

# Chapter 4:

# Solving Literal

# Equations







*Solving for a variable using division:*

7) $3x = 45$	8) $3x = y$ solve for $x$
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**Quick Check for Understanding**

9) $2x + y = 9$ solve for $y$	10) $-3b - c = d$ solve for $b$	11) $P = mv$ solve for $m$
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**Application**

12) The formula  $d = rt$  relates the distance an object travels,  $d$ , to its average rate of speed  $r$ , and amount of time  $t$  that it travels.

a) Solve the formula  $d = rt$  for  $t$ .

b) How many hours would it take for a car to travel 150 miles at an average rate of 50 miles per hour?

**Homework:** Independent Practice/Homework

Solve for the variable indicated.



<p>1) <math>d = rt</math> Solve for <math>r</math></p> $\frac{d}{t} = \frac{rt}{t}$ <p>Isolate the variable by dividing both sides by <math>t</math>.</p> $\boxed{\frac{d}{t} = r}$ <p>Leave the other side.</p>	<p>2) <math>E = IR</math> Solve for <math>R</math></p>
<p>3) <math>s = r\theta</math> Solve for <math>r</math></p> $\frac{s}{\theta} = \frac{r\theta}{\theta}$ <p>Isolate the variable by dividing both sides by <math>\theta</math>.</p> $\boxed{\frac{s}{\theta} = r}$ <p>Leave the other side.</p>	<p>4) <math>C = \pi d</math> Solve for <math>d</math></p>
<p>5) <math>I = prt</math> Solve for <math>r</math></p> $\frac{I}{pt} = \frac{prt}{pt}$ <p>Isolate the variable by dividing out the others.</p> $\boxed{\frac{I}{pt} = r}$ <p>Leave the other side.</p>	<p>6) <math>PV = nRT</math> Solve for <math>R</math></p>
<p>7) <math>P = a + b</math> Solve for <math>b</math></p> $\begin{array}{r} P = a + b \\ -a \quad -a \\ \hline \end{array}$ <p>The variables are joined by addition. Isolate the variable by subtracting</p> $\boxed{P - a = b}$ <p>Leave the other side</p>	<p>8) <math>T = M + N</math> Solve for <math>N</math></p>

9)  $Ay + B = C$  Solve for  $y$

$$\begin{array}{r} Ay + B = C \\ -B \quad -B \\ \hline \end{array}$$

Isolate the term  
 $y$  is in by subtracting  
 $B$ .

$$\frac{Ay}{A} = \frac{C-B}{A}$$

Divide both sides  
by  $A$ , the coefficient.

$$\boxed{y = \frac{C-B}{A}}$$

Leave the other  
side.

10)  $y = mx + b$  Solve for  $x$

11)  $5m + n = 10$  Solve for  $m$

$$\begin{array}{r} 5m + n = 10 \\ -n \quad -n \\ \hline \end{array}$$

Isolate the term by  
subtracting  $n$ .

$$\frac{5m}{5} = \frac{10-n}{5}$$

Divide by the  
coefficient.

$$\boxed{m = \frac{10-n}{5}}$$

Leave the other  
side.

12)  $P = 2l + 2w$  Solve for  $l$

13)  $E = mc^2$  Solve for  $c^2$

$$\frac{E}{m} = \frac{mc^2}{m}$$

~~Divide~~ Isolate the  
variable by dividing  
both sides by  $m$ .

$$\boxed{\frac{E}{m} = c^2}$$

Leave the other side.

14)  $C = 2t^2$  Solve for  $t$

Self-Evaluation:

I understand this topic well.

I need more practice.

I don't understand this at all.

A-REI.3 Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters.

Warm-Up

If  $3ax + b = c$ , then  $x$  equals

1  $c - b + 3a$

2  $c + b - 3a$

3  $\frac{c - b}{3a}$

4  $\frac{b - c}{3a}$

Model Problems Solving Proportions

Solve for  $x$  in each equation.

*Linear Equations:*

*Literal Equation:*

1)  $\frac{x}{3} = 9$

2)  $\frac{x}{3} = y$

3)  $2x - 1 = \frac{3}{2}$

4)  $\frac{x}{2} = m - 6$

$$5) 10 = \frac{2}{3}(x - 4)$$

$$6) D = \frac{11}{5}(x - 15)$$

Application

7) The formula to convert Celsius to Fahrenheit is given by  $C = \frac{5}{9}(F - 32)$ .

a) Solve this formula for  $F$ .

b) The boiling point of water is  $100^{\circ}\text{F}$ . What is the Fahrenheit equivalent of this temperature?



8) Check for Understanding Solve for the given variable.

a) $d = \frac{c}{n}$ solve for $n$	b) $A = \frac{a+b}{2}$ solve for $b$	c) $F = \frac{Gm_1m_2}{r^2}$ solve for $m_1$
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- d) The formula for the mean (average)  $A$  of two numbers  $y$  and  $z$  is one-half their sum, or  $A = \frac{1}{2}(y + z)$ .  
If the average of two numbers is 7 and one of the numbers is 4, find the other number.



Homework

<p>1) <math>\frac{m}{k} = x</math> for <math>k</math></p> <p><math>\frac{m}{k} = \frac{x}{1}</math> Put right side over 1 to put into structure.</p> <p><del><math>\frac{m}{k} = \frac{x}{1}</math></del> Cross-multiply.</p> <p><math>\frac{m}{x} = \frac{kx}{x}</math> Divide by <math>x</math> on both sides.</p> <p><math>\frac{m}{x} = k</math> Leave the other side.</p>	<p>2) <math>\frac{p}{a} = y</math> for <math>a</math></p>
<p>3) <math>s = \frac{1}{2}gt^2</math> for <math>g</math></p> <p><math>\frac{s}{1} = \frac{1gt^2}{2}</math> Put left side over 1, Put anything on the side on top. (Proportion structure)</p> <p><del><math>s = \frac{1gt^2}{2}</math></del> Cross-multiply.</p> <p><math>2s = gt^2</math></p> <p><math>\frac{2s}{t^2} = \frac{gt^2}{t^2}</math> Divide both sides by <math>t^2</math></p> <p><math>\frac{2s}{t^2} = g</math> Leave the other side.</p>	<p>4) <math>V = \frac{1}{3}Ah</math> for <math>A</math></p>
<p>5) <math>s = \frac{w-10e}{m}</math> for <math>w</math></p> <p><math>\frac{s}{1} = \frac{w-10e}{m}</math> Put the equation into proportion structure.</p> <p><del><math>s = \frac{w-10e}{m}</math></del> Cross-multiply.</p> <p><math>sm = w - 10e</math>  <math>+10e \quad +10e</math></p> <p><math>sm + 10e = w</math> Isolate the <math>w</math>-term by adding <math>10e</math>. Leave the other side.</p>	<p>6) <math>\frac{x-y}{7} = t</math> for <math>x</math></p>

$$7) \frac{x}{7} - y = t \text{ for } x$$

$$\frac{x}{7} - y = t$$

$$+y \quad +y$$

$$\frac{x}{7} = t + y$$

$$\frac{x}{7} = \frac{t+y}{1}$$

~~$$\frac{x}{7} = \frac{t+y}{1}$$~~

$$x = 7(t+y)$$

Put into proportion structure: move  $y$  over and place binomial over 1.

Cross-multiply.  
Binomial goes in parentheses.

$$8) q = \frac{p}{5} - r \text{ for } p$$

$$9) q + r = \frac{p}{5} \text{ for } p$$

$$\frac{q+r}{1} = \frac{p}{5}$$

~~$$\frac{q+r}{1} = \frac{p}{5}$$~~

$$5(q+r) = p$$

Put into proportion structure.

Cross-multiply.  
Binomials go in parentheses.

$$10) a + b = \frac{c}{6} \text{ for } c$$

$$11) \frac{x-y}{7} = t \text{ for } y$$

$$\frac{x-y}{7} = \frac{t}{1}$$

~~$$\frac{x-y}{7} = \frac{t}{1}$$~~

$$x - y = 7t$$

$$\begin{array}{r} x - y = 7t \\ -x \quad -x \\ \hline \end{array}$$

$$\begin{array}{r} -y = 7t - x \\ \hline \end{array}$$

Put into proportion structure.

Cross-multiply.

Isolate  $y$  by subtracting  $x$ .

Divide by  $-1$

$$12) R = \frac{c-s}{t} \text{ for } S$$

$$y = -7t + x$$

$$\text{or } x - 7t$$

13)  $V = \frac{2}{3}(x + 2y)$  for  $x$

$$\frac{V}{1} = \frac{2(x+2y)}{3}$$

Put into proportion structure.  
Remember: parentheses go up top!

Cross-multiply.

$$3V = 2(x + 2y)$$

$$3V = 2x + 4y$$

Distribute.

$$\begin{array}{r} -4y \\ \hline 3V - 4y = 2x \end{array}$$

Isolate term w/  $x$  init.

$$\begin{array}{r} 3V - 4y = 2x \\ \hline 2 \end{array}$$

Divide by 2.

14)  $V = \frac{6}{5}(p + 2q)$  for  $p$

$$\frac{3V - 4y}{2} = x$$

Leave the other side.

Self-Evaluation:

I understand this topic well.

I need more practice.

I don't understand this at all.

A-REI.3 Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters.

Warm-Up

The formula  $P = 2(L + W)$  is the formula for the perimeter of a rectangle. Solve this formula for L. What is the length of a rectangle whose perimeter is 48 and whose width is 6?

Distribution and Reverse distribution

- 1) When there is a common factor in all terms of an expression, we can use the distributive property in reverse to write it in factored form.

<u>Simplest form</u>	<u>Factored Form</u>
a) $2L + 2W$	$2(L + W)$
b) $3a - 3b$	
c) $2lw + 2l$	
d) $fb + fa$	
e) $2\pi rh + 2\pi r^2$	

- 2) Model Problem *Using the Distributive Property in Reverse*

Solve for  $c$  in terms of  $a$  and  $b$ :  $ac + bc = ab$

3) Practice

<p>a)</p> <p>If <math>a + ar = b + r</math>, the <u>value of <math>a</math></u> in terms of <math>b</math> and <math>r</math> can be expressed as</p> $\frac{a+ar}{a} = \frac{b+r}{a} \leftarrow \text{Pull out the } a \text{ from both terms.}$ $a(1+r) = b+r$ $\frac{a(1+r)}{1+r} = \frac{b+r}{1+r}$ <div style="border: 1px solid black; padding: 5px; width: fit-content; margin: 10px auto;"><math display="block">a = \frac{b+r}{1+r}</math></div>	<p>b)</p> <p>If <math>k = am + 3mx</math>, the value of <math>m</math> in terms of <math>a</math>, <math>k</math>, and <math>x</math> can be expressed as</p>
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4) Using Rational Equations: Model Problem

The formula  $\frac{1}{a} + \frac{1}{b} = \frac{1}{f}$  relates an object's distance,  $a$ , and its image's distance,  $b$ , to the focal length of the lens,  $f$ . Solve this formula for  $f$ .

5) Practice

<p>The total resistance in a circuit is given by the formula <math>\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2}</math>. Solve this formula for <math>R_1</math>.</p>
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Cumulative Practice/Homework Day 3 Solve for the requested variable.



<p>1) <math>r = pn</math> for <math>n</math></p> <p>(Day 1) (p.5 #1)</p>	<p>2) <math>v = v_0 + at</math> for <math>v_0</math></p> <p>(Day 1) (p.5, #7)</p>
<p>3) <math>V = \frac{1}{3}Bh</math> for <math>B</math></p> <p>(Day 2) (p.10 #3)</p>	<p>4) <math>s = \frac{2x+t}{r}</math> for <math>x</math></p> <p>(Day 2) (p.10 #5)</p>
<p>5) <math>J = ma - mb</math> for <math>m</math></p> <p>(Day 3) (p.13 MP #2)</p>	<p>6) <math>D = xc + xd</math> for <math>x</math></p> <p>(Day 3) (p.13 MP #2)</p>

<p>7) Which of the following is an irrational number?</p> <p>(A) <math>-\frac{3}{5}</math>  (B) <math>7.\overline{2}</math>  (C) <math>\sqrt{8}</math>  (D) 0</p>	<p>8) Which number is in the solution set of <math>-4.5x &lt; -9</math>?</p> <p>(A) 0  (B) 1  (C) 2  (D) 3</p>
<p>9) Solve for <math>x</math>:</p> $-(1 + 7x) - 6(-7 - x) = 36$	<p>10) Solve for <math>a</math>:</p> $\frac{a}{4} - \frac{a + 5}{12} = \frac{a}{6}$

Self-Evaluation:

I understand this topic well.

I need more practice.

I don't understand this at all.



A-REI.3 Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters.

### Warm-Up

The formula for the volume of a pyramid is

$$V = \frac{1}{3} Bh. \text{ What is } h \text{ expressed in terms of } B \text{ and}$$

$V$ ?

1  $h = \frac{1}{3} VB$

2  $h = \frac{V}{3B}$

3  $h = \frac{3V}{B}$

4  $h = 3VB$

### Mini-Lesson: Using Square Roots

To solve for a squared variable, take its square root.

<p><i>Linear Equation:</i>      <math>64 = 16x^2</math></p>	<p><i>Literal Equation:</i>      <math>A = \pi r^2</math> solve for <math>r</math></p>
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### Check for Understanding      Solve for the indicated variable.

<p>1) The formula for kinetic energy is <math>K = \frac{1}{2}mv^2</math>. Write an expression for <math>v</math> in terms of <math>K</math> and <math>m</math>.</p>	<p>2) The gravitational force <math>F</math> that two planetary bodies exert on one another is given by <math>F = \frac{Gm_1m_2}{r^2}</math>. Solve this formula for <math>r</math>.</p>
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**Cumulative Practice/Homework – Day 4** Solve for the value of the indicated variable.



1) $V = lwh$ Solve for $h$ (DAY 1) (p. 5 #5)	2) $q - 3r = 2$ Solve for $r$ (DAY 1) (p. 6 #11)
3) Solve $R = \frac{l+3w}{2}$ for $w$ (DAY 2) (p. 10 #5)	4) Rewrite $K = \frac{3}{2}kT$ solved for $T$ in terms of $k$ and $T$ . (DAY 2) (p. 10 #3)
5) In $a + ax = b$ , what is $a$ in terms of $x$ and $b$ ? (DAY 3) (p. 14 #3a)	6) $A = 2xy + 2y$ Solve for $y$ (DAY 3) (p. 14 #3a)

<p>7) <math>a_c = \frac{v^2}{r}</math> Solve for <math>v</math> (DAY 4)</p> <p>(Hint: Get rid of the fraction first!)</p>	<p>8) <math>s = \frac{1}{2}at^2</math> solve for <math>t</math> (DAY 4)</p> <p>(Hint: Get rid of the fraction first!)</p>
<p>9) The formula for the volume of a cone is <math>V = \frac{1}{3}\pi r^2 h</math>. The radius, <math>r</math>, of the cone may be expressed as</p> <p>1 <math>\sqrt{\frac{3V}{\pi h}}</math></p> <p>2 <math>\sqrt{\frac{V}{3\pi h}}</math></p> <p>3 <math>3\sqrt{\frac{V}{\pi h}}</math></p> <p>4 <math>\frac{1}{3}\sqrt{\frac{V}{\pi h}}</math></p>	<p>10) The distance a free falling object has traveled can be modeled by the equation <math>d = \frac{1}{2}at^2</math>, where <math>a</math> is acceleration due to gravity and <math>t</math> is the amount of time the object has fallen. What is <math>t</math> in terms of <math>a</math> and <math>d</math>?</p> <p>1 <math>t = \sqrt{\frac{da}{2}}</math></p> <p>2 <math>t = \sqrt{\frac{2d}{a}}</math></p> <p>3 <math>t = \left(\frac{da}{d}\right)^2</math></p> <p>4 <math>t = \left(\frac{2d}{a}\right)^2</math></p>

Self-Evaluation

I understand this topic well.

I need more practice.

I don't understand this at all.

Which topic(s) do you feel that you understand well? \_\_\_\_\_

\_\_\_\_\_

Which topic(s) do you feel that you need more practice or help on? \_\_\_\_\_

\_\_\_\_\_

A-REI.3 Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters.

Look for **STRUCTURE** in equations:

<i>One-Step Equations</i>	
1) $I = prt$ Solve for $r$ .	2) $T = M - N$ Solve for $M$ .
<i>Two-Step Equations</i>	
3) $5t - 2r = 25$ Solve for $t$ .	4) $vt - 16t^2$ Solve for $v$ .

**Proportions**

5)  $F = \frac{lt}{d}$  Solve for  $l$ .

6)  $P = \frac{144p}{y}$  Solve for  $p$ .

7)  $A = \frac{1}{2}h(a + b)$  Solve for  $a$ .

8)  $m = \frac{y_2 - y_1}{x_2 - x_1}$  Solve for  $y_2$

**Reverse Distribution**

9)  $S = R - rR$  Solve for  $R$ .

10)  $ax = bx + c$  Solve for  $x$ .

**Square Roots**

11)  $K = \frac{1}{2}mv^2$  Solve for  $v$ .

12)  $V = \frac{1}{3}\pi r^2 h$  Solve for  $r$ .

- 13) **Applications.** The surface area of a sphere is given by the formula  $S = 4\pi r^2$ . Solve this formula for  $r$ .  
What is the radius of a sphere whose surface area is  $201 \text{ cm}^2$ ?

## Homework Answers

<p><b>Day 1 pages 5-6</b></p> <p>2) <math>R = \frac{E}{I}</math></p> <p>4) <math>d = \frac{c}{\pi}</math></p> <p>6) <math>R = \frac{PV}{nT}</math></p> <p>7) <math>N = T - M</math></p> <p>10) <math>x = \frac{y - b}{m}</math></p> <p>12) <math>l = \frac{P - 2w}{2}</math></p> <p>14) <math>t = \sqrt{\frac{c}{2}}</math></p>	<p><b>Day 2 pages 10-12</b></p> <p>2) <math>a = \frac{p}{y}</math></p> <p>4) <math>A = \frac{2v}{h}</math></p> <p>6) <math>x = 7t + y</math></p> <p>8) <math>p = 5q + 5r</math></p> <p>10) <math>c = 6a + 6b</math></p> <p>12) <math>s = c - Rt</math></p> <p>14) <math>p = \frac{5V - 12q}{6}</math></p>	<p><b>Day 3</b></p> <p>2) <math>v_0 = v - at</math></p> <p>4) <math>x = \frac{sr - t}{2}</math></p> <p>6) <math>x = \frac{D}{c + d}</math></p> <p>7) <math>C</math></p> <p>8) <math>D</math></p> <p>9) <math>5</math></p> <p>10) <i>No solution</i></p>	<p><b>Day 4</b></p> <p>1) <math>h = \frac{V}{lw}</math></p> <p>2) <math>r = \frac{q - 2}{3}</math></p> <p>3) <math>w = \frac{2r - l}{3}</math></p> <p>4) <math>T = \frac{2K}{3k}</math></p> <p>5) <math>a = \frac{b}{1 + x}</math></p> <p>6) <math>y = \frac{A}{2x + 2}</math></p> <p>7) <math>v = \sqrt{a_c r}</math></p> <p>8) <math>t = \sqrt{\frac{2s}{a}}</math></p> <p>9) (1)</p> <p>10) (2)</p>
			<p><b>Day 5</b></p> <p>1) <math>r = \frac{l}{pt}</math></p> <p>2) <math>T = P + N</math></p> <p>3) <math>t = \frac{25 + 2r}{5}</math></p> <p>4) <math>v = 16t</math></p> <p>5) <math>l = \frac{Fd}{t}</math></p> <p>6) <math>p = \frac{Py}{144}</math></p> <p>7) <math>a = \frac{2A}{h} - b</math></p> <p>8) <math>y_2 = m(x_2 - x_1) + y_1</math></p> <p>9) <math>R = \frac{s}{1 - r}</math></p> <p>10) <math>x = \frac{c}{a - b}</math></p> <p>11) <math>v = \sqrt{\frac{2k}{m}}</math></p> <p>12) <math>r = \sqrt{\frac{3V}{\pi h}}</math></p> <p>13) <math>v = \sqrt{\frac{s}{4\pi}}</math>; about 4 cm.</p>

