}{1}$$~~

Cross-multiply to eliminate the fraction.

$$x = 3(x+4)$$

Write as a product, coefficient in front.

$$x = 3x + 12$$

Distribute.

$$-2x = 12$$

Isolate the variable to solve.

$$x = \frac{12}{-2}$$

$$\boxed{x = -6}$$

$$8) \frac{y}{6} = y + 5$$

$$9) \frac{7}{6}(6p - 24) + 9 = 23$$

$$\frac{7}{6}(6p - 24) = 14$$

$$\frac{7(6p - 24)}{6} = \frac{14}{1}$$

$$\frac{7(6p - 24)}{6} = \frac{14}{1}$$

$$7(6p - 24) = 84$$

$$42p - 168 = 84$$

$$42p = 252$$

$$\boxed{p = 6}$$

Isolate the term with the parentheses.

Put the () on top of the fraction.

Put the whole number over 1.

Cross-multiply to eliminate the fraction.

Distribute.

Isolate the variable to solve.

$$10) \frac{4}{5}(m - 2) - 3 = 9$$

$$11) \frac{1}{2}(2h - 1) = \frac{1}{3}(2h - \frac{1}{2})$$

$$\frac{1(2h - 1)}{2} = \frac{1(2h - \frac{1}{2})}{3}$$

$$\frac{1(2h - 1)}{2} = \frac{1(2h - \frac{1}{2})}{3}$$

$$3(2h - 1) = 2(2h - \frac{1}{2})$$

$$6h - 3 = 4h - 1$$

$$2h = 2$$

$$\boxed{h = 1}$$

Put the () on top of the fraction.

Cross-multiply to eliminate the fraction.

Write as a product, coefficient in front.

Distribute.

Isolate the variable terms to solve.

$$12) \frac{1}{5}(2x - 10) = \frac{1}{3}(x - 1)$$

Day 2: Equations with Sums or Differences

A-REI.3 Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters.

Warm-Up Solve for the value of the variable:

$$1) 3b - 4 = \frac{8b-11}{2}$$

$$2) 9 - 2d = \frac{2d-4}{2}$$

Structure of Sums and Differences

$$\frac{\square}{\square} \pm \frac{\square}{\square} = \frac{\square}{\square}$$

To solve: Get a common denominator, then cross them out.
Solve the resulting equation. Check.

Model Problem A Numerical Denominators

$$\frac{x}{5} + \frac{x}{2} = 14$$

Steps:

Guided Practice A

$$\frac{2x}{3} + \frac{x}{6} = 5$$

Rewrite the Steps:

Model Problem B *Variables on the Side*

$$\frac{3}{4}w + 8 = \frac{1}{3}w - 7$$

Steps:

Guided Practice B

$$1 - \frac{5}{8}x = 2 - \frac{2}{3}x$$

Rewrite the Steps:Model Problem C *Binomials in the Numerator*

$$\frac{t-3}{6} - \frac{t-25}{5} = 4$$

Steps:

Guided Practice C

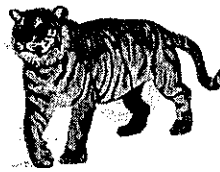
$$\frac{a-4}{2} - \frac{a-6}{4} = 12$$

Steps:

Summary of Strategies

- Structure of Sums and Differences: $\frac{\square}{\square} \pm \frac{\square}{\square} = \frac{\square}{\square}$
- Solving Method: Get a common denominator. Cross off denominators and solve.
 - Variables/negatives/parentheses on side: Put them up top.
 - Binomials in numerator: Put parentheses on them.
 - Variables in denominator: Break the monomial down. Multiply by missing parts.

Homework Chapter 3, Day 2



Study the examples below. They will help you solve the problems on the right.

<p>1) $\frac{x}{3} + \frac{x}{7} = 10$</p> $\frac{x}{3} + \frac{x}{7} = \frac{10}{1}$ <p>LCD = 21</p> $\frac{7 \cdot x}{7 \cdot 3} + \frac{3 \cdot x}{3 \cdot 7} = \frac{10 \cdot 21}{1 \cdot 21}$ $\frac{7x}{21} + \frac{3x}{21} = \frac{210}{21}$ $7x + 3x = 210$ $10x = 210$ $x = 21$ <p>Give any integers a denominator of 1.</p> <p>Find the LCD.</p> <p>Create the LCD by multiplying top + bottom by missing factor.</p> <p>Cross out denominators</p> <p>Solve remaining equation.</p>	<p>2) $\frac{x}{3} + \frac{x}{2} = 5$</p>
<p>3) $\frac{3x}{20} + \frac{1}{10} = \frac{x}{4} - \frac{1}{5}$</p> <p>LCD = 20</p> $\frac{3x}{20} + \frac{2}{2} \cdot \frac{1}{10} = \frac{x}{4} \cdot \frac{5}{5} - \frac{1}{5} \cdot \frac{4}{4}$ $\frac{3x}{20} + \frac{2}{20} = \frac{5x}{20} - \frac{4}{20}$ $\frac{3x + 2}{20} = \frac{5x - 4}{20}$ $3x + 2 = 5x - 4$ $6 = 2x$ $3 = x$ <p>Find the LCD.</p> <p>Create the LCD by multiplying top + bottom by missing factor. Then cross out denominators</p> <p>Solve remaining equation.</p>	<p>4) $\frac{2x}{6} + \frac{4}{3} = \frac{14}{3}$</p>

$$5) \frac{3y}{4} - 6 = \frac{y}{8} + 4$$

$$\frac{3y}{4} - \frac{6}{1} = \frac{y}{8} + \frac{4}{1}$$

Give any integers a denominator of 1.

$$\text{LCD} = 8$$

Find the LCD.

$$\frac{3y \cdot 2}{4 \cdot 2} - \frac{6 \cdot 8}{1 \cdot 8} = \frac{y}{8} + \frac{4 \cdot 8}{1 \cdot 8}$$

Create the LCD by multiplying top + bottom by missing factor. Then, cross out the denominators.

$$\frac{6y}{8} - \frac{48}{8} = \frac{y}{8} + \frac{32}{8}$$

$$\frac{6y}{8} - \frac{48}{8} = \frac{y}{8} + \frac{32}{8}$$

$$6y - 48 = y + 32$$

$$5y = 80$$

$$\boxed{y = 16}$$

Solve the remaining equation.

$$6) \frac{7y}{8} - 4 = \frac{y}{2} + 2$$

$$7) \frac{x-2}{3} - \frac{2x-4}{4} = 5$$

$$\frac{x-2}{3} - \frac{2x-4}{4} = \frac{5}{1}$$

Give any integers a denominator of 1.

$$\text{LCD} = 12$$

Find the LCD.

$$\frac{4(x-2)}{4 \cdot 3} - \frac{3(2x-4)}{3 \cdot 4} = \frac{5 \cdot 12}{1 \cdot 12}$$

Create the LCD by multiplying top + bottom by missing factors. DO NOT DISTRIBUTE to any binomials.

$$\frac{4(x-2)}{12} - \frac{3(2x-4)}{12} = \frac{60}{12}$$

$$\frac{4(x-2)}{12} - \frac{3(2x-4)}{12} = \frac{60}{12}$$

cross out denominators.

$$4(x-2) - 3(2x-4) = 60$$

Solve remaining equation.

$$4x - 8 - 6x + 12 = 60$$

$$-2x + 4 = 60$$

$$-2x = 56$$

$$\boxed{x = -28}$$

$$8) \frac{x+6}{3} - \frac{x-3}{4} = 2$$

Day 3: Practice Solving Rational Equations

A-REI.3 Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters.

Remember the Basic Structures:

<u>A) Proportion</u>	<u>B) Sums and Differences</u>
$\frac{\square}{\square} = \frac{\square}{\square}$	$\frac{\square}{\square} \pm \frac{\square}{\square} = \frac{\square}{\square}$

Match each equation with its basic structure. Write (A) or (B). Then solve each equation in the space provided.

1) $\frac{6}{t+3} = \frac{4}{t}$

3) $\frac{5}{4}(r - 3) = -\frac{5}{8}$

2) $\frac{a}{4} + \frac{1}{2} = \frac{2}{3}$

4) $x + \frac{3}{4} = \frac{12x-7}{12}$

<u>Proportions</u>	<u>Sums and Differences</u>

Practice

1) $\frac{4}{y} = \frac{1}{y+6}$

(see p. 5, #3)

2) $-\frac{4}{5}t + \frac{2}{5} = \frac{2}{3}$

(see p. 11, #1)

3) $\frac{y}{3} - 8 = 1$

(see p. 4, #3)

4) $\frac{14-c}{6} = c - 7$

(see p. 5, #7)

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$$5) \frac{1}{4}p + \frac{2}{5}p = \frac{1}{2}p - \frac{9}{20} \quad (\text{see p. 11, \#3})$$

$$6) \frac{x}{2} - \frac{x+3}{8} = 5 \quad (\text{see p. 12, \#7})$$

7) Study this example to help you complete #8:

$$-\frac{4}{3}\left(\frac{5}{3} + n\right) = -\frac{16}{9}$$

$$-\frac{4\left(\frac{5}{3} + n\right)}{3} = \frac{-16}{9}$$

Put the parentheses on top. Do not distribute the fraction!

~~$$-\frac{4\left(\frac{5}{3} + n\right)}{3} = \frac{-16}{9}$$~~

Cross-multiply. Don't distribute in the same step.

$$-36\left(\frac{5}{3} + n\right) = -48$$

$$-60 - 36n = -48$$

Distribute.

$$-12 = 36n$$

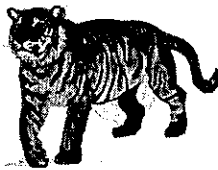
$$-\frac{1}{3} = n$$

Solve remaining equation.

$$8) -\frac{1}{2} = \frac{1}{4}\left(\frac{1}{2}x + 4\right)$$

Challenge!

Solve: $\frac{3x+1}{3} = \frac{4x+5}{4} - \frac{8+x}{6} + \frac{2x+5}{8}$



Homework Chapter 3, Day 3

1) $\frac{2x+1}{3} = \frac{6x-9}{5}$

$\frac{2x+1}{3} = \frac{6x-9}{5}$

Cross-multiply
to eliminate
the fraction.

$5(2x+1) = 3(6x-9)$

Write as a product,
coefficient in
front.

$10x+5 = 18x-27$

Distribute.

$32 = 8x$

Isolate the
variable to
solve.

$4 = x$

2) $\frac{x+9}{5} = \frac{x-7}{10}$

$$3) \frac{3}{4} = \frac{3}{8}x - 4$$

$$\frac{3}{4} = \frac{3x}{8} - \frac{4}{1}$$

Give any integers
a denominator of
1.

$$\text{LCD} = 8$$

Find the LCD.

$$\frac{3}{4} \cdot \frac{2}{2} = \frac{3x}{8} - \frac{4}{1} \cdot \frac{8}{8}$$

Create the LCD by
multiplying top +
bottom by the
missing factor.

$$\frac{6}{8} = \frac{3x}{8} - \frac{32}{8}$$

$$\frac{6}{8} = \frac{3x}{8} - \frac{32}{8}$$

Cross out
denominators.

$$6 = 3x - 32$$

Solve the
remaining
equation.

$$38 = 3x$$

$$\boxed{\frac{38}{3} = x}$$

$$4) \frac{r}{3} - \frac{r}{6} = 2$$

$$5) -\frac{2}{5}y - 6 = 14$$

$$+6 \quad +6$$

$$-\frac{2}{5}y = 20$$

Collect the
integers on
one side
to get rid of
the +/-.

$$-\frac{2y}{5} = \frac{20}{1}$$

$$\frac{-2y}{5} = \frac{20}{1}$$

Cross-multiply.

$$-2y = 100$$

Solve remain-
ing equation.

$$\boxed{y = -50}$$

$$6) -\frac{4}{3}y - 6 = 18$$

$$7) \frac{t-3}{6} - \frac{t-25}{5} = 4$$

$$\frac{t-3}{6} - \frac{t-25}{5} = \frac{4}{1}$$

Give any integers
a denominator of 1.

$$\frac{5}{5} \cdot \frac{t-3}{6} - \frac{6}{6} \cdot \frac{t-25}{5} = \frac{4}{1} \cdot \frac{30}{30} \quad \text{Find LCD} = 30$$

$$\frac{5(t-3)}{30} - \frac{6(t-25)}{30} = \frac{120}{30}$$

Create LCD
by multiply-
ing top +
bottom by
same factor.

$$\frac{5(t-3)}{30} - \frac{6(t-25)}{30} = \frac{120}{30}$$

$$5(t-3) - 6(t-25) = 120$$

Solve
remaining
equation.

$$5t - 15 - 6t + 150 = 120$$

$$-t + 135 = 120$$

$$-t = -15 \quad \boxed{t = 15}$$

$$8) \frac{x-6}{x} - \frac{x+4}{x} = 1$$

$$9) \frac{3}{5}(x+4) = 2(x-2)$$

$$\frac{3(x+4)}{5} = \frac{2(x-2)}{1}$$

Put parenthe-
ses up top.
Cross-
multiply.

$$3(x+4) = 10(x-2)$$

$$3x + 12 = 10x - 20$$

Solve
remaining
equation.

$$32 = 7x$$

$$\boxed{\frac{32}{7} = x}$$

$$10) \frac{8}{9}(x+6) = 2(x-3)$$

Day 4: Review

A-REI.3 Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters.

Review of the Rules

	<u>Proportions</u> $\frac{\square}{\square} = \frac{\square}{\square}$	<u>Sums and Differences</u> $\frac{\square}{\square} \pm \frac{\square}{\square} = \frac{\square}{\square}$
<p><i>If the structure matches, just apply the rule.</i></p>	<p>1) $\frac{3x}{4} = \frac{12}{20}$</p>	<p>2) $\frac{a}{2} + \frac{a}{3} + \frac{a}{4} = 26$</p>
<p><i>Always put binomials in parentheses.</i></p>	<p>3) $\frac{5}{9} = \frac{h-26}{h+21}$</p>	<p>4) $\frac{y+2}{4} - \frac{y-3}{3} = \frac{1}{2}$</p>

	<u>Proportions</u> $\frac{\square}{\square} = \frac{\square}{\square}$	<u>Sums and Differences</u> $\frac{\square}{\square} \pm \frac{\square}{\square} = \frac{\square}{\square}$
<i>Variables, negatives, and parentheses go up top.</i>	5) $\frac{3}{2}(x + 4) = 2(x - 2)$	6) $-\frac{1}{2}z + 1 = \frac{3}{2}$

Level A

1) $\frac{2}{5}r = 8$

2) $\frac{x}{3} + \frac{x}{4} = 12$

3) $\frac{x-6}{x} = \frac{x+4}{x} + 1$

Level B

Level C

4) $\frac{3y+1}{2} = \frac{2(44-y)}{5}$

5) $2x - 1 - \frac{2x-2}{2} = \frac{3x+1}{5} + \frac{x+1}{4}$

Answers to Independent Practice and Homework Exercises

p. 4 HW

- 2) $\frac{5}{2}$
- 4) -15
- 6) -25
- 8) -6
- 10) $\frac{23}{4}$
- 12) 25

p. 11 HW

- 2) 6
- 4) 10
- 6) 16
- 8) -12

p. 14 Practice

- 1) -8
- 2) $-\frac{1}{3}$
- 3) 27
- 4) 8
- 5) -3
- 7) $-\frac{1}{2}$
- 8) -12

p. 16 HW

- 2) -21
- 4) 12
- 6) -18
- 8) -10
- 10) 10.2

Day 4 Review

- 1) $\frac{4}{5}$
- 2) 24
- 3) 84.75
- 4) 12
- 5) 20
- 6) -1

Level A:

- 1) 20
- 2) $\frac{144}{7}$
- 3) -10

Level B: $\frac{171}{11}$

Level C: 3

